Breaking Tradition in the Mathematics Classroom: Making Mathematics Real, Relevant, and Personal (Project)

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Breaking Tradition in the Mathematics Classroom: Making Mathematics Real, Relevant, and Personal
by
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July 2013

Master’s Project
Submitted to the College of Education
At Grand Valley State University
In partial fulfillment of the
Degree of Master of Education
Abstract

Traditional mathematics education is failing students as many are unable to comprehend and master advanced and abstract concepts (Rakes, Valentine, McGatha, & Ronau, 2010). Struggling students, including those at Knapp Charter Academy, face a disadvantage in subsequent mathematics classes and limited opportunities in higher education and beyond. The importance of mathematics, as a gatekeeper to opportunity, makes increasing students’ success in mathematics a necessity. The mathematics failure rates coupled with the proven ineffectiveness of traditional mathematics teaching and learning additionally establish the need for meaningful reform. This project analyzes educational theory and research to determine the effect of certain reform constructs on students’ mathematical comprehension and achievement. A supplementary algebra curriculum was created based on research to improve the plight of struggling mathematics students at Knapp Charter Academy. Authentic real world connections, differentiation for meeting student needs, active learning, and an emphasis on conceptual understanding and flexible problem solving are key features of the supplementary algebra curriculum because research found these constructs linked to increased student achievement in mathematics.
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Abstract

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Chapter One: Introduction

Problem Statement

Students often struggle to comprehend mathematics taught in traditional context. The nationwide failure rate for Algebra I is more than sixty percent of students (Gates, 2008). Referring to algebra failure, Rakes, Valentine, McGatha, and Ronau (2010) asserted that “improving the teaching and learning of algebra is critical to improving these long-term trends” (p. 372). Newton, Star, and Lynch (2010) correlated student mathematical struggles with the inability to solve problems flexibly, which may result from traditional modes of instruction that emphasize a singular method of solving. Palm (2008) identified a disconnect between student mathematical learning and application of real world knowledge, which is exacerbated by the lack of authenticity in traditional mathematics problems. Nontraditional teaching modes and enriched presentation of curriculum may be a solution to increasing students’ comprehension of mathematics, especially in algebra and other abstract higher level mathematics courses.

Rationale

Student comprehension of mathematics is important because understanding previous concepts helps students successfully learn and master new concepts. A strong conceptual foundation is necessary to efficiently learn novel, and especially abstract, mathematical concepts (Ketterlin-Geller & Chard, 2011). In regard to abstract courses like algebra, Greer (2008) stated that struggling to comprehend leads to many more years of frustration in higher mathematics courses. Sciarra (2010)
proved that success begets success with data showing previous achievement in mathematics has a strong statistical correlation to future achievement in mathematics. Sciarra also found past achievement scores in mathematics to be especially predictive of what future mathematics courses students would successfully complete.

Students’ struggles to succeed in mathematics can significantly impact the outcome of their K-12 education. Across the country, many states and individual school districts require all students to pass Algebra I to earn a high school diploma (Ketterlin-Geller & Chard, 2011). The state of Michigan currently requires successful completion of Algebra I, Geometry, Algebra II, and an additional mathematics course to qualify for high school graduation (Geno, 2010). Morgatto (2008) correlated the high school algebra requirement to the goal for student equality in college admission. However, the goal of student equality assumes student success in required mathematics courses. The desired equality is jeopardized when students do not successfully learn mathematics due to traditional teaching and learning or other factors. Morgatto explained that struggling students are at risk to “become frustrated and perhaps withdraw from high school because they cannot pass an algebra class” (p. 217).

Success in mathematics, or a lack thereof, can also affect students’ futures beyond high school. Spielhagen (2010) denoted the importance of algebra by describing it as “the gatekeeper course to advanced study in both mathematics and science” (p. 214). Research has proven a connection between successful completion of advanced mathematics courses in high school and completion of a college degree
Greer (2008) linked significant detrimental effects in education and life to struggling with mathematics because of the position of “algebra at the entrance to a national educational and economic gated community” (p. 425). Competency in mathematical calculation and algebraic reasoning is considered vital for admission to higher education, achievement in mathematics and science classes, success in some industries, and the ability to solve everyday problems (Ketterlin-Geller & Chard, 2011).

**Background of Study**

Student difficulty in the mastery of mathematics is a longstanding and multi-faceted problem. Failure in mathematics has generated significant attention for over thirty years because of the implications of mathematics failure on individual students, the institution of education, and the nation’s economy and industries (Buckley, 2010). According to Buckley, the national concern for struggling students is warranted and urgent “if it is assumed that mathematical understanding is imperative not only for economic but also for democratic participation” (p. 52). Wheland, Konet, and Butler (2003) agreed with their assertion that “in an increasingly global economy the poor mathematics performance of American students has become a topic of national concern” (p. 18).

The concern regarding students’ mathematics performance and mastery is widespread just as it is longstanding. Rylands and Coady (2008) found that universities in several countries including Australia, the United Kingdom, and Ireland report that many students have not mastered enough content in grade school
mathematics to be prepared for higher education mathematics. In South Africa, current ability deficits in certain professions are attributed to high levels of student failure in mathematics (Mji & Mwambakana, 2008).

The origin of students’ struggles in mathematics is multi-faceted. Lack of prior knowledge and preparedness is one key reason many students find it hard to be successful in mathematics. A fundamental understanding of the basics of mathematics is essential to successfully learn new and more advanced content (Ketterlin-Geller & Chard, 2011). Wheland et al. (2003) explained that mathematics generally builds upon prior knowledge so “performance in intermediate algebra does have an impact on performance in subsequent math classes” (p. 22). Sciarra (2010) found that a lack of comprehension and mastery is a significant problem as students advance to new mathematics courses because research shows academics, specifically mathematics and general achievement, are a strong predictor of future success in mathematics.

Evidence suggests another main contributor to student failure is traditional mathematics instruction that does not meet student needs (Rakes et al., 2010). Rakes et al. (2010) stated that “traditional instructional practices have led students to view mathematics as a set of disjointed algorithms” (p. 378). Traditional methods of teaching and learning mathematics are described in Smith and Geller (2004) as the teacher simply demonstrating the procedure for solving a problem and then students practicing the procedure. With traditional methods, there is a lack of attention paid to conceptual understanding, relevant and meaningful problems, student differences,
learning tools (Smith & Geller, 2004). As a result, students with disabilities and other struggling students “may not be able to meet the standards necessary because teachers either lack or do not implement effective strategies to foster understanding of basic concepts of algebra” (Smith & Geller, 2004, p. 23).

Exacerbating the problem of traditional teaching and learning is the very nature of certain courses in mathematics, which intensify the problem. Rakes et al. (2010) explained that the traditional modes of teaching for mastery of arithmetic and concrete mathematical concepts are less effective in algebra because it includes abstract reasoning, new mathematical language, and novel structure. Rakes et al. described algebra as full of many new foundational understandings and stated that “teaching methods that focus on skill or procedural levels on cognitive demand fail to address these foundational understandings and therefore fall short of providing students the tools necessary to find their way once they waiver from a scripted path” (p. 375).

Regardless of the cause, the problem with the lack of student comprehension is that understanding mathematics, algebra in particular, could prove vitally important for students’ futures. “There is agreement that algebra is critically important to the success of students throughout high school and college” (Rakes et al., 2010, p. 373). Algebra is a way of thinking and generalizing relationships that is essential in mathematics and also transcends mathematical situations (Ketterlin-Geller & Chard, 2011). Researchers from the National Council of Teachers of Mathematics explained that “knowing algebra opens doors and expands opportunities, instilling a broad range
of mathematical ideas that are useful in many professions and careers” (NCTM, 2008, para. 1). Additionally, research has proven that taking algebra increases achievement in mathematics for all students regardless of ability level (Gamoran & Hannigan, 2000).

With the importance of mathematics in the forefront of the minds of legislators, Michigan passed 2006 education legislation to mandate that “students in this state are required to take 4 credits of math to include Algebra 1, Geometry, Algebra 2 and a fourth year math-related class” (Geno, 2010, para. 2). Michigan students, and many other students nationwide, are required to take algebra, which makes the immediate question how best to help the students who struggle to comprehend algebra and risk failure as a result. Non-traditional teaching methods may be one solution.

Gamoran and Hannigan (2000) supported the use of nontraditional teaching methods by suggesting that students, especially those more apt to have difficulty in mathematics, need “more diverse and thought-provoking instructional methods than are typically offered in high school algebra” (p. 250). Additionally, “teachers need to be able to address the diverse abilities of our student population, while making it relevant and holding the kids before them to high standards” (Geno, 2010, para. 17). Meta-analysis results found that altering instruction from traditional methods to emphasize conceptual understanding and connections within the content yielded a positive and statistically significant increase in student achievement (Rakes et al., 2010). Therefore, using non-traditional methods of instruction combined with
emphasis on conceptual understanding is one way to work toward increasing student comprehension in mathematics.

Statement of Purpose

The purpose of this project is to increase student comprehension by creating a supplementary algebra curriculum. This curriculum will include original lessons, activities, projects, worksheets, and assessments that are designed in a nontraditional format. The nontraditional components will have emphasis on conceptual understanding, meaningful real world connections, and differentiated learning. This project will not be comprehensive in terms of content coverage, so it will be designed to use in addition to a textbook or standard curriculum that presents the entire subject of algebra in individual chapters and lessons.

Students who struggle to learn algebra through traditional methods of instruction will be exposed to non-traditional modes of instruction and learning through the use of this curriculum supplement. Teachers have no control over the level of readiness in which student enter their mathematics courses, therefore the intent of this project is to modify the methods of teaching and learning that fall within the teacher’s control. The implementation of this curriculum is intended to increase student achievement in algebra by incorporating methods of teaching and learning that are different than what students customarily experience in a secondary mathematics classroom. The aim of this project is to improve the plight of students who would struggle to comprehend abstract mathematical concepts taught through traditional modes.
Objectives

- The learner will be given opportunities to increase his/her comprehension of algebraic concepts and reasoning through nontraditional lessons and activities.
- The learner will experience relevant mathematics problems that are meaningful and demonstrate connections within mathematics and beyond.
- The learner will have opportunities to learn and practice mathematical concepts tailored to meet his/her interests at his/her current level of mastery of algebraic content through differentiated instruction and practice.
- The teacher will have lesson plans and activities to implement that depart from traditional methods to include other proven methods of increasing student comprehension.

Definitions of Terms

Abstract mathematics- Abstract mathematics are mathematical concepts that are not able to be easily modeled with concrete manipulatives. The concepts are intangible and require a more theoretical, versus practical, understanding.

Algebra- “Algebra is an abstract representation of mathematical relationships that goes beyond computation of concrete numbers to focus on common properties, statements or equations” (Ketterlin-Geller & Chard, 2011, p. 66).

Authenticity- Authenticity in regards to real world connections in mathematics requires that the problem “must represent some task situation in real life, and
important aspects of that situation must be simulated to some reasonable degree” (Palm, 2008, p. 40).

**Differentiated instruction**- Differentiation is a form of teaching and learning that includes varied content, learning process, or assessment according to students’ needs in the areas of academic level, learning preference, or interest (Chamberlin, 2011). **Nontraditional teaching methods**- Nontraditional teaching methods are instructional strategies that focus on conceptual understanding, use flexible problem solving for procedural understanding, and provide multiple representations of content (Rakes et al., 2010).

**Traditional mathematics**- “Traditional view of mathematics as procedural and sequential, as a set of facts and algorithms that students needed to master with computational fluency” (Buckley, 2010, p. 72).

**Traditional teaching methods**- Traditional teaching features direct instruction on procedural skills and little instruction on conceptual understanding and connections (Rakes et al., 2010).

**Scope of the Project**

This project will provide specific and detailed lesson ideas, activities, projects, and assessments that differ in context and format from a conventional approach of providing direct instruction centered on procedural fluency to passive student learners. The supplementary curriculum will add depth to textbooks and standard curriculum by providing enriching materials designed to focus on conceptual understanding and making abstract concepts more relevant, meaningful,
differentiated, straightforward, and connected. One philosophy included in this project, differentiated instruction, will allow students to experience novel algebraic concepts in ways that are tailored to their needs and interests. As a result of the interplay of these elements, student achievement in algebra should increase regardless of the level of knowledge and prior understanding students bring to the course.

The level of readiness in which students enter the algebra course is beyond the control of this project. Consequently, some students may enter into an algebra course without basic skills necessary for a foundational understanding of algebra or the motivation or work ethic to pursue mastery. While this project aims to increase comprehension, it cannot guarantee that all students who experience the supplementary curriculum will pass algebra. Nor can it be guaranteed that all students will willingly participate and engage in the supplementary lessons and activities. This project will not replace algebra textbooks or traditional curriculums in the classroom; the components in this supplementary curriculum were not designed to meet all state and national standards required for algebra courses or specific grade levels. It is also not intended to replace direct instruction techniques in the classroom. Instead it aims to change the nature of direct instruction by providing interactive lesson ideas and ensuing activities, projects, and assessments that will make algebra comprehension easier for struggling students by way of addressing their needs and interests.
Chapter Two: Literature Review

Introduction

Students’ difficulty with mathematical comprehension has plagued this country for decades, and yet traditional methods of teaching and learning continue to persist in the mathematics classroom. According to Stemhagen and Smith (2008), much of American education modernized in policy and practice a century ago under the guidance of visionary reformers and philosophers, such as John Dewey, while mathematics education has largely withstood reform. A mathematics system that results in extensive student failure is a significant problem as mathematics is recognized as a gatekeeper to individual educational success and future achievement (Greer, 2008; Spielhagen, 2010). Student success in mathematics is also vital “if it is assumed that mathematical understanding is imperative not only for economic but also for democratic participation” (Buckley, 2010, p. 52).

To address the problem of students’ mathematical struggles, this literature review examines educational theories and research focused on successful teaching and learning. While philosophies on teaching and learning abound, the educational reform theories of the late philosopher John Dewey are featured due to their relevance and applicability to the problem of student failure in traditional mathematics classrooms. Based on Dewey’s philosophies, several ideas within the scope of teaching and learning are identified as having the potential to increase student comprehension of mathematics. This literature review critically examines current research from the field on the topics of authentic learning, differentiation, conceptual
understanding, flexible problem solving, and active learning and reports on their effectiveness in impacting student learning.

**Theory/Rationale**

There are undoubtedly many interwoven factors that affect whether a child succeeds or fails in comprehending all subject matter, including mathematics. Dewey, a revolutionary American philosopher, was concerned not only with the learning and success of the child, but also with the health of the education system and its ability to promote democracy (Platz & Arellano, 2011). Dewey’s philosophies on education were both narrow and broad in spectrum, but Dewey’s ideas could be summarized into an overarching educational theory that addressed many aspects of education and advocated for the simultaneous benefit of the child and society. Dewey’s theory of education was that learning must support democracy, be meaningful to students, and put students in an active role (Wyett, 1998).

In reference to making learning meaningful to students, Wyett (1998) stated that Dewey believed that content was only powerful to students when delivered or experienced through purposeful pursuits. According to Wyett, Dewey acknowledged that learning curricular facts is often necessary in academic studies; however those facts must be attached to meaning, connected to the child’s life, or used for reasoning and making judgments to become tools of real learning and student growth. Dewey felt that teachers must purposefully structure learning to allow for exploration, questioning, and making connections between subject matter and real life (Platz &
Arellano, 2011). In regard to how the educational system of his day failed students by presenting curriculum that was detached from reality, Dewey (2002) wrote:

A continual angle of refraction and distortion is introduced in viewing existing studies, through the fact that they are looked at from an artificial standpoint. Even those studies which are popularly regarded as preparing distinctively for life rather than for college cannot get their full meaning, cannot be judged correctly, until the life for which they are said to be a preparation receives a fuller and more balanced representation in the school. While, on the other hand, the more scholastic studies, if I may use the expression, cannot relate themselves properly so long as the branches which give them their ultimate raison d’etre and sphere of application in the whole of life are non-existent in the curriculum. (Dewey, 2002, p. 115)

In addition to meaningful curriculum, Platz and Arellano (2011) described active student learning as another main component of Dewey’s educational theory. Dewey believed that students could not truly learn when classrooms required them to passively absorb information in a sterile environment removed from the engagement and involvement that naturally comes with living (Voparil, 2008). Referencing ineffective traditional classrooms with regard to Dewey’s theories, Voparil (2008) stated that “students are confronted with alien information, occupying a wholly external relation to themselves and their interests, often presented in abstract form with little hope of engaging the emotions” (p. 42).
Despite the obvious need for educational reform a century ago, Dewey was careful to never place the blame on students for the failings of the educational system (Voparil, 2008). Platz and Arellano (2011) described Dewey’s opinion of students as naturally curious and eager to learn if learning applied and connected with their lives. Instead of attributing poor behavior to classroom management or imperfect student character, “Dewey regarded the all too common student displays of apathy and inattention not as illnesses to be cured with medications, but as direct consequences of a flawed context of learning” (Voparil, 2008, p. 38).

To resolve the flaws in education, Dewey (2001) believed that reform must reach beyond the classroom and to the system itself. Instead of education focusing on memorization of facts and learning a specific set of information, Voparil (2008) explained that Dewey favored individual progress and authentic understanding as the goals of education. Dewey recognized that curriculum is dictated from outside the classroom and explained that in this system average teachers will devote their energy to the execution of the set curriculum instead of devoting it to making connections between the content and the life of the child. Dewey (2001) lamented this fact since “it is through the teacher that the value even of what is contained in the text-book is brought home to the child” (p. 397). Dewey also felt ignoring natural and potential connections and instead teaching curriculum in isolation skews reality and goes against the body of education being unified and forming a collective whole.

Dewey (2002) ultimately advocated for what he felt was in the best interests of students and for the educational system as a whole. Dewey (2002) believed in
addition to making sure that education supported the ideals of democracy, education
must also “discover what the nature of the individual is and what his best growth calls
for” (p. 118). Dewey believed in and pushed for reform to make education a system
where content matter was connected to real life and other subjects, created
meaningful growth and understanding in each individual child, and reached the
democratic goal of developing each child for participation in society.

Dewey’s theories on education extended specifically to mathematics as well.
Referring to the need for meaning in learning, Dewey (2002) wrote “for a certain type
of mind algebra and geometry are their own justification. But to another type of mind
these studies are relatively dead and meaningless until surrounded with a context of
obvious bearings” (p. 116). Stemhagen and Smith (2008) stated that to Dewey the
value in mathematics education stemmed from functionality of the content and
developing the ability to solve authentic problems. Mathematics education, according
to Stemhagen and Smith per Dewey’s ideals, should not be about learning arbitrary
information for the sake of knowledge, but instead about acquiring meaningful
knowledge that students can relate to and deem relevant for their lives. The authors
further stated that mathematics education should provide tools for inquiry and
genuine problem solving that positively impacts student life and a democratic society.
Stemhagen and Smith concluded that although Dewey’s century old reform ideas
never properly reformed the subject area of mathematics, that the need for reform is
still present due to the extensive arbitrary and disconnected presentation of
mathematical concepts in mathematics classrooms.
Research/Evaluation

Students often struggle to comprehend mathematics taught in traditional context, and there are varying ideas about the possible causes and solutions to the problem of student failure in mathematics. According to Stemhagen and Smith (2008), much of the problem in mathematics education is that mathematics has not sufficiently updated the traditional ways of teaching and learning in the midst of reform efforts by education experts and philosophers, such as John Dewey. In recent years, research on concepts within Dewey’s theories of teaching and learning has been conducted and analyzed. This research provides empirical evidence for the necessity of change in mathematics education in order to improve teaching and learning and address the problem of extensive student failure in mathematics.

Real World Connections in Mathematics

One of the ways that the mathematics classroom is still considered traditional is in presentation of the content as absolute stand-alone truths, whose justification for learning is for the sake of knowing mathematics (Stemhagen & Smith, 2008). However, there has recently been a paradigm shift towards attitudes in favor of linking mathematics education with real world connections. Confirming the mindset shift coupled with a lack of actual classroom change, Gainsburg (2008) wrote “the mathematics-education community stresses the importance of real-world connections in teaching. The extant literature suggests that in actual classrooms this practice is infrequent and cursory” (p. 199). Palm (2008), citing previous research, claimed that in traditional classrooms students do not believe that their answers must make sense
in a real world context and instead look for a single solution that can be gleaned from straightforward procedural solving. Cooper and Harries (2002) agreed stating that both mathematics teachers and curriculum, due to standardized test pressure or other factors, do not encourage thinking realistically or going beyond mechanical computation.

In original field research, Gainsburg (2008) surveyed 62 secondary mathematics teachers and conducted follow up observations of some to determine the extent of real world connections in the mathematics classroom. The researcher found that 36 percent of teachers made what they considered real world connections on a monthly basis or less frequently. While the remaining teachers claimed to make connections on a daily or weekly basis, Gainsburg’s research showed that the majority of the connections teachers counted consisted simply of a realistic word problem, teacher reference, or example within a lesson. In summary, Gainsburg (2008) stated “secondary mathematics teachers count a wide range of practices as real-world connections. Teachers make connections frequently, but most are brief and many appear to require no action or thinking on the students’ part” (p. 215).

Gainsburg’s (2008) research explored other aspects of real world connections in mathematics as well. The researcher found that approximately 84 percent of the responding teachers indicated that the real world connections they use in their classrooms they create themselves, while only nine percent cited the textbook or curriculum as a source. Additionally, Gainsburg explained that the observed teachers lamented that their textbooks and curriculums were simply not sufficient in providing
realistic connections for their content. When questioned on the purpose of using real world connections, the researcher stated that more than 75 percent of responding teachers indicated that motivating students and increasing comprehension of content were good reasons. Gainsburg reported that just over 50 percent indicated they use connections to link mathematics with real life in a meaningful way that helps students understand a deeper purpose of the content in relationship to the world. Conversely, when asked about the primary reasons they do not use real world connections in their mathematics classrooms, Gainsburg found that the primary reasons were that teachers felt they do not have enough time and adequate resources.

Several conclusions can be drawn from Gainsburg’s research. Gainsburg (2008) states that “research points to the urgency of connecting school mathematics to the outside world. Yet many students fail to see the utility of mathematics” (p. 200). The researcher found that although many teachers are using real world connections in their mathematics classes, these connections are often superficial and lack authenticity. According to Gainsburg, the real world connections are used for grabbing attention or generating interest instead of making meaningful ties between mathematics and real life. Gainsburg (2008) summarized that “these findings are consistent with the conclusion that teachers’ main goal is to impart mathematical concepts and skills, and that developing students’ ability and disposition to recognize applications and solve real problems is lower priority” (p. 215).

**Authenticity of Real World Mathematical Connections**
Like Gainsburg, other researchers have also been interested in studying the prevalence and authenticity of the real world connections that are presented in mathematics classrooms. Before beginning their research, Cooper and Harries (2002) noted that “previous research has shown that children, in the context of a mathematics test, typically do not pay attention to realistic considerations when constructing their response to short word problems which embed some arithmetical operation in a textually represented everyday context” (p. 2). Cooper and Harries conducted their own research with a single authentic word problem to see if students were using their real world knowledge when forming answers. The researchers used a problem about an elevator at rush hour. The technical answer to the mathematics problem posed by the researchers was a decimal answer, so the researchers were examining whether students knew how to create a realistic answer by rounding this decimal.

Cooper and Harries (2002) had a list of other possible answer options to present to students who had used realistic thinking when given the problem originally. The researchers would ask the students to consider whether the new options could be real solutions to this problem. It is important to note that the researchers designed the new options to reflect situations like the elevators being overcrowded or not full to capacity so students would have to use real world knowledge to identify them as realistic possibilities.

The students who were the subjects of Cooper and Harries (2002) research were eleven and twelve year old children from England with varied backgrounds and skills. The researchers found that under 50 percent of the students succeeded in
providing a realistic answer to the elevator problem. Fifty-five students, of the total 121 participants, who gave the realistic rounded answer were also given new options to consider by Cooper and Harries. The researchers noted that approximately 38 percent of those students recognized that it was possible for the elevator to take additional trips, while only around 22 percent thought it was possible for the elevator to take fewer than expected trips because it would break the maximum capacity rule.

Cooper and Harries (2002) drew several conclusions from their research regarding traditional classrooms, standardized tests, and lack of current emphasis on real world connections in mathematics classrooms. In regard to traditional classrooms, the researchers found that students are not adequately encouraged to think realistically and many students become solely reliant on procedural solving to get one answer to a given mathematics problem. On the topic of standardized tests, Cooper and Harries (2002) criticized that “high stakes assessment in mathematics has continued to employ short stereotypical test items which set mathematical operations within restricted and artificial contexts” (p. 1). In conclusion to the research and the implications moving forward, Cooper and Harries (2002) stated that “a serious commitment to encouraging children to contribute to the solution of problems drawn from everyday life will also need to increase the permeability of the boundary between children’s everyday knowledge and experience and their more purely mathematical knowledge” (p. 21).

Palm (2008) also conducted research on authenticity of real world connections in mathematics. Palm’s research intended to determine whether the authenticity of
realistic mathematics problems affects the level of realism in the answers students provide. Before beginning original research, Palm studied previous research on real world mathematic problems and examined the effects of adding realistic elements, such as interaction with real professionals, concrete manipulatives, or a real life setting outside of the classroom. Palm (2008) concluded that “these studies show that substantial changes in the task solving conditions may influence students’ modeling behavior” (p. 38).

Palm’s (2008) original research to test the effect of problem authenticity consisted of 161 fifth graders from classes chosen randomly in a town in Sweden. The researcher created two forms of a test, one with authentic word problems and the other with traditional word problems, that both contained identical numerical content and required identical mathematical operations to solve. The results, according to Palm, showed that 51 percent of students given the authentic version of the test answered the questions realistically compared to only 33 percent of students who had the traditional word problem version. The researcher concluded that the authenticity of problem increases realistic responses was confirmed and proven statistically significant according to the Mann-Whitney test.

Palm (2008) also noted that while task authenticity increased realistic responses, almost half of the students tested with the authentic word problems still did not provide realistic answers. The researcher attributed this lack of connection between mathematical concepts and reality to the format of traditional mathematics classrooms where realistic thinking is not fostered because it is not required to
compute answers to conventional mathematics problems. According to Palm, another factor encouraging students to separate mathematics from reality is their inherent beliefs about mathematics. Palm (2008) stated:

> These beliefs do not include the requirement that school mathematics and real life outside school must be consistent. On the contrary, they do include the ideas that all tasks have a solution, that the solution is attainable for the students, and that the answer is a single number. (Palm, 2008, p. 52)

It is evident, based on Palm’s research, results, and analysis that two main ideas emerge for advancing mathematics in regard to making real world connections. Palm (2008) explained that while increasing problem authenticity does increase student realistic responses, there is a lack of task authenticity in traditional mathematics classrooms. Additionally, the researcher stated that realistic thinking in mathematics is not being developed in students due to an emphasis on procedural problem solving involving mechanical computation of numbers with little regard to compliance with reality. Palm (2008) concluded by advising that “given the students’ focus on superficial solution strategies, shown in this paper and elsewhere, and the many descriptions of learning environments consisting mainly of stereotyped tasks an improvement of this purposeful choice of learning tasks would be of significant importance” (p. 56).

**Developing Students’ Conceptual Understanding of Mathematics**

Real world connections, and the authenticity of those connections, are just one component of teaching and learning that has been researched for its resulting impact
on student achievement. Just as it is believed that real world connections support student learning, other researchers tie increased student achievement to an increased level of conceptual understanding. Kazemi and Stipek (2008) explained that to create meaningful and necessary reform in the classroom, mathematics teachers must go beyond making superficial changes to teaching and learning and find ways for students to develop deep conceptual understanding of mathematics concepts. Based on this belief, Kazemi and Stipek conducted original research focused on examining classroom practices that could lead to high conceptual understanding of mathematics in students.

The research conducted by Kazemi and Stipek (2008) centered on examining four upper elementary classrooms, in part of a larger research project, chosen specifically as proven examples of classrooms with high or relatively low conceptual understanding in students. The research of Kazemi and Stipek was mainly qualitative in nature, but it was found that there was a quantitative and statistically significant positive correlation between rigorous engagement in mathematical thinking and increased student conceptual understanding of the mathematics concepts. The primary qualitative differences found by the researchers between classrooms with high and low conceptual understanding involved justification, multiple strategies, use of errors, and collaborative learning.

Kazemi and Stipek (2008) found that in classrooms with high levels of conceptual understanding students were always asked to explain, justify, and describe mathematical thinking and reasoning behind solving processes instead of only
describe solving procedures. The researchers stated that classrooms with low conceptual understanding tended to feature teacher corrected errors and collaborative groups with no individual accountability whereas highly conceptual classrooms encouraged students to learn from errors by rigorously examining them and every member of each collaborative group was accountable for the mathematical thinking in the group. In conclusion of their research, Kazemi and Stipek indicated that to increase students’ conceptual understanding teachers must focus more on concepts than procedures, make student error a learning springboard, and expect all students to be doing mathematical thinking inside collaborative teams.

Without regard to classroom practices that may factor into student learning, Khairani and Sahari Nordin (2011) researched levels of students’ mathematical competencies and relationships between them. The researchers assessed competency in conceptual understanding, procedural fluency, and strategic competence in nearly 600 fourteen-year-old students with the Mathematics Proficiency Test they developed. The researchers analyzed results with the Rasch Model calibration and conducted factor analysis. Khairani and Sahari Nordin found that students’ proficiency was highest in conceptual understanding and lowest in procedural fluency. The researchers also found the most variability occurred within the range of students’ conceptual understanding whereas procedural fluency had the least dispersion. Khairani and Sahari Nordin stated that both of these results are in line with the research confirmed link between level of mathematical proficiency and mastery of each strand studied.
Within a synopsis of their research and results, Khairani and Sahari Nordin (2011) stated their belief that the traditional emphasis of rote procedural fluency in mathematics classrooms must be reformed for modern education. The researchers also stated, in analysis of their data, that students were mainly successful on straightforward problems but showed weakness when asked to make connections, approach novel problems, or use prior knowledge. Khairani and Sahari Nordin (2011) wrote that “today, students must understand the Mathematics that they are learning; that is, Mathematics involves various components or strands that are interdependent and interwoven” (p. 33). In reference to using their data to inform future mathematics teaching and learning, Khairani and Sahari Nordin (2011) wrote:

Regarding mathematics pedagogy in schools, the findings clearly showed the importance of making connections within and among topics to nurture proficiency. Mathematics cannot and should not be taught as an isolated construct; rather, mathematics should be interwoven and interdependent among topics or strands. Successful learning can be characterised by comprehension of mathematical ideas. (p. 46)

**Active Learning and Conceptual Understanding**

Samuelsson (2010) also researched the mathematical strands of conceptual understanding, strategic competency, and procedural fluency. However, Samuelsson focused on their development from classroom practices. In Samuelsson’s research, two traditionally taught classrooms of eleven year olds in Sweden were compared to two similar classrooms that were taught with an active problem solving approach.
According to Samuelsson, teachers in the traditionally taught classrooms gave direct instruction to passive learners whose participation consisted of practicing the procedural methods demonstrated whereas the problem solving classrooms focused on discussion, negotiation, and collaborative work with thought provoking mathematical concepts. The researcher examined student test data with t-tests and according to Cohen’s $d$ differences and found no statistically significant correlation in procedural fluency ability of students from the two types of classrooms. However, Samuelsson reported a significant positive correlation between the active problem solving format of teaching and an increase in students’ conceptual understanding, strategies competence, and adaptive reasoning. To summarize the research, Samuelsson (2010) wrote “in this study, it is obvious that different teaching approaches have different impacts on different aspects of students’ mathematical proficiency. Problem-based learning is significantly better for improving students’ performances in conceptual understanding, strategic competence and adaptive reasoning” (p. 71).

**Flexible Problem Solving in Mathematics**

Research has also been conducted on students’ ability to solve problems flexibly, by using multiple procedures, strategies, and representations, which is thought to be vital in mathematics and linked to mathematical understanding. Newton, Star, and Lynch (2010) stated that while flexibility might be considered an aspect of procedural fluency, the ability to be flexible comes “from a merging of both procedural and conceptual knowledge” (p. 283). To study flexibility, Newton et al.
conducted original research with data from a three week summer algebra class focused on flexible problem solving for struggling students. In regards to the content of the course, the researchers wrote “it is important to note that in order to promote flexibility, various strategies were introduced and discussed with an emphasis on meaning” (Newton et al., 2010, p. 287). To study the results, the researchers held interviews and gave an identical pretest and posttest of 55 questions designed to test students ability to solve problems with more than one method.

The three goals of the research of Newton et al. (2010) were to determine how a course promoting flexibility would affect students’ knowledge and use of various solving methods, if prior knowledge of one solving method hinders or aides in developing flexibility, and if struggling students can successfully develop flexibility. The researchers’ results showed that over the three week course, students did gain knowledge of flexible problem solving, but students primarily used multiple methods only when presented with difficult or tedious problems. Newton et al. also found that having one previous familiar strategy of problem solving inhibited developing flexibility because students instinctually used the known method. Lastly, the researchers found that struggling students were not overwhelmed by learning multiple solving methods and instead felt “knowing more than one way to solve a problem may enable them to choose a method that maximizes the accuracy of their solution and/or that is most clear or understandable to them” (Newton et al., 2010, p. 303).

Ross, Reys, Chávez, McNaught, and Grouws (2011) also researched student flexibility in using various mathematical strategies effectively. Propelling their
research was the belief that “a central goal of secondary mathematics is for students to learn to use powerful algebraic strategies appropriately. Research has demonstrated student difficulties in the transition to using such strategies” (Ross et al., 2011, p. 389). The researchers collected data on several thousand eighth, ninth, and tenth grade students from different schools over the course of three years to examine their strategy choices for solving the same algebraic problem. Ross et al. reported that the percent of students who answered the problem correctly averaged approximately 20 percent over three years and showed only a modest gain each year. The researchers were surprised that the majority of the students used no official strategy and did not improve their performance or choose a more efficient strategy after three years of higher level mathematics instruction.

Ross et al. (2011) explained that they expected to report that more students used an algebraic solving method each year and that students had increasing success with various methods. However, the researchers found that many students used the same strategy all three years and of the three recognized methods the algebraic method had the lowest percent of correct answers. Ross et al. (2011) described their conclusions by stating that “considerable time and effort is spent in high school working to provide students with the algebraic tools to solve problems. Our findings suggest that current approaches are not equipping students well” (p. 397). The researchers suggested that moving forward, mathematics teachers need to focus explicitly on helping students develop conceptual understanding, the ability to find and represent mathematical meaning in problems, a reflective capacity to monitor
one’s own work for errors, and the flexibility to use various strategies on familiar and novel problems.

Stylianou (2011) also researched mathematical flexibility in students with a specific focus on mathematical representations within solving methods and strategies. According to Stylianou (2011) “representation is viewed as central to mathematical problem solving. Yet, it is becoming obvious that students are having difficulty negotiating the various forms and functions of representations” (p. 265). In order to draw conclusions on the role and function of students’ mathematical representations, the researcher examined data in the form of interviews and mathematic problem solving from expert mathematicians and students from a sixth grade classroom from a large city. Stylianou’s research, which was part of a larger research project, was evaluated with a grounded theory paradigm.

Results of Stylianou’s (2011) research showed that “both experts and students use representations as tools towards the understanding, exploration, recording, and monitoring of problem solving” (p. 276). The primary difference between expert and student use was the concrete nature of students’ representations which “points to the need to equip students with strategies for more efficient use of their representation as exploration tools” (Styliano, 2011, p. 277). The researcher also recommended that teachers explicitly demonstrate uses of representations, different purposes of representations, and how representations can be used to generalize, deduce, and make mathematical connections. Referencing the overall importance of students’ ability to use representations, Stylianou (2011) stated “it is argued that using various (or
several representations in a flexible manner has the potential of making the learning of mathematics more meaningful and effective” (p. 277).

**Differentiation in Teaching and Learning**

Researchers have also studied flexibility from other standpoints, including a possible correlation with differentiated teaching and learning. Sellars (2011) conducted such research with three Australian classrooms of ten to twelve year old students to study the effects of the implementation of a differentiation program. The researcher described the program as a multi-subject intervention system employed over six months in three graduated cycles. Sellars also explained that the program had two main components consisting of increased student choice, according to learning preference, and modified delivery of content from direct instruction to teacher facilitation of students acquiring knowledge. The researcher stated that teachers were required to mentor in an individualized manner while getting involved in each student’s personal learning.

The data conducted by Sellars (2011) was analyzed with t tests, and indicated that students made statistically significant positive progress with flexible thinking skills. The researcher found that 85 percent of studied students “demonstrated the cognitive capacities of thinking flexibly” (Sellars, 2011, p. 105). In analyzing these results, Sellars concluded that this type of differentiation can be an effective method of developing flexibility in student thinking by increasing choice and engagement in student learning. “It is suggested, however, that the significant increase in flexible thinking that became evident to the teachers was the result of both of these changes to
the more traditional pedagogy that dominated the customary teaching and learning activities and teacher-student interaction” (Sellars, 2011, p. 107).

In addition to increasing flexible thinking, researchers have sought to prove that differentiated teaching and learning increase academic achievement. Mastropieri et al. (2006) researched the academic impact on students in classrooms taught with traditional methods compared to those taught with differentiated instruction. The researchers studied 13 eighth-grade science classrooms, randomly assigned to either experimental or control conditions, with over 200 students. Mastropieri et al. collected data throughout the twelve week study and described the control conditions as traditional teacher-directed classrooms with lectures and worksheets. The experimental conditions were described by the researchers as similar in instruction but with differentiated, primarily collaborative, activities in place of the worksheets. The researchers found that the experimental conditions “statistically facilitate learning of middle school science content on posttests and on state high-stakes tests for all students and that students enjoyed using the activities” (Mastropieri et al., 2006, p. 130).

Other studies have also proven the effectiveness of differentiation on academic achievement, including a meta-analysis of data several decades ago. Kulik and Kulik (1984) used a meta-analysis of 31 studies to report on the statistical effects of ability grouping, an early and rudimentary form of differentiation, on achievement. The researchers found that ability grouping raised student achievement .19 standard deviations, which is approximately equivalent to two months of additional learning.
However, Kulik and Kulik also found that when ability grouping involved differentiation, which they labeled enriched instruction, the result was a moderate and significant gain of .49 standard deviations. The results on differentiation within ability groups “produced especially clear effects” (Kulik & Kulik, 1984, para. 1).

The effects of differentiation have been explored in higher education as well. Chamberlin and Powers (2010) conducted research on the implications of differentiated instruction on the student achievement and attitudes toward learning of first-year teacher preparatory students. The researchers examined data from ten mathematics courses, five traditionally taught and five taught using differentiation, from two universities. Chamberlin and Powers reported that the study included the data from 224 voluntary participants. The researchers conducted t tests on achievement data in addition to using a multivariate data analysis on survey items.

Instruction in the five experimental classes, according to Chamberlin and Powers (2010), was differentiated from the comparison classes with regard to readiness, interest, or learning profiles for approximately one-third of the course material. The researchers found that “both the t-test results and the examination of effect sizes reveal greater growth on the post-test for the treatment students than for the comparison students” (Chamberlin & Powers, 2010, p. 125). In addition, Chamberlin and Powers reported that survey results showed a statistically significant recognition of differentiated elements by participants in the treatments group. In a conclusion of the study, Chamberlin and Powers (2010) wrote “both the quantitative
and qualitative results provide evidence that the differentiated instruction supported
the mathematical learning of students” (p. 130).

Using a different survey portion of the same study, Chamberlin (2011) conducted research on how experiencing differentiation would impact the future teaching plans of the teacher preparation students in the treatment classes. Chamberlin described that the original goals for the research were to meet the needs of students while helping them plan for differentiation in their future classrooms. The researcher reported that over 90 percent of respondents indicated their plans to differentiate in their future instructional mode to accommodate students. Chamberlin found that almost 94 percent of respondents also planned to differentiate content for future students. The researcher’s strong conviction that differentiation is beneficial for all students is evident in a concluding statement that “by experiencing differentiated instruction in a mathematics content course, prospective teachers have the opportunity to begin enhancing their future mathematics teaching while also having their own diverse instructional needs met” (Chamberlin, 2011, p. 154).

While some insight on differentiation can be gained from pre-service teachers, Logan (2011) chose to research data on differentiation from practicing teachers. Logan’s research goal was to determine if teachers correctly understood the various components of differentiation and how to implement it successfully. The researcher claimed that many teachers are ill-prepared to differentiate, by teacher education programs and their employing schools, and that differentiation is not as prevalent as it needs to be in middle school classrooms.
Logan (2011) surveyed 141 middle school teachers from an urban public school district in Georgia. The results of the survey showed that over 90 percent of responding teachers understood that differentiation “must show respect for their learners’ commonalities and differences in many ways” and “should be responsive to individual student differences” (Logan, 2011, p. 8). Logan found over 85 percent of teachers felt differentiated classrooms concentrate on vital content and skills, have teacher-student collaboration, assess readiness, interests, and learning styles of students, and constantly modify content, materials, processes, and assessments.

However, a significant percentage of teachers responded incorrectly to the myths that differentiation “is only individualized instruction,” “does not use whole group instruction,” and “does not prepare students to compete in the real world” (Logan, 2011, p. 9). In response to these results, Logan suggested that more teachers must learn what differentiation is, and is not, in order to adequately meet the diverse needs of their students. To conclude, Logan (2011) stated that “differentiated instruction belongs in middle school because this is where student differences are more apparent. Thus it is there, where teachers can be instrumental in helping students to reach their heights and potential” (p. 11). As evidenced by the research cited in this literature review, many researchers would argue that differentiation and education reform in general is vital for any educational setting.

Summary

Educators, researchers, politicians, and experts agree that “the importance of both practical and theoretical aspects has earned the subject of Mathematics a pivotal
place in teaching and learning” (Khairani & Sahari Nordin, 2011, p. 34). The large percentage of students who struggle to comprehend algebraic content and master the use of algebraic strategies even after multiple years of algebra courses is particularly alarming considering the proven importance of mathematics proficiency (Ross et al., 2011). When contemplating the reform that must happen in mathematics to remedy this situation, Samuelsson (2010) suggested that although there are many diverse ideas for reform it is vital to remember that “what happens in the classroom has an impact on students’ opportunity to learn” (p. 61).

Dewey, a prominent philosopher from a century ago, had ideas and theories for general and mathematics specific education reform. Dewey (2001) recognized that for education to fulfill its role for each child and in society, the primary reform must come within the sphere of teaching and learning. Dewey’s theory of education was that learning must be meaningful, purposeful, child-centered, encourage choice and flexibility, and support democratic values in order to positively influence society (Wyett, 1998). While Dewey’s ideals were received as revolutionary and resulted in reform in many educational areas, Stemhagen and Smith (2008) argued that Dewey’s education reform never reached mathematics teaching and learning. To reform mathematics under Dewey’s ideals, mathematical learning must change to “demonstrate the human intentional facets of mathematics, the ways in which mathematics can be used to empower individuals, and the ways in which mathematics can relate to and even impact personally meaningful aims or ends” (Stemhagen & Smith, 2008, p. 37).
With the purpose of justifying Deweyan reform in mathematics, this literature review researched the components of Dewey’s educational theories that were applicable to improving the teaching and learning of mathematics. Dewey advocated for making learning purposeful, and Palm (2008) found that a statistically significant positive correlation between problem authenticity and realistic responses from students. In regard to active and meaningful learning, researchers found that a significant correlation exists between active mathematical thinking, discussion, and problem solving in the classroom and increased conceptual understanding of the mathematical concepts (Kazemi & Stipek, 2008; Samuelsson, 2010). Research addressing the topic of flexibility showed that increases in student flexibility resulted when flexibility was specifically emphasized in the learning process whereas in traditional learning no such increases were found (Newton et al., 2010; Ross et al., 2011). Differentiation, the final component to fit with Deweyan ideals of mathematics reform, was shown to increase student achievement across grade levels and subject matter (Kulik & Kulik, 1984; Mastropieri et al., 2006; Chamberlin & Powers, 2010).

**Conclusions**

Students’ struggles to comprehend mathematics, especially in courses at higher levels and with abstract content, is a longstanding problem that requires reform. While many areas of education have reformed and evolved with the times, mathematics education has stayed stagnantly within the realm of traditional teaching and learning (Stemhagen & Smith, 2008). Many in education agree that this
traditional teaching and learning of mathematics is neither viable nor effective for the needs of today’s students and society (Khairani & Sahari Nordin, 2011). As a result, improvements must be made to the traditional mathematics classroom.

Within this literature review it was shown that research supports the effectiveness of several teaching and learning constructs that fit with Dewey’s educational theories for increasing student comprehension and achievement. Therefore, reform must center on implementing these constructs into the mathematics classroom. To accomplish this reform, a supplementary algebra curriculum will be developed in order to implement authentic real world connections, differentiation for student needs, and an emphasis on conceptual understanding and flexible problem solving into the traditional algebra classroom. Based on the previously cited research, it is expected that implementing these constructs will improve students’ comprehension of mathematical concepts and their academic success with algebraic content.
Chapter Three: Project Description

Introduction

For many years mathematics has been dominated by traditional teaching and learning (Stemhagen & Smith, 2008). The traditional lecture-style instruction, passive learning, and emphasis on mechanical procedures have become obsolete because “today, students must understand the mathematics that they are learning” (Khairani & Sahari Nordin, 2011, p. 33). Without reform, many students will continue to struggle to comprehend mathematics taught in traditional context. Real reform requires changing outdated traditional components and implementing new constructs that can increase student comprehension of mathematics and meaningfully connect the mathematics classroom to the real world.

The purpose of this project is to create a supplementary curriculum for implementing research-based ideas for reformed teaching and learning of mathematics. The intended setting for the implementation is the eighth grade algebra classroom at Knapp Charter Academy in Grand Rapids, Michigan. The supplementary curriculum is designed for use in conjunction with the current algebra curriculum to meet required academic standards while promoting student engagement in purposeful mathematics. The addition of the supplementary curriculum should help meet students’ learning needs while making meaningful connections between mathematics and the real world. Based on the previously cited research, the author intends for the supplementary curriculum to increase overall student comprehension and mastery of algebra.
The supplementary algebra curriculum will be described in this chapter starting with the components of the created curriculum. The justification for the layout of the components and the research-based ideas behind the components will also be addressed in the forefront of the chapter. Next, the evaluation protocol for the project will be described followed by research conclusions regarding the reform of traditional mathematics teaching and learning. The chapter will conclude with the plans for implementation of this project, ideas for using the project beyond the intended platform, and suggestions for further research.

**Project Components**

The supplementary curriculum designed to reform the traditional eighth grade algebra classroom at Knapp Charter Academy includes original lessons, activities, projects, worksheets, and assessments that were created by the author in a nontraditional format. Although it is possible that the algebra classroom at the heart of this project is already less traditional than many, the goal of this supplementary curriculum is to reinvent the abstract algebraic concepts that many students find dull, meaningless, and disconnected from reality.

The supplementary materials for this project were created by the author using knowledge of the algebraic concepts, curriculum and student needs based on teaching experience in an algebra course. The components of this project seek to connect students to algebra in an engaging and meaningful way. The lessons, activities, projects, worksheets, and assessments in this project purposefully unite procedural fluency with conceptual understanding and present the abstract algebraic concepts in
a relevant, meaningful, and differentiated context. An emphasis on flexible problem solving is featured in several of the components as well.

To devise the supplementary curriculum, educational theory and research was analyzed to determine how certain reform constructs affect effectiveness of teaching and learning in mathematics. A literature review examined empirical studies in the field and identified ideas linked to positive impact on students’ comprehension and achievement. The author found that research, previously cited in the paper, supports the effectiveness of authentic real world connections, differentiation for meeting student needs, active learning, and an emphasis on conceptual understanding and flexible problem solving.

Based on these findings, the current eighth grade algebra curriculum at Knapp Charter Academy was analyzed for the presence of these research-based constructs. The author also analyzed past student performance in algebra at Knapp Charter Academy to determine problematic units in terms of student comprehension. These results were used to decide what units could most benefit from the addition of nontraditional lessons, activities, projects, worksheets, and assessments. As a result, numerous supplementary materials were created for some algebra units and few materials were created for others.

The supplementary curriculum begins with a document featuring the new conceptual understanding goals for the enhanced curriculum. This document also explains how the enhanced curriculum will continue to meet procedural fluency goals while placing a new emphasis on conceptual understanding for all algebra units.
through instruction, discussion, and formative assessment. The supplementary curriculum features a full unit on quadratic functions and many materials for both the functions unit and unit on systems of linear equations and inequalities. The basis for the prevalence of materials in these units was that Knapp Charter Academy students have historically struggled to comprehend the unit concepts. Additional reasoning came from the format of the current curriculum for these units, which supported passive learning of difficult concepts with emphasis on procedural fluency.

The supplementary quadratic unit includes full lesson plans and more than a dozen activities and assessments. Many of the activities are differentiated to meet students’ readiness needs and provide choice to boost student interest. The functions unit contains numerous activities and assessments designed to incorporate real world connections with a conceptual emphasis that was previously lacking. There are also several activities differentiated for student choice. The unit on systems of linear equations and inequalities is similar to the function unit, although there is an additional emphasis on flexible solving. Real world connections and differentiation are also prevalent in the unit components. The remaining units in the curriculum are all represented in the supplementary curriculum but to lesser degrees. The author chose specific problematic concepts from the curriculum to focus on for these remaining units and created activities and lessons to address these targeted areas.

**Project Evaluation**

To evaluate the effectiveness of the new supplementary curriculum, test results of students who have experienced the enhanced curriculum will be compared
to test results from previous students who experienced the traditional curriculum and
to nationwide normative growth data. These results will only be viewed and analyzed
by the author to preserve student confidentiality. The test results that will be
compared are from the Northwest Evaluation Association’s Measures of Academic
Progress (MAP) computerized adaptive test. Knapp Charter Academy students take
the MAP test three times each academic year.

In the fall, winter, and spring, the MAP test measures Knapp Charter
Academy students’ comprehension and level of ability in different strands of
mathematical proficiency including algebra. Testing reports with detailed student
data, past and present, are available online and through emailed reports to all teachers
who proctor the MAP test for each testing session. Data is also given to proctoring
teachers regarding nationwide growth norms, score percentiles, and standardized
levels of achievement.

For the purpose of evaluating this project, the fall to spring growth, overall
and algebra specific, for students who went through the supplementary curriculum
will be compared to past student fall to spring growth rates and current nationwide
normative data. The supplementary curriculum will be viewed as effective for
increasing student comprehension and achievement if spring student growth rates are
higher than the comparison groups. The level of effectiveness will be determined
based on the amount of difference in average student growth. However, specific
attention will also be given to examining the results of students who were considered
“at risk” at the beginning of the course. It is possible that the supplementary
curriculum will increase achievement for these “at risk” students while not having a significant impact on students entering the course with a successful mathematics background. All possibilities will be explored with the data to find any and all correlations between the supplementary curriculum and student achievement.

**Project Conclusions**

Students’ struggle to comprehend mathematics is not a new or isolated problem. Nationwide, the failure rate for algebra exceeds fifty percent (Gates, 2008). While the predicament is complex, students’ difficulty in comprehending mathematics can be at least partially attributed to the ineffective traditional nature of mathematics teaching and learning (Rakes, Valentine, McGatha, & Ronau, 2010). While many subjects have undergone reform to improve teaching and learning, mathematics has largely remained stagnant for a century (Stemhagen & Smith, 2008). As to the reason behind the lack of reform, Stemhagen and Smith (2008) conclude “the unexamined, common-sense version of mathematics as objective, neutral, and extra-human has fostered resistance to the aforementioned pedagogical shifts that have taken place in other subject areas” (p. 30).

There has been much research done on the topic of student comprehension and achievement in mathematics and several teaching and learning constructs have emerged as potential ways to increase student understanding. Palm (2008) found that increasing problem authenticity increases realistic considerations in students. Researchers also found that conceptual understanding can be increased by emphasizing active mathematical thinking, discussion, and problem solving in the
classroom (Kazemi & Stipek, 2008; Samuelsson, 2010). The ability to solve problems flexibly increased when flexibility was stressed in the learning process (Newton et al., 2010; Ross et al., 2011). Differentiation, the final research-based construct that addressed the original problem statement, was found to increase comprehension in multiple educational settings (Kulik & Kulik, 1984; Mastropieri et al., 2006; Chamberlin & Powers, 2010).

Based on the research results, a key for combating student difficulty in mathematics, especially abstract courses such as algebra, is to focus on nontraditional teaching modes and enriched presentation of curriculum. For this project, a supplementary algebra curriculum was developed to implement nontraditional components of teaching and learning into the mathematics classroom. This nontraditional curriculum features authentic real world connections, differentiation for meeting student needs, and an emphasis on conceptual understanding and flexible problem solving. Based on the previously cited research, it is expected that implementing this supplementary curriculum will directly improve the plight of students experiencing difficulty in comprehending mathematics.

**Plans for Implementation**

The supplementary curriculum created for this project will be implemented into the eighth grade algebra classroom at Knapp Charter Academy beginning in the fall of 2013. The nontraditional emphasis will be woven into instruction methods and the created components will be presented alongside the current curriculum. The effectiveness of the supplementary curriculum will be evaluated, as previously
described, and edited accordingly before use each academic year. If it is proven effective for increasing student achievement, the research conclusions and project will be shared with other algebra teachers at National Heritage Academies via email correspondence for their use as desired. Additionally, the results will be documented and shared with the middle school mathematics team at Knapp Charter Academy in an effort to spur mathematics reform and increase student mathematical comprehension throughout the building.

While research shows that the constructs being implemented in this project statistically improve student achievement, the author suggests new research that studies the effect of combining all of the constructs as is being done in this project. A study comparing traditional mathematics methods without these constructs with reformed teaching and learning could be helpful in proving the necessity of overall mathematics reform throughout the education system. Such research may help to bring about the change needed by struggling mathematics students in this country. Ultimately, without proven need for mathematics reform many students could continue to face mathematics failure in a world where mathematical achievement is a gateway to opportunity and full participation in society.
References


Appendix A:

Teaching & Learning Emphasis for All Algebra Units

Includes:

Plan for Implementation

New Conceptual Understanding Goals (Conceptual Understanding)

Traditional Procedural Fluency Goals
Teaching & Learning Emphasis for All Algebra Units

Plan for Implementation: Every algebra unit has new conceptual understanding goals, which will be addressed along with traditional procedural fluency goals. The teaching and learning emphasis will switch from straightforward procedural solving to uniting the solving procedures with the underlying conceptual meaning. The conceptual understanding goals will be addressed and discussed during instruction and while learning the procedures that correspond. There will also be exit tickets given to formatively assess conceptual understanding of current topics. Embedded in lessons and activities there will be assigned peer discussion time to converse about the conceptual meaning in the featured lesson or activity. There will also be a conceptual understanding component added to assessments so that the students’ level of conceptual understanding on a topic can be measured and so that students understand that conceptual understanding is valued as much as procedural fluency.

Solving Equations & Inequalities Unit

NEW Conceptual Understanding Goals

I can justify why Order of Operations must be used to simplify and/or evaluate an expression.
I can describe the components of an equation/inequality.
I can explain what it means to solve an equation/inequality.
I can translate a real-world situation into an equation/inequality.
I can explain how and why equations/inequalities are useful representations of real world situations.
I can justify the order of the steps you must follow to solve an equation/inequality.
I can judge whether there is more than one way to solve an equation/inequality and justify my answer.
I can explain why identities and contradictions exist and describe the implications of these solutions.
I can justify the two reasons for switching the inequality sign when solving.

Traditional Procedural Fluency Goals

I can solve one-step, two-step, and multi-step equations/inequalities by using inverse operations.
I can solve equations/inequalities that are not simplified.
I can solve equations/inequalities with variables on both sides.
I can solve multivariable equations for a variable.
I can graph solutions to inequalities.
Systems of Linear Equations & Inequalities Unit

NEW Conceptual Understanding Goals

I can explain what constitutes a system of linear equations/inequalities.
I can explain how the two primary forms of linear equations (standard and slope-intercept) are formed from real-world situations.
I can analyze a system of linear equations to find the most efficient way to solve it.
I can justify the existence of systems of linear equations with no solutions or infinitely many.
I can explain what it means to solve a system of linear equations/inequalities.
I can explain why the solving methods (graphing, substitution, and elimination) work to find a solution to systems of linear equations.
I can explain why only the solving method of graphing works to find a solution to systems of linear inequalities.
I can describe and explain the difference between the solution(s) for a system of linear equations versus inequalities.

Traditional Procedural Fluency Goals

I can identify solutions of linear systems.
I can solve linear systems by graphing.
I can solve linear systems using substitution.
I can solve linear systems using elimination.
I can identify solutions of linear inequalities.
I can graph linear inequalities.
I can graph and solve systems of linear inequalities.

Exponents, Proportions, & Percents Unit

NEW Conceptual Understanding Goals

I can explain the concept of a ratio and give real-world examples.
I can explain the concept of a proportion.
I can explain why and how a proportion is used to solve problems with ratios.
I can explain how and why ratios and proportions are useful tools for representing and solving real world situations.
I can explain how percentage is a comparison to a whole.
I can explain the relationship between percentage and the base ten system.
I can evaluate whether there is more than one way to solve a percent problem and justify my answer.
I can explain why different situations require either an exact or estimated percent answer.
I can explain the difference between percent and percent change and why you cannot solve them using the same methods.
I can explain the meaning and describe the numeric representation of integer exponents.
I can explain why a negative exponent does not result in a negative answer.
I can justify the existence and usefulness of scientific notation.
I can describe situations that require converting between scientific and standard notation.
I can explain why the properties of exponents work.

*Traditional Procedural Fluency Goals*

I can write and compare ratios.
I can calculate proportions for a missing variable.
I can calculate percents.
I can estimate percents.
I can calculate percent increase and decrease.
I can evaluate and simplify expressions with integer exponents.
I can evaluate scientific notation and convert between scientific and standard notation.
I can use properties of exponents to evaluate and simplify expressions.

*Polynomials Unit*

*NEW Conceptual Understanding Goals*

I can describe the importance of like terms when adding and subtracting polynomials.
I can justify why coefficients change and exponents do not when adding and subtracting polynomials.
I can justify why it is important to change subtraction to adding a negative when working with polynomials.
I can describe which types of polynomials correspond to each multiplication method (commutative, FOIL, distributive, box) and explain why the other methods will not work.
I can describe the relationship between factoring and division.
I can describe which type of polynomial corresponds to each factoring method (GCF/leftovers, Reverse FOIL, and special products) and explain why the other methods will not work.
I can describe how special products are different than typical polynomial products and why it is important to have separate multiplying and factoring methods.

*Traditional Procedural Fluency Goals*

I can classify polynomials according to term and degree.
I can add polynomials.
I can subtract polynomials.
I can multiply monomials using the commutative property.
I can binomials polynomials using the FOIL method.
I can multiply polynomials using the distributive property.
I can multiply polynomials using the box/lattice method.
I can recognize and apply special binomial products.
I can factor polynomials using greatest common factor.
I can factor polynomials using reverse FOIL.
I can factor special products.

Functions Unit

*NEW Conceptual Understanding Goals*

I can explain how a function differs from a relation that is not a function.
I can describe the relationship between the $x$ and $y$ values in a function.
I can justify whether a relation is a function by its equation, ordered pairs, or graph.
I can explain the relationship between a function and its equation, ordered pairs, and graph.
I can identify different types of functions and give proof to justify my assertion.
I can describe the components of an exponential function and what they represent.
I can explain why an exponential curve results from graphing an exponential function.
I can translate an appropriate real-world situation into an exponential function.
I can explain how and why exponential functions are useful representations of certain real world situations.
I can describe the components of an inverse variation function and what they represent.
I can explain why discontinuous curve results from graphing an inverse variation function.
I can translate an appropriate real-world situation into an inverse variation function.
I can explain the mathematical/scientific principal of inverse variation.

*Traditional Procedural Fluency Goals*

I can identify a mathematical function by viewing the equation or ordered pairs.
I can find the domain and range of functions.
I can identify a function using the Vertical Line Test.
I can evaluate a function for specific domain.
I can identify the independent and dependent variables.
I can identify a function as continuous or discrete.
I can compare linear, quadratic, and exponential models and write equations to fit each model.
I can evaluate, identify, and graph exponential functions.
I can identify, write, and graph inverse variation functions.

Linear Functions Unit

*NEW Conceptual Understanding Goals*

I can explain the format of a linear function and how it differs from other functions.
I can define the potential real-world meaning of x- and y-intercepts.
I can explain the relationship between the slope of a line and the graphed line.
I can explain the relationship between the y-intercept of a line and the graphed line.
I can explain the various graphing methods and identify the situations in which each is most efficient.
I can describe the components of an exponential function and what they represent.
I can explain why a line results from graphing a linear function.
I can translate an appropriate real-world situation into a linear function.
I can explain how and why linear functions are useful representations of certain real world situations.
I can explain the real world meaning of the slope and y-intercept for a given real world situation.
I can explain why and how changing the slope and/or y-intercept affects the linear function.

**Traditional Procedural Fluency Goals**

I can identify linear functions by viewing the equation, ordered pairs, or graph.
I can graph linear functions.
I can find x- and y- intercepts of linear functions.
I can graph linear functions using intercepts.
I can find the slope of a line.
I can write and graph a linear equation in slope-intercept form.
I can write a linear equation given two points.
I can transform a linear function using slope and y-intercept.

**Quadratic Functions Unit**

**NEW Conceptual Understanding Goals**

I can explain the format of a quadratic function and how it differs from other functions.
I can define the term solutions and explain the potential real-world meaning of solutions.
I can describe the components of a quadratic function and what they represent.
I can explain the relationship between the components of the equation and the graphed parabola.
I can explain why and how changing the components of the equation affects the quadratic function.
I can explain why a parabola results from graphing a quadratic function.
I can translate an appropriate real-world situation into a quadratic function.
I can explain how and why quadratic functions are useful representations of certain real world situations.
I can explain the various solving methods and identify the situations in which each is most efficient.
I can explain the difference between a quadratic function and a quadratic equation and the purpose of each.
I can prove why only the discriminant is needed to determine the number of solutions for a quadratic.

**Traditional Procedural Fluency Goals**

I can identify a quadratic function.
I can identify and define the basic elements of a parabola (minimum or maximum, axis of symmetry, domain, range, y-intercept, zeros, and vertex).
I can graph quadratic functions.
I can identify the solutions of a quadratic function.
I can find the axis of symmetry and vertex of a parabola.
I can transform quadratic functions by changing a, b, and/or c values.
I can solve quadratic equations by graphing.
I can solve quadratic equations by factoring.
I can solve quadratic equations using square roots.
I can solve quadratic equations by using the quadratic formula.
I can determine the number of solutions (of a quadratic equation) by using the discriminant.

**Sequences Unit**

**NEW Conceptual Understanding Goals**

I can explain the relationship between an arithmetic sequence and a linear function.
I can use my knowledge of the format of an arithmetic sequence to derive the formula for finding the nth term for an arithmetic sequence.
I can explain the relationship between a geometric sequence and an exponential function.
I can use my knowledge of the format of a geometric sequence to derive the formula for finding the nth term for a geometric sequence.

**Traditional Procedural Fluency Goals**

I can recognize an arithmetic sequence and extend the pattern.
I can find a given term of an arithmetic sequence.
I can write a linear function from an arithmetic sequence.
I can write an arithmetic sequence from a linear function.
I can recognize and extend geometric sequences.
I can find the nth term of a geometric sequence.
I can write an exponential function from a geometric sequence.
I can write a geometric sequence from an exponential function.
Appendix B:
Solving Equations & Inequalities Unit

Includes:

Equation Solving Activity (Flexible Solving & Real World Connections)
Equation Solving Activity-

*Emphasis on Flexible Solving and Real World Connections*

**Flexible Equation Solving:**

Find two mathematically correct ways to solve the following equations. Then justify which method was more efficient.

1.) Solving method #1
   
   \[ 4(x + 2) = 32 \]
   
   Solving method #2
   
   \[ 4(x + 2) = 32 \]

   Which method was more efficient? ___ Explain. _____________________________

_____________________________________________________________________

2.) Solving method #1

   \[ \frac{20}{x} = 5 \]

   Solving method #2

   \[ \frac{20}{x} = 5 \]

   Which method was more efficient? ___ Explain. _____________________________

_____________________________________________________________________

3.) Solving method #1

   \[ \frac{2}{3}x + \frac{1}{3} = 2 \]

   Solving method #2

   \[ \frac{2}{3}x + \frac{1}{3} = 2 \]

   Which method was more efficient? ___ Explain. _____________________________

_____________________________________________________________________
Real World Connections:

4.) Write equations for each of the following situations. They might have multiple variables.
   a. For eight hours of work, Pam made $40. Equation: _____________________

   b. At the bowling alley, it costs $20 total for two games of bowling and a $5 shoe rental. Equation: _____________________

   c. To be a member of the Y, it costs $15 to join and then $25 a month. Equation: _____________________

   d. You are buying party supplies. Chips cost $3.99 a bag and pop costs $1.99 per 2 liter bottle and you have a $20 budget. Equation: _____________________

5.) Mark spent 32 dollars at the movie theater. He spent 8 dollars on snacks plus he bought four tickets. Write an equation for this situation using the variable t for tickets. Then solve for t to figure out how much each ticket cost.

   Equation                          cost of each ticket (t) = $ _______

                                        ________ = __________

6.) Mario is comparing two cell phone plans. Plan A has a twenty dollar activation fee plus five cents per minute. Plan B has no fee and costs fifteen cents per minute. For what amount of minutes would these plans cost the same amount? Write an equation using m for minutes. Then solve for m (minutes). Hint: five cents = .05

       Plan A = Plan B

   Equation: = Answer: _____ minutes
Appendix C:

Systems of Linear Equations & Inequalities Unit

Includes:

Tiered Graduation Assignment (Real World Connections & Differentiation)

Systems of Equations vs. Inequalities Venn Diagram (Conceptual Understanding)

Cell Phone Activity (Real World Connections)

Unit Project (Differentiation)

Tiered Practice Activity (Differentiation)

Chicago Trip Performance Task (Real World Connections)

Flexible Solving Activity (Flexible Solving)

Systems Assessment (Flexible Solving & Real World Connections)
Graduation Assignment

The year is flying by and we need your help planning the graduation ceremony and reception. We want to choose the best value for the DJ, the cake, and the chair rental.

Directions:

1. Make each option into a linear equation. Doing this will create a system of linear equations for each category (the band, the cake, and the chair rental).
2. Solve each system of linear equations using graphing, substitution, or elimination. Show your work! If you prefer to work on a separate sheet, staple the sheet(s) with your work to this packet. Fully complete each solution section.
3. After solving, decide which option our school should choose if we plan on having 350 people attend the graduation festivities and we want three hours of music.
4. Choose one of the following options.
   a. Write a letter to Mr. Turcotte explaining your choices.
   b. Figure out how much money our school will save by using the options you chose instead of the other options. You must turn in an organized copy of your calculations.
   c. Make a song or a rap about what you did in steps one through three. Write down your lyrics to turn in.

Disc Jockey Options:
The Music Man charges a flat fee of $200. Amazing DJ Service charges $40 per hour.

Linear equation (The Music Man)  _________________
Linear equation (Amazing DJ Service)  _________________

Work Space:

Solution:
The cost for The Music Man and Amazing DJ Service are the same for _____ hours.
The Music Man is cheaper for ____________ hours.
Amazing DJ Service is cheaper for ____________ hours.
For our graduation, we should hire ________________.
**Cake Options:**
Cakes Galore charges $1.00 per slice for their cakes and a $50 delivery fee.
Yum Cakes charges $1.25 per slice with no delivery fee.

Linear equation (Cakes Galore) ___________________
Linear equation (Yum Cakes) ___________________

Work Space:

**Solution:**
The cost for Cakes Galore and Yum Cakes are the same for _____ slices.
Cakes Galore is cheaper for _______ slices.
Yum Cakes is cheaper for ________ slices.

For our graduation, we should hire ________________.

**Chair Rental Options:**
Jon’s Table and Chair Rental charges $0.60 per chair with no fees.
Chairs Unlimited charges $0.50 per chair plus a delivery fee of $20.

Linear equation (Jon’s Table and Chair Rental) ___________________
Linear equation (Chairs Unlimited) ___________________

Work Space:

**Solution:**
The cost for Jon’s Table and Chair Rental and Chairs Unlimited are the same for _____ chairs.
Jon’s Table and Chair Rental is cheaper for ________ chairs.
Chairs Unlimited is cheaper for ________ chairs.

For our graduation, we should hire ________________.
Graduation Assignment

The year is flying by and we need your help planning the graduation ceremony and reception. We want to choose the best value for the DJ, the cake, and the chair rental.

Directions:
1. Make each option into a linear equation. Doing this will create a system of linear equations for each category (the band, the cake, and the chair rental).
2. Solve each system of linear equations using graphing, substitution, or elimination. Show your work! If you prefer to work on a separate sheet, staple the sheet(s) with your work to this packet. Fully complete each solution section.
3. After solving, decide which option our school should choose if we plan on having 350 people attend the graduation festivities and we want three hours of music.
4. Choose one of the following options.
   a. Write a letter to Mr. Turcotte explaining your choices.
   b. Figure out how much money our school will save by using the options you chose instead of the other options. You must turn in an organized copy of your calculations.
   c. Make a song or a rap about what you did in steps one through three. Write down your lyrics to turn in.

Disc Jockey Options:
The Music Man charges a set-up fee of $50 and $20 per hour of music. Amazing DJ Service charges a set-up fee of $40 and $24 per hour of music.

Linear equation (The Music Man) 
Linear equation (Amazing DJ Service) 

Work Space:

Solution:
The cost for The Music Man and Amazing DJ Service are the same for _____ hours. The Music Man is cheaper for _________ hours. Amazing DJ Service is cheaper for _________ hours.

For our graduation, we should hire ________________.
**Cake Options:**
Cakes Galore charges $1.15 per slice for their cakes plus a $20 delivery fee and a $20 set-up fee.
Yum Cakes charges $1.25 per slice plus a $10 delivery fee.

Linear equation (Cakes Galore)  _______________
Linear equation (Yum Cakes)  _______________

Work Space:

**Solution:**
The cost for Cakes Galore and Yum Cakes are the same for _____ slices.
Cakes Galore is cheaper for ________ slices.
Yum Cakes is cheaper for __________ slices.

For our graduation, we should hire ________________.

**Chair Rental Options:**
Jon’s Table and Chair Rental charges $0.85 per chair.
Chairs Unlimited charges $0.75 per chair plus a delivery fee of $10 and a set-up fee of $15.

Linear equation (Jon’s Table and Chair Rental)  _______________
Linear equation (Chairs Unlimited)  _______________

Work Space:

**Solution:**
The cost for Jon’s Table and Chair Rental and Chairs Unlimited are the same for _____ chairs.
Jon’s Table and Chair Rental is cheaper for ___________ chairs.
Chairs Unlimited is cheaper for ___________ chairs.

For our graduation, we should hire ________________.
Graduation Assignment

The year is flying by and we need your help planning the graduation ceremony and reception. We want to choose the best value for the DJ, the cake, and the chair rental.

Directions:
1. Make each option into a linear equation. Several are done for you. Doing this will create a system of linear equations for each category (the band, the cake, and the chair rental).
2. Solve each system of linear equations using graphing, substitution, or elimination. Show your work! If you prefer to work on a separate sheet, staple the sheet(s) with your work to this packet. Fully complete each solution section.
3. After solving, decide which option our school should choose if we plan on having 350 people attend the graduation festivities and we want three hours of music.
4. Choose one of the following options.
   a. Write a letter to Mr. Turcotte explaining your choices.
   b. Figure out how much money our school will save by using the options you chose instead of the other options. You must turn in an organized copy of your calculations.
   c. Make a song or a rap about what you did in steps one through three. Write down your lyrics to turn in.

Disc Jockey Options:
The Music Man charges a flat fee of $200.
Amazing DJ Service charges $40 per hour.

Linear equation (The Music Man) \( y = 200 \) \( y \) is cost, \( x \) is hours
Linear equation (Amazing DJ Service) \( y = 40x \)

Work Space:
To solve \( y = 200 \) and \( y = 40x \) using substitution, fill in the \( y \) value in \( y = 40x \) with a value you know is equal to \( y \) (200). The resulting equation is \( 200 = 40x \). Solve for \( x \).

Solution:
The cost for The Music Man and Amazing DJ Service are the same for _____ hours.
The Music Man is cheaper if the DJ plays more / less hours. (circle one)
Amazing DJ Service is cheaper if the DJ plays more / less hours. (circle one)

For our graduation, we should hire ______________ since we need three hours of music.
**Cake Options:**
Cakes Galore charges $1.00 per slice for their cakes and a $50 delivery fee.
Yum Cakes charges $2.00 per slice with no delivery fee.

Linear equation (Cakes Galore) \( y = 50 + x \)
Linear equation (Yum Cakes) \( y = 2x \)

**Work Space:** Solve \( y = 50 + x \) and \( y = 2x \) using graphing, substitution, or elimination.

**Solution:**
The cost for Cakes Galore and Yum Cakes are the same for _____ slices.
Cakes Galore is cheaper if you buy more / less slices. (circle one)
Yum Cakes is cheaper if you buy more / less slices. (circle one)

For our graduation, we should hire ________________ since we need 350 slices.

**Chair Rental Options:**
Jon’s Table and Chair Rental charges $220 for an unlimited number of chairs and delivery.
Chair Masters charges $0.50 per chair plus a delivery fee of $20.

Linear equation (Jon’s Table and Chair Rental) \( y = 220 \)
Linear equation (Chair Masters) \( y = \)

**Work Space:**

**Solution:**
The cost for Jon’s Table and Chair Rental and Chair Masters are the same for _____ chairs.
Jon’s Table and Chair Rental is cheaper if you rent more / less chairs. (circle one)
Chair Masters is cheaper if you rent more / less chairs. (circle one)

For our graduation, we should hire ________________ since we need 350 chairs.
**Systems of Linear Equations & Inequalities Activity**

**Compare – How are things alike and different?**

We have been studying systems of linear equations and systems of linear inequalities in this unit. Obviously, linear inequalities have an inequality sign instead of an equals sign. How else are systems of linear equations and systems of linear inequalities alike and different? Address the aspects listed below and other features you think of as you fill out the Venn Diagram and answer the questions that follow.

Consider:
- Equation/inequality structure
- Methods of solving
- Steps in graphing
- Appearance on graph
- Solutions
- Real-world application

---

**Linear Systems**

<table>
<thead>
<tr>
<th>Systems of Linear Equations</th>
<th>Systems of Linear Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image_url" alt="Venn Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

---
Follow-up questions about systems of linear equations vs. systems of linear inequalities

1. Is it easier to solve a system of linear equations or linear inequalities? Why?

2. On a graph, are the solutions easier to locate for a system of linear equations or inequalities? Why?

3. Which are more useful for real-world applications? Why?
**Systems of Linear Equations- Real World Connection Activity**

Congratulations, you are getting a new cell phone! The only problem is that you have to pay half of the monthly bill for your new service plan. (Your parents/guardians have graciously agreed to pick up the other half of the bill.) You need to decide which plan is right for you, your budget, and your needs. If you do not have the money for any plan, you’ll have to do chores to earn it. Check out the three plans below, and use the information to fill out the chart comparing them. Then use the chart to make a decision of which plan is right for you. You should have several logical reasons for your choice. Fill out the solution report with your choice. Then examine and evaluate Mark’s choice, which is featured below.

**Plan #1- The Unlimited Plan**  
Cost: $100 per month  
Details: Unlimited calling, texting, internet surfing, etc.  
Contract: 2 years – If you break your contract early, you pay a $250 fee.

**Plan #2- The Basic Plan**  
Cost: $20 monthly fee + $0.10 per minute you talk/text message  
Details: There is a monthly fee regardless of how much you use this phone. You also get charged per minute for each call/text message you send. You cannot surf the internet with this phone.  
Contract: 2 years – If you break your contract early, you pay a $150 fee.

**Plan #3- The Average Plan**  
Cost: $40 monthly fee + $0.05 per minute you talk/text message  
Details: There is a monthly fee regardless of how much you use this phone. You get charged per minute for each call/text message you send. However, you do receive free nights (after 7 p.m.) and weekends with this plan, which means you can call/text for free during that time. You cannot surf the internet with this phone.  
Contract: 2 years – If you break your contract early, you pay a $200 fee.

<table>
<thead>
<tr>
<th>Cell Phone Plans</th>
<th>Plan #1– Unlimited</th>
<th>Plan #2– Basic</th>
<th>Plan #3– Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear equation–</td>
<td>Write a linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Write a linear</td>
<td>equation that</td>
<td></td>
<td></td>
</tr>
<tr>
<td>equation that</td>
<td>represents the</td>
<td></td>
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<tr>
<td>monthly cost for</td>
<td>monthly cost for</td>
<td></td>
<td></td>
</tr>
<tr>
<td>each plan.</td>
<td>each plan.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price &amp; fees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contract</td>
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<td></td>
<td></td>
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<tr>
<td>----------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Your usage &amp; needs</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(Hint: You can plug the number of minutes you plan to talk per month into the linear equation to see a monthly cost.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other factors to consider</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>My choice is plan #</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The reasons I chose this plan are:</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mark chose plan #2. Mark *loves* to talk on the phone, but he does not have a lot of money to spend on a phone service plan. He decided to get plan #2 because he says it costs less. Did Mark make a smart choice? Please evaluate Mark’s reasoning.
Systems of Linear Equations and Inequalities

Directions: Choose three of the activities below to complete during this unit. You may work on them in class when you finish an activity early. Work not completed during class time must be finished outside of class. At the end of the unit, you must turn in three completed activities.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sea Breeze &amp; Salt Water</strong></td>
<td>There are two boats. The first boat is 50 miles due north of the second. The first boat is traveling south-east at 20 miles per hour. The second boat is traveling northeast at 10 miles per hour. Will their paths cross? If so, how long will it take? Answer this question and then make up three of your own similar problems and solve them. Show your work. Hint: If you are stuck, model the problem on the coordinate plane.</td>
</tr>
<tr>
<td><strong>Mow, Mow, Mow the Lawn</strong></td>
<td>Mowing the lawn, planting a garden, and hiking in the woods are activities that can involve linear paths. (You can plant the seeds in a straight line.) Imagine the three activities as three systems of linear equations. How many solutions would each activity have (none, one, infinitely many). Explain your answers in detail and explain your reasoning. Include a sketch of the linear paths for each activity.</td>
</tr>
<tr>
<td><strong>Slow Down</strong></td>
<td>Interview three people about speed limits. Pose questions to them about what speed limits mean. For example, on the expressway the signs say the maximum speed is 70 mph and the minimum speed is 45 mph. How fast can you go? Record their answers. Summarize all of the interviews. Compare the information about speed limits to solutions of linear inequalities and system of linear inequalities.</td>
</tr>
<tr>
<td><strong>Combination Chaos</strong></td>
<td>You go to a store to buy soda and chips with $20. Sodas cost $1.25 each and a bag of chips costs $2.50. You need at least 1 bag of chips and four sodas because you have three friends with you. How many different combinations of sodas and chips can you buy without going over your $20? Create a graph of this situation. Find all of the possible combinations and list them as ordered pairs.</td>
</tr>
<tr>
<td><strong>Belt It Out</strong></td>
<td>Write your own song or rap about solving systems of linear equations. The lyrics must include information on each of the three methods of solving. Spice it up by using the information in a story line. For example, maybe you lost your best friend because of an argument about the best way to solve a system of linear equations.</td>
</tr>
<tr>
<td><strong>What a Chore</strong></td>
<td>A teenager has the following options for an allowance. Option A = $10 per week Option B = $1.50 per chore Option C = $5 per week + $0.75 per chore Find the number of chores per week that a teenager would have to do to want option A. Repeat this process for option B and option C. Show your work and write a paragraph (at least) explaining your answers.</td>
</tr>
<tr>
<td><strong>The Third Dimension</strong></td>
<td>Build three 3-D models for a system of linear equations with no solutions, one solution, and infinitely many solutions. The model should include the coordinate plane and should be conceptually accurate. Numerical precision is not required.</td>
</tr>
<tr>
<td><strong>Act I</strong></td>
<td>Create a skit about this unit. It should include detailed and accurate information about systems of linear equations and systems of linear inequalities. Use props or your own movement to demonstrate the concepts. Present your skit and turn in a script outline.</td>
</tr>
<tr>
<td><strong>For Sale By Owner</strong></td>
<td>You are trying to sell the “secret” methods used to solve systems of linear equations. Write an article and make a flyer about your products. Combine accurate information and word choice to persuade potential buyers.</td>
</tr>
</tbody>
</table>
Linear Inequalities

Directions: Cut on the light lines of the squares below, and fold on the dark lines to form a cube. Tape the cube together. Each side of the cube has directions for a task. You and an assigned partner will take turns rolling the cube. You must complete whatever task the cube lands on. You get one chance to roll again if you are not happy with the roll. If the cube lands on a previously completed task, roll again. Record all of your work on a separate sheet.

<table>
<thead>
<tr>
<th>Explain It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>You have $10 to buy school supplies. Pencils cost $0.20 and notebooks cost $1.00. Explain how to find all of the possible combinations of pencils and notebooks you can buy.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sketch It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use the equation ( y = 2x + 3 ) to create four different inequalities. Sketch each inequality and label each sketch with the correct inequality.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connect It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of two real-world situations that would be graphed using a linear inequality. Briefly explain both situations. Create linear inequalities for both situations and list several possible solutions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compare and Contrast It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare and contrast real-world problems with linear equations and linear inequalities. For example, what indicators would you find in a problem that could help you decide whether to use an equation or an inequality.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fix It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>A student has proposed that when graphing an inequality with a less than sign you shade to the left of the line because the arrow ( (&lt;) ) points that direction. Is the student correct? Explain your reasoning with examples.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design a way to help your classmates remember the three steps for graphing linear inequalities. Some ideas: song, joke sequence, comic strip, letters and phrase (like PEMDAS), arm movements, etc.</td>
</tr>
</tbody>
</table>
Linear Inequalities- Version B

Directions: Cut on the light lines of the squares below, and fold on the dark lines to form a cube. Tape the cube together. Each side of the cube has directions for a task. You and an assigned partner will take turns rolling the cube. You must complete whatever task the cube lands on. You get one chance to roll again if you are not happy with the roll. If the cube lands on a previously completed task, roll again. Record all of your work on a separate sheet.

---

Explain It!
Explain what you would do to answer the following question.
Is (2, 7) a solution of 
y < 5x + 1 ?

Connect It!
Think of two real-world situations that would be graphed using a linear inequality. Briefly explain both situations. (You may use your book for ideas, but create your own situation.)

Compare and Contrast It!
Compare and contrast the graphs of linear equations and linear inequalities.

Sketch It!
Sketch four inequalities with the following features.
1. dashed border, shade up
2. solid border, shade up
3. solid border, shade down
4. dashed border, shade down
Label each sketch with the correct inequality sign.

Fix It!
Sam was graphing the linear inequality y > 2x and made one mistake. Find her mistake and explain to her how to fix it.
Points on the line: (0,0) (1,2) (2,4) (3,6)
Boundary: Solid
Shading: Upward

Design It!
Design a way to help your classmates remember which type of boundary lines and shading match which inequality signs.
Ideas: song, joke sequence, comic strip, letters and phrase (like PEMDAS), arm movements, etc.
### Systems of Linear Equations Performance Task

**Objective:** The goal is to determine the most profitable, yet feasible, fundraiser for the eighth grade graduation trip to Chicago.

You will use a system of linear equations to compare possible profits for various fundraisers. Based on your analysis, you will decide which is the most profitable, yet feasible, fundraiser. You will present this information in a report that includes rationale for your choice.

**Task:** As you know, we are going to Chicago in June! We want to hold a fundraiser to raise enough money to buy each student a one-hour unlimited gaming pass for ESPN Zone. The total amount we need to raise to purchase these game cards is $560. Your job is to figure out the most profitable, yet feasible, fundraiser. Check out the proposed fundraising ideas. Choose two fundraising options to investigate. The investigation must include calculation of each fundraiser’s potential profits by using a linear equation and corresponding graph. Then combine the individual linear equations into a system of linear equations. Use the system to compare the profit potential of both fundraisers. Next, analyze the feasibility of each and make a decision of which is the best. You will make a report based on your research and calculations. Volunteers will present their reports to the class. As a class, we will vote on which fundraiser we would like to do in order to raise money for those awesome game cards.

Fundraiser #1  Pop Can Drive- Profit: $0.10 per can  Cost: $20 Incentives

Things to consider: How long would we collect cans at school? Who would return the cans to a store for the money? We might need to provide incentives for bringing in cans, such as an ice cream party for the class in our school that collects the most. Incentives cost money that will come out of our profits. What is the potential total profit?

Fundraiser # 2   Candy Sale- Profit: $0.50 on each candy bar sold (assuming we buy them for $0.50 and sell them for $1.00)  Cost: One-time $40 Sam’s Club Fee

Things to consider: Could we sell candy bars at lunch on a long-term basis? Who would keep track of the money? What is the potential total profit?

Fundraiser # 3 Car Wash-
Profit: $5 per car  Cost: Supplies $25

Things to consider: Where would we hold the car wash? When would we hold the car wash? We would need adult chaperones, permission slips, rules, and a dress code.
Systems of Linear Equations Activity

Directions: Solve each system of linear equations using graphing, substitution, or elimination. Explain why you chose your solving method. You must use at least two of the three methods in this activity.

1.) \[ y = x + 1 \]
   \[ y = -\frac{5}{2}x + 8 \]

   Solution (   ,    ) What method did you use to solve? ________________
   Why did you choose to use this method? ________________________
   _________________________________________________________
   _________________________________________________________

2.) \[ y = 4x \]
   \[ 3x + y = 14 \]

   Solution (   ,    ) What method did you use to solve? ________________
   Why did you choose to use this method? ________________________
   _________________________________________________________
   _________________________________________________________
3.) \[8x + 5y = 42\]
\[3x - 5y = 2\]

Solution ( , ) What method did you use to solve? ________________
Why did you choose to use this method? ________________________
__________________________________________________________________________

4.) \[y = \frac{1}{2}x\]
\[y = 2x - 7\]

Solution ( , ) What method did you use to solve? ________________
Why did you choose to use this method? ________________________
__________________________________________________________________________


Name:________________

Systems of Linear Equations Assessment

Objective: I can write and solve linear systems.  Score: ___/3

1. Solve the following system of linear equations using graphing, substitution, or elimination.

   \[ x = x + 4 \]
   \[ 5x + 2y = 29 \]

   Solution (   ,    )

2. Solve the following system of linear equations using graphing, substitution, or elimination.

   \[ 6x - 3y = 9 \]
   \[ 4x - 3y = -1 \]

   Solution (   ,    )
The year is flying by and the 8th grade class trip to Chicago will be here before you know it! We need your help deciding which charter buses to hire to drive us to Chicago. We want to choose the best value.

3. **Write each option as a linear equation.** Doing this will create a system of linear equations.

4. **Solve the system** of linear equations using graphing, substitution, or elimination. Show your work! Answer the questions for # 4.

**Busing Options:**

Blue Jay Buses charges $50 per student and a $225 cleaning fee.

Voyager Buses charges $55 per student with no cleaning fee.

3. Blue Jay equation _______________

   Voyager equation___________________

4. Work Space-Solve the system:

Solution: \( x = \)

The cost for Blue Jay Buses and Voyager Buses is the same for _____ students.

Blue Jay Buses is cheaper for _______________ students.

Voyager Buses is cheaper for _______________ students.

If we have 60 students going on the Chicago trip, we should hire ____________.
Appendix D:
Exponents, Proportions, & Percents
Unit

Includes:

Unit Take Home Project (Real World Connections)

Lesson for Scientific Notation & Properties of Exponents (Real World Connections)

Math Wonder Activity (Active Learning & Real World Connections)
Unit Take Home Project

for Proportions, Percents, and Exponents

Directions: Complete one activity from each section.

Section I: Rates/Ratios/Proportions

Option #1 Go to the gas station with a family member who drives. Be a sweetie and pump the gas. While you’re at it, write down the amount of gasoline the vehicle took. Use the amount of gas and the number of miles (make sure to trip meter gets set each time after a fill up) to calculate the miles per gallon rate that the car gets. Before you fill up next time, use the miles the car has traveled (see trip meter or odometer) to predict how many gallons of gas you will need based on your last miles per gallon calculated rate. Obviously it varies depending on how the driver drives the car (think racecar driving versus a cautious Grandma), but see how close you can get. Record all your work and calculations (include details about the car and label all data) to turn in.

Option #2 Take five food items with price tags (or it’s also fine if you have the receipts or find the prices at the grocery store or online) and figure out the cost per ounce of each food. Then figure out how many cookies, crackers, slices of cheese, or whatever are in the package and figure out the cost for each cookie, cracker, etc. If you are using a bag of chips or something, you may have to guess how many are inside. Lastly, use a proportion to predict the cost of the item if you had 150% of the original amount of food inside. Record all your work and calculations (include details like exactly what kind of food and what size package) to turn in.

Option #3 Take your cell phone bill (or a family member’s or friend’s if you don’t have one) and use the monthly cost and the total number of minutes (talking, texting, & browsing) to figure out the cost per minute for the month. Even if your bill already has it broken down by cost per minute there are probably additional fees and surcharges. So, your job is to take the grand total (with all fees and surcharges) and divide it up amongst the total minutes to see how much each minute actually costs. After doing that, figure out how much your next monthly bill would be if you talked exactly two hours a day each day in the 30 day month at the same rate per minute you calculated. Show all work.
Section II: Percents

Option #1 When you go out to eat figure out the tip for your server (without your cell phone or a calculator). Round the total bill to the nearest dollar and then move the decimal point once to the left to get 10%. Double that to get 20%. Then find 15% of the bill. If you left $3.00 what percent would that be? How about $7.00? Include the total bill amount, the tip amount, and all your calculations for your project.

Option #2 When you buy an item estimate the sales tax before purchasing and justify the exact amount (it is listed on your bill so you already know what it is) with calculations when you get home. Michigan sales tax is 6%. To estimate you may want to use a rounded total and find 5% by finding 10% and cutting it in half. To find an exact answer use the exact total and move decimal point twice to the left for 1% then multiply by six. What if you bought the same item in Washington where the tax is 8%. Find the estimated and exact tax for the same purchase made in Washington. Record the item, item price, estimations, and exact sales tax calculations in your report.

Option #3 Keep track for a day (with a daily log- include times) how much time you spend sleeping, eating, watching tv or playing video games, doing homework, getting ready (includes bathing), and talking on the phone. Then figure out what percentage of the day you spending doing each of those things. Record the amounts of time that you do each of the activities and the calculations to find percentage.

Section III: Exponents

Option #1 Pick a fractional amount from your day (recipe or eating part of something) and write the amount as a power with an integer exponent. Describe the original amount and explain how you transferred it to a power.

Option #2 Convert five very large and/or small numbers that are part of your daily life into scientific notation. List the original numbers (and what they came from) and show the conversion process.

Option #3 Think of a situation that you have encountered outside of school that naturally uses exponents. Describe the situation and show how exponents are used.
Lesson Idea for Scientific Notation & Properties of Exponents

**Entrance Activity:** Write down the smallest and largest living things you can think of on the planet. Estimate their weights (in pounds).

**Introduction:** Students will be introduced to scientific notation by discussing their ideas about large and small living things.

**Lesson:** To teach about scientific notation, the distances between the planets will be used for examples. In scientific notation, a number is written as a product with a power of ten. The number must be converted to a single digit number with the remaining value behind the decimal point. For example, if two planets are 389,000,000 miles apart you could rewrite that amount in scientific notation as $3.89 \times 10^8$ miles apart.

The size of bacteria will also be used to illustrate how to convert from standard notation to scientific notation and the opposite. To convert from scientific notation, students will move the decimal point in the number the amount of times indicated by the exponent. The direction they move the decimal will depend on the sign of the exponent (positive exponent = move right, negative exponent = move left). For example, if a bacteria particle is $2.127 \times 10^{-3}$ mm long you would move the decimal point three times to the left to get the number 0.002127 mm.

**Practice:** Guided practice on writing and converting scientific notation will be done whole class and additional practice will be done in partners.

**Activity:** For real life connection, the national debt of the United States will be used for the activity. As a class, we will go online and look up the current national debt. Students will round that number to the trillions and write it in scientific notation. We will also look up the population of the United States and students will write the rounded population (in millions) in scientific notation. Lastly, we will do the same for the number of taxpayers in the United States.

Then, in partners, students will be asked to find the amount of money that each member of the population owes if we split the national debt equally. They will also find out how much each taxpayer would owe if the debt were split only amongst taxpayers. Students will write answers in both scientific and standard notation.

The partners will also answer the following question. If the government accumulates an average of 5 trillion in debt each year, how much debt will there be (don’t forget to add what we already have) when you turn 80? Use scientific notation and exponent multiplication rules to solve.

We will go over the activity and answers as a class. We will also discuss how scientific notation is useful for real world applications.
The Math Wonder- Proportion Activity

Congratulations! Your dream of becoming a famous movie star is coming true. You just got cast to play the lead role as the superhero in the new movie The Math Wonder Saves the Day. (Okay, so it might not be the best movie ever made.) You are preparing yourself for all that comes with being famous, and your first surprise is that they are making a Math Wonder action figure replica of you! The only problem is that the factory needs the dimensions of the action figure and since you are the Math Wonder it is your job to figure it out.

**Your task:** To find the measurements of your personalized action figure.

First- You need to first measure yourself and record the measurements on the left side.

<table>
<thead>
<tr>
<th>Real Life Math Wonder (YOU)</th>
<th>Math Wonder Action Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (inches) _______</td>
<td>Height (in) _______</td>
</tr>
<tr>
<td>Brain Size (Head Circumference- cm) _______</td>
<td>Brain Size (cm) ______</td>
</tr>
<tr>
<td>Arm Span (inches) _______</td>
<td>Arm Span (in) _______</td>
</tr>
<tr>
<td>Calculator Length (cm) _______</td>
<td>Calculator Length (cm) _____</td>
</tr>
</tbody>
</table>

Next- Pick a height for your action figure (somewhere between 5 and 10 inches) and put this number on the right side column above. Now use the action figure height and your real life dimensions to form proportions to find the rest of the missing action figure dimensions.

**Done?** Check your work with a partner and then get it approved by Mrs. Sanders. Then pick up a container of play dough and create your very own action figure! Make sure to use the measurements that you calculated in the activity.
Appendix E:

Polynomials Unit

Includes:

Tiered Unit Review Worksheet (Differentiation)
Practice Makes Perfect- Operations with Polynomials

Directions: There are three sections to this assignment. Section I focuses on adding and subtracting polynomials. Section II focuses on multiplying polynomials. Section III focuses on factoring polynomials. Please show your work for each problem.

SECTION I: Adding and Subtracting Polynomials.
1. Add the following polynomials using either the vertical or horizontal method.
   \[(11x^3 + x^2 + 8x) + (2x^3 + 7x + 4)\]

2. Subtract the following polynomials using either the vertical or horizontal method.
   \[(10x^3 - 3x^2 + 7) - (-12x^3 - 6x^2 + 5)\]

SECTION II: Multiplying Polynomials.
3. Multiply using the correct method: commutative, distributive, FOIL, or boxes.
   \[(x^2 + 2x + 4)(5x - 6)\]
4. Multiply using the correct method: commutative, distributive, FOIL, or boxes.
   \[ 7x(4x^3) \]

5. Multiply using the correct method: commutative, distributive, FOIL, or boxes.
   \[ (x - 3)(x - 6) \]

6. Multiply using the correct method: commutative, distributive, FOIL, or boxes.
   \[ 2xy(x^2 - 6y) \]

---

**SECTION III: Factoring Polynomials**

7. Find the GCF (Greatest Common Factor) of the following pair of monomials.
   \[ 8x^3 \text{ and } 20x \]
8. Factor the polynomial using GCF(leftovers).

\[ 15x^3 + 6x^2 - 12x^4 \]

9. Factor the trinomial using reverse FOIL.

\[ x^2 - 13x + 36 = (\quad)(\quad) \]

10. Factor the trinomial using reverse FOIL.

\[ 2x^2 + 10x + 12 = (\quad)(\quad) \]

11. Without using reverse FOIL, use the special binomial products to factor the following:

\[ x^2 + 10x + 25 = (\quad)(\quad) \]
\[ x^2 - 6x + 9 = (\quad)(\quad) \]
\[ x^2 - 49 = (\quad)(\quad) \]

12. Factor the polynomial using multiple methods.

\[ 4x^3 - 36x^2 + 72x \]
Practice Makes Perfect- Operations with Polynomials

Directions: There are three sections to this assignment. Section I focuses on adding and subtracting polynomials. Section II focuses on multiplying polynomials. Section III focuses on factoring polynomials. Please show your work for each problem.

SECTION I: Adding and Subtracting Polynomials.
1. Add the following polynomials using either the vertical or horizontal method.
   
   \[(11x^3 + x^2 + 3x) + (2x^3 + 7x + 4x)\]  Hint: Add the coefficients of the like terms.

2. Subtract the following polynomials using either the vertical or horizontal method.  Hint: Change the subtraction sign to addition and change all the signs in the second polynomial to compensate.
   
   \[(10x^3 + 9x^2 + 7) - (2x^3 - 6x^2 + 5)\]

SECTION II: Multiplying Polynomials.
   
   \[(x^2 + 2x + 4)(5x + 6)\]
4. Multiply using the commutative property.
\[7x(4x^3)\]

5. Multiply using FOIL.
\[(x + 3)(x + 6)\]

6. Multiply using the distributive property.
\[2xy(x^2 + 6y)\]

SECTION III: Factoring Polynomials

7. Find the GCF (Greatest Common Factor) of the following pair of monomials. Use a factor tree and then write prime factorization. The matches make the GCF.
\[8x^3 \text{ and } 20x\]

8. Factor the polynomial using GCF(leftovers). Remember to circle the matches and underline the leftovers. Matches make the GCF and leftovers are underlined.
\[15x^3 + 6x^2 + 12x\]
9. Factor the trinomial using reverse FOIL.
\[ x^2 + 13x + 36 = (\quad)(\quad) \]

10. Factor the trinomial using reverse FOIL.
\[ 2x^2 + 10x + 12 = (\quad)(\quad) \]

11. Without using reverse FOIL, use the special binomial products to factor the following:
\[ x^2 + 10x + 25 = (\quad)(\quad) \text{ Remember } a^2 + 2ab + b^2 \]
\[ x^2 - 6x + 9 = (\quad)(\quad) \text{ Remember } a^2 - 2ab + b^2 \]
\[ x^2 - 49 = (\quad)(\quad) \text{ Remember } a^2 - b^2 \]

12. Factor the polynomial. Steps: GCF(leftovers) then reverse FOIL the leftovers.
\[ 4x^3 - 36x^2 + 72x \]
Practice Makes Perfect- Operations with Polynomials

Directions: There are three sections to this assignment. Section I focuses on adding and subtracting polynomials. Section II focuses on multiplying polynomials. Section III focuses on factoring polynomials. Please show your work for each problem.

SECTION I: Adding and Subtracting Polynomials.
1. Add the following polynomials using either the vertical or horizontal method.
   \[(11x^3 + x^2 - 8x) + (2x^3 - 7x + 4)\]

2. Subtract the following polynomials using either the vertical or horizontal method.
   \[(10x^3 - 3x^2 + 7) - (-12x^3 - 6x^2 + 5x + 5)\]

SECTION II: Multiplying Polynomials.
3. Multiply.
   \[(2x^2 - 3x + 4)(5x - 6)\]
$7x^5(4x^3)(-2x)$

5. Multiply.
$(3x - 3)(x - 6)$

$2x^3y(x^2 - 6y)$

SECTION III: Factoring Polynomials

7. Find the GCF (Greatest Common Factor) of the following monomials.

$8x^3$ and $20x$ and $56x^4$

8. Factor the polynomial using GCF(leftovers).

$-15x^3 - 6x^2 - 12x^4$
9. Factor the trinomial using reverse FOIL.

\[ x^2 - 5x - 36 = (\quad )(\quad ) \]

10. Factor the trinomial using reverse FOIL.

\[ -2x^2 + 4x + 12 = (\quad )(\quad ) \]

11. Without using reverse FOIL, use the special binomial products to factor the following:

\[ 4x^2 + 20x + 25 = (\quad )(\quad ) \]
\[ x^2 - 6xy + 9y^2 = (\quad )(\quad ) \]
\[ 16x^2 - 49 = (\quad )(\quad ) \]

12. Factor the polynomial using multiple methods.

\[ 4x^3 - 36x^2 + 72x \]
Appendix F:

Functions Unit

Includes:

Formative Assessment (Real World Connections & Conceptual Understanding)

Lesson Plan- Functions (Conceptual Understanding)

Choice Activity- Functions (Differentiation)

Independent and Dependent Variables Activity (Real World Connections)

Formative Assessment: Continuous vs. Discrete (Conceptual Understanding)

Performance Assessment (Real World Connections)

Cubic Functions Activity (Differentiation & Conceptual Understanding)

Unknown Functions Activity (Active Learning)

Exponential Functions Activity (Differentiation)
FUNCTIONS: Money and Cell Phones

Objective:
Throughout this unit we have been learning all about functions. Over the past several days, we have worked on writing and evaluating functions. This assessment will help us see whether you have mastered these two learning objectives. It is important to know if you need more practice on these targets because you must be able to write and evaluate functions before you can graph functions. You guessed it…graphing functions is where we are heading next in this unit.

Directions:
♦ Please put your name, class period, and the date in the upper right hand corner of each test page.
♦ There are two questions on this assessment. Each question has its own set of directions and its own scoring rubric. Make sure that you read the directions for each question carefully. Check out each scoring rubric before answering the question so you know what I expect in your answer. The scoring rubrics are located at the end of the assessment.
♦ Please write your answers in the space provided to you after each question. If you need more room for your answer, you may use a separate piece of paper and staple it to the assessment.
♦ You will have until the end of this class period (approximately twenty minutes) to complete this assessment. If you need more time, please see me to set up a time for you to come in and finish.
♦ If you have a question, please raise your hand and I will come to you at your seat.
♦ When you are finished, please place this assessment in the turn in tray for your class period. Then quietly return to your seat and work on your tic-tac-toe board for this unit.

Please take your time and do your best work. I am excited to see the excellent answers and great explanations you come up with!
Questions 1: Prompt

In this unit we have become familiar with functions and the ways they can be displayed. While it is true that functions are usually written as equations, sometimes functions are written as word problems or shown as raw data. For this question, you will be analyzing a data table in order to write a function (in equation form) for the data. You must write the correct equation in function notation and provide a detailed explanation of how you used the data to figure out your answer. You must also describe the relationship between Zeke’s pay and the function equation by identifying what each number and variable represent. Looking for a relationship between Zeke’s hours and his earnings might be a good place to start.

Function for Zeke’s Pay: _____________________

How did you figure out your answer? What part of Zeke’s pay does each variable and number represent?

<table>
<thead>
<tr>
<th>Zeke’s Pay Over Six Weeks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Week</td>
<td>Hours Worked</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>14.5</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>37.5</td>
</tr>
<tr>
<td>6</td>
<td>26</td>
</tr>
</tbody>
</table>
Question 2: Scenario

We recently learned how to evaluate functions by plugging in input values and solving. You will find that this knowledge can be extremely useful in everyday situations. Imagine your friend just got a cell phone (woo-hoo!) and needs your help deciding which calling plan to sign up for. He is looking for a plan that lets him talk to you and his other friends each month for a low price. There are two plans that he can get with his phone. The first plan costs ten dollars a month plus ten cents per minute that you talk. The second plan costs twelve cents per minute without an additional monthly charge. These two plans are written in function notation in the tables below. Evaluate the function for each plan using input values of 0, 225, 500, 850, and 1,000 minutes. Then analyze the data and decide which calling plan your friend should get if he will talk 500 minutes or less per month. In the space provided, tell which plan your friend should choose and explain why. Make sure to use data to support your decision.

<table>
<thead>
<tr>
<th>Calling Plan # 1</th>
<th>Calling Plan # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function:</strong> f(x) = .1x + 10</td>
<td><strong>Function:</strong> f(x) = .12x</td>
</tr>
<tr>
<td><strong>Number of Minutes</strong></td>
<td><strong>Number of Minutes</strong></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>225</td>
<td>225</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>850</td>
<td>850</td>
</tr>
<tr>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Which calling plan should your friend choose? ____________

Explain why.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
**ANSWER KEY**

**Question #1:**

Function for Zeke’s Pay: \( f(x) = 9.5x \)

How did you figure out your answer? Answers may vary. However, students must mention something about determining how much Zeke made per hour by using the total pay and the corresponding number of hours.

What part of Zeke’s pay does each variable and number represent?

- \( f(x) \) is the output, which is Zeke’s total pay per week.
- \( x \) is in the input, which is the number of hours Zeke works in a week.
- 9.5 is the amount of money Zeke gets paid per hour.

**Question #2:**

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$10.00</td>
</tr>
<tr>
<td>225</td>
<td>$32.50</td>
</tr>
<tr>
<td>500</td>
<td>$60.00</td>
</tr>
<tr>
<td>850</td>
<td>$95.00</td>
</tr>
<tr>
<td>1,000</td>
<td>$110.00</td>
</tr>
</tbody>
</table>

Calling Plan #1

Function: \( f(x) = .1x + 10 \)

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
</tr>
<tr>
<td>225</td>
<td>$27.00</td>
</tr>
<tr>
<td>500</td>
<td>$60.00</td>
</tr>
<tr>
<td>850</td>
<td>$102.00</td>
</tr>
<tr>
<td>1,000</td>
<td>$120.00</td>
</tr>
</tbody>
</table>

Calling Plan #2

Function: \( f(x) = .12x \)

Which calling plan should your friend choose? **Calling Plan #2**

Explain why. The data tables show that the two calling plans cost the same amount of money ($60) for 500 minutes. Calling Plan #1 is cheaper when you talk for more than 500 minutes. For example, to talk 850 minutes with Calling Plan #1 costs $95 and it would cost $102 with Calling Plan #2. However, Calling Plan #2 is cheaper for less than 500 minutes. For example, 0 minutes costs $0 with Calling Plan #2, but it costs $10 with Calling Plan #1. If my friend plans to talk 500 minutes or less, he will save money by using Calling Plan #2.
General Lesson Idea for Conceptual Understanding: What’s Your Function?

I can explain how a function differs from a relation that is not a function.
I can describe the relationship between the x and y values in a function.

Idea: In order to get more familiar with the definition and properties of a function we need to dissect one! 😊

First: Students will turn to the glossary in the back of your book and copy down the definition they have for “function”.
Next: Students put the definition from above in their own words in their notes.
Last: Partner discussion about definition of a function followed by whole group discussion.

Instruction: A function is like a machine that does the same thing every time. So… can you input the same thing twice and have something different come out the second time? Of course not! If you put cookie dough in the oven today it makes cookies. If you put the exact same dough in the oven tomorrow, it will not make a casserole. Use several real life examples of how the same input (a specific person) cannot have multiple outputs (their age). Cupcakes are also a good analogy for functions. Functions are like cupcakes because each input must have only one output just like people should only take one cupcake. We all know that it is wrong to lick a cupcake and then set it back down on the tray and take a different cupcake. It is the same idea with functions. It is wrong for an input to have one output and then try to have a second (different) output. More than just being wrong, it is mathematically impossible! Then use numeric examples of a function “machine” that calculates numbers and cannot calculate differently for the same input, but can calculate the same output for two different inputs if it involves squaring or absolute value.

Partners fill in the blanks to finish these sentences.

In a function, there cannot be repeat ____ values (because that means that an input has more than one output). However, it does not matter if the ____ values repeat.

Bottom Line: The ____ value depends on the ____ value, so the same ____ value cannot produce more than one ____ value!

What does it look like? Go over the four ways that a function can be displayed. Then provide an example of each. Have students copy down the table and add their own example.

- Equation
- Set of ordered pairs
- Table or Mapping Diagram
- Graph
Choice Activity: Are You a Function?
Objective: I can identify a function.

Directions: To be able to identify a function, you need to know all about the vertical line test. What better way to get familiar with it then to explain it to someone else. You need to choose one of the options below. For all the options, you must thoroughly explain what the vertical line test is and how and why it works to identify functions. Also, you need to include at least one example of a function and one example of data that is not a function. Be creative if you want! If you choose an option like making a television commercial, I need to see your script.

Choose one of the following:
♦ Write a letter to a friend
♦ Write a song or rap
♦ Make a television commercial
♦ Write a script for a play
♦ Make a poster (must still have the explanation)
♦ Write the dictionary entry (using your OWN words)
♦ Write a newspaper article or interview
♦ Do you have a different idea? If so, come talk to me.
Activity: Real Connections to Independent and Dependent Variables

Objectives: I can identify the independent and dependent variables of a function.
I can describe the relationship between the x and y values in a function.

Directions: Look up the words independent and dependent in your textbook glossary and in the dictionary. Then write a definition for both terms in your own words. Next, work through the practice problems by filling in the blanks. Last, create two of your own functions and see if a friend/classmate can figure out the independent and dependent variables. 😊

Independent means _________________________________.
Dependent means _________________________________.

Example: An all you can eat buffet costs $11.99 per person.
The total cost depends on how many people eat, so total cost is dependent.
Independent variable (input): # of people eating Dependent variable (output): total cost

1. Garden mulch costs $4.99 per bag.
   __________ depends on ___________, so ___________ is dependent.
   Independent variable: Dependent variable:

2. Brielle runs at a pace of five miles per hour.
   __________ depends on ___________, so ___________ is dependent.
   Independent variable: Dependent variable:

3. Pete takes two hours to wash and wax a car.
   __________ depends on ___________, so ___________ is dependent.
   Independent variable: Dependent variable:

4. Mia’s car gets 25 miles per gallon.
   __________ depends on ___________, so ___________ is dependent.
   Independent variable: Dependent variable:

5. Water is leaking out of a bucket at a rate of one gallon per minute.
   __________ depends on ___________, so ___________ is dependent.
   Independent variable: Dependent variable:

Your Turn: Make two of your own and quiz a friend/classmate.
**Formative Assessment: Continuous vs Discrete Functions**

Objective: I can classify a function as continuous or discrete.

*What you need to know:*
To classify a graph as continuous or discrete, you must look at the graph and examine the points. Continuous means connected, so a continuous function is one whose points are connected to make a line. A discrete function, on the other hand, has separate points.

To classify a function written in word form, you must think about the input values. A common reason that a function is discrete is that you must have whole number input values. Example: *Jasmine is buying CDs for ten dollars each.* She can not buy part of a CD. So, the input values must be whole numbers. This is a discrete function. Some ordered pairs on the graph would be (0 CDs, $0) (1 CD, $10) (2 CDs, $20). These points would not be connected because you cannot buy part of a CD. You could not have the ordered pair (2.5 CDs, $25).

*Activity Directions:* You are going to write three continuous functions and three discrete functions (in word form). Then graph each function using at least four input values. An example of a discrete function is written above. A continuous function might be something like *Erin runs three miles an hour.* This is continuous because you can run for part of an hour. So, in .5 hours Erin runs 1.5 miles.

*Continuous Function #1:*

______________________________
______________________________

Ordered Pairs \{ ( , ) ( , ) ( , ) ( , ) \}

*Continuous Function #2:*

______________________________
______________________________

Ordered Pairs \{ ( , ) ( , ) ( , ) ( , ) \}
Continuous Function #3: ___________________
______________________________________
______________________________________
Ordered Pairs \{ ( , ) ( , ) ( , ) ( , ) \}

Discrete Function #1: ___________________
______________________________________
______________________________________
Ordered Pairs \{ ( , ) ( , ) ( , ) ( , ) \}

Discrete Function #2: ___________________
______________________________________
______________________________________
Ordered Pairs \{ ( , ) ( , ) ( , ) ( , ) \}

Discrete Function #3: ___________________
______________________________________
______________________________________
Ordered Pairs \{ ( , ) ( , ) ( , ) ( , ) \}
Objective:
In this unit we have been learning all about functions. Most recently, we learned how to write and evaluate functions. Over the next several days, we will learn how to graph and compare functions. While we are learning these skills, we will also be practicing them by applying them to a real life situation. The situation is set up as a three day assessment that will help us see whether you have mastered the ability to write, evaluate, graph, and compare functions. It is important to know if you need more practice on these targets because we have reached the end of the unit and soon we will be taking the summative Performance Task Assessment.

The Task:
Throughout this unit we have seen how functions occur frequently in everyday life. The functions in real life are no different than those in our textbook. We can write, evaluate, graph, and compare them. That is exactly what you will need to do in order to get yourself a new set of wheels. That’s right… A CAR!

Most of you are going to be taking driver’s training soon, and it is never too soon to start thinking about your first car. You have been saving up money for years, and now you just need to find a good deal that you can afford. That is where this assessment comes in. You will be choosing two cars and writing functions to describe their total cost. You will then evaluate how much each car will cost to own over the next few years. Then you will make a graph of each car’s cost and another graph that shows both together. Finally, you will be comparing the cost of the two cars and using your data to decide which of the two cars is the best deal.

Final Product:
Your final product will be a letter to someone at home (parent, grandparent, guardian, etc) who needs to drive you to the dealership and help sign the papers with you. Your letter should convince the reader that you have done enough research and calculations to make a good decision. It must contain data tables and graphs to prove that the car you have decided to buy is a good deal. You will also need to write an explanation of the calculations you have done, the conclusions of the data, and your opinion on why the car you have chosen is worth buying.
**Scoring Criteria:**
You will be given a score for each section of the assessment according to a scoring rubric. Each score will be out of five possible points. You will be scored on the following:

- your ability to write functions using a given story problem.
- your ability to evaluate functions with specified input values.
- your ability to graph functions correctly.
- your ability to compare two functions.

**Directions:**

- Please put your name, class period, and the date in the upper right hand corner of each page of the assessment.
- There are four different sections on this assessment. Each section has its own set of directions and its own scoring rubric. Make sure that you read the directions for each section carefully. Check out each scoring rubric before answering the question so you know what I expect in your answer. The scoring rubrics are located at the end of the assessment.
- Please write your answers and show your work in the space provided to you in each section. If you need more room for your answer, you may use a separate piece of paper and staple it to the assessment.
- For the next three days, you will have approximately thirty minutes of each class period to work on the appropriate section(s) of the assessment. If you need more time, please see me to set up a time for you to come in and finish.
- If you have a question about the directions or the section you are working on, please raise your hand and I will come to you at your seat.
- When you are finished with the section(s) for each day, please place your paper in the turn in tray for your class period. Then quietly return to your seat and work on your tic-tac-toe board for this unit. Please note: You will get your assessment back with my feedback the following day.

*Please take your time and do your best work. I know that you will do a fantastic job!* ☺
Day 1: I can write a function in function notation using given information and/or data. I can evaluate a function.

Today, you will be given a flyer from a used car dealership. The dealership is offering a special deal on five cars. You will choose two vehicles that you are interested in purchasing and write a function that represents the cost of owning each vehicle (cost to buy the car + gas money). Once you have written a function for each car, you will be evaluating both functions for a given number of miles. Basically, you will be figuring out how much the car will cost you over the next few years since you will need to pay for gas each year. You will find more specific directions in Sections 1 & 2. When you finish each section (1 & 2) you must do a self-evaluation. You will use the scoring rubric provided to score your work. The scoring rubric is located on the last page of the assessment. Then you will turn in your paper to receive feedback from me as to whether you are ready to continue on to Day 2.

Day 2: I can graph a function.

Today, you will use the data that you calculated yesterday (in your two data tables) to make a graph of each car’s cost. You will also be making a combined graph that features both cars. These graphs are going to give you visual evidence of how much each car costs to own. You will find more specific directions in Section 3. When you finish this section you must do a self-evaluation. You will use the scoring rubric provided to score your work. Then you will turn in your paper to receive feedback from me as to whether you are ready to continue on to Day 3.

Day 3: I can compare functions.

Today, you will be using your data tables and your graphs to compare the functions for the two cars. You will use your functions to analyze each car’s starting cost, cost from gas, and total cost. You will then do a comparison to determine which car is a better deal. Once you decide which car is a better deal, you will write a letter to someone at home (who is over 18 years old) that will help you buy the car. Remember, the goal of all this hard work is to get you behind the wheel! You will find more specific directions in Section 4. When you finish this section you must do a self-evaluation. You will use the scoring rubric provided to score your work. Then you will turn in your assessment packet to receive feedback from me as to whether your task is complete.

Note: Every day you must bring the entire assessment packet with you to class!
Name_________________________Period_________________________Date_________________________

**Day 1: Writing and Evaluating Functions**

The closer you get to turning sixteen, the more you probably think about driving. How much would you love to have your own car? Well, do something about it! A flyer just came in the mail today from a used car dealership. You have worked hard to save up some money, but you want to make sure that the car you buy is a good deal. Take a look at the flyer below and pick two cars you like. (Remember you are going to end up comparing these cars to determine which to buy.)

### HOT DEALS! COOL WHEELS!
**HERE AT KARL’S CARS WE WANT TO GET YOU DRIVING!**

*At Karl’s Cars, we do more than just show you the year, the price, and the miles per gallon of our cars. We break it down to tell you exactly how much it will cost to drive your car. Using $3.00 per gallon, we have calculated how much it costs to drive one mile in each of our cars. We hope this helps you decide which car is right for you. Now let’s go shopping!*  

<table>
<thead>
<tr>
<th>Make/Model</th>
<th>Year/Miles Per Gallon/Color</th>
<th>Cost to Drive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeep Grand Cherokee</td>
<td>1996 / 15 mpg / White</td>
<td>$5,000 to buy + $0.20 per mile to drive</td>
</tr>
<tr>
<td>Honda Civic</td>
<td>2002 / 30 mpg / Blue</td>
<td>$8,000 to buy + $0.10 per mile to drive</td>
</tr>
<tr>
<td>Chevy Malibu</td>
<td>2001 / 20 mpg / Black</td>
<td>$7,500 to buy + $0.15 per mile to drive</td>
</tr>
<tr>
<td>Toyota Prius</td>
<td>2000 / 40 mpg / Silver</td>
<td>$10,000 to buy + $0.075 per mile to drive</td>
</tr>
<tr>
<td>Ford F150</td>
<td>1998 / 10 mpg / Red</td>
<td>$6,000 to buy + $0.30 per mile to drive</td>
</tr>
</tbody>
</table>
HERE WE GO… Day 1 / Section # 1

It is time to choose two cars you would be interested in purchasing from the flyer above. You will write down your choices on the lines below. There is also a line provided for you to write a function for each car’s cost. You need to use the information from the flyer to help you write your functions. Remember, this function will represent the total cost to drive the car. Hint: The “Cost to Drive” section of the flyer has the information you will need to write your functions. Please use function notation.

PICK TWO:

Car # 1: _____________________

Cost to buy the car: $_______
Cost to drive the car per mile: $_______

Function for this car’s cost: __________________________

Car # 2: _____________________

Cost to buy the car: $_______
Cost to drive the car per mile: $_______

Function for this car’s cost: __________________________

Self-Evaluation:
Now it is time to think about how you did on Section # 1. Did you prove that you can write functions in function notation correctly? Please use the scoring rubric for Section # 1 (located on the last page of the assessment packet) to score your work.

I think that I deserve a score of ______ on Section # 1 because…

_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
________________________________________________________________________
You may not have realized this but cars are expensive! Buying the car is only part of the total cost for the vehicle. Once you own the vehicle you have to pay for the gas to drive it! Some cars can drive more miles per gallon of gasoline than others, and the more miles you get per gallon means the less money you spend on gas. You do not have to worry about calculating the miles per gallon of the two cars you chose because the dealership did the math for you and figured out the gasoline cost per mile of driving for each car. All you have to do is plug in the number of miles you plan to drive to figure out your cost in gasoline. The average driver drives approximately 12,000 miles per year, but we will say that you will drive 10,000 miles per year with your new car. Now it is time to figure out how much (total money) each of the two cars you chose would cost over the next five years if you drove 10,000 miles per year. Use the mileage as your input.

**Evaluate your functions:**

<table>
<thead>
<tr>
<th>Car # 1: _______________</th>
<th>Function:________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of years you own the car</td>
<td>Total Number of Miles Driven (input)</td>
</tr>
<tr>
<td>1</td>
<td>10,000</td>
</tr>
<tr>
<td>2</td>
<td>20,000</td>
</tr>
<tr>
<td>3</td>
<td>30,000</td>
</tr>
<tr>
<td>4</td>
<td>40,000</td>
</tr>
<tr>
<td>5</td>
<td>50,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Car # 2: _______________</th>
<th>Function:________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of years you own the car</td>
<td>Total Number of Miles Driven (input)</td>
</tr>
<tr>
<td>1</td>
<td>10,000</td>
</tr>
<tr>
<td>2</td>
<td>20,000</td>
</tr>
<tr>
<td>3</td>
<td>30,000</td>
</tr>
<tr>
<td>4</td>
<td>40,000</td>
</tr>
<tr>
<td>5</td>
<td>50,000</td>
</tr>
</tbody>
</table>
**Self-Evaluation:**
Now it is time to think about how you did on Section # 2. Did you prove that you can evaluate functions correctly? Please use the scoring rubric for Section # 2 (located on the last page of the assessment packet) to score your work.

I think that I deserve a score of ______ on Section # 2 because…

_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

____________________________________

Congratulations, you are finished with Day # 1 of the assessment! However, there are a few things you need to do before you can turn in your paper. Please reread both sections and examine each of your answers. Then (if you have not already done this) check out the scoring rubrics for each question and use them to score your answers.

Once you have checked and scored your work, you may turn in your assessment packet to the turn in tray for your hour. When you get your packet tomorrow, you will need to turn back to this page to see my feedback. One of the options below will be checked so that you know what to do tomorrow.

______ Awesome! You are ready to move on to Day 2: Section # 3

______ Oops! It looks like you might have made a silly mistake or two. Please go back and check over your work before continuing on to Day 2: Section # 3. If you can not find your mistake(s) please ask a neighbor to help or raise your hand for my assistance.

______ Wait! There seems to be a problem with one or both of your sections from Day 1. Please raise your hand (when you see that Mrs. Sanders is available) so that we can go over your work. It might take a few minutes for me to get to you, so please start looking over your work from Day # 1. Feel free to look ahead to Day # 2. Don’t worry, we will take care of the problem and get you going again in no time! 😊
GRAPH IT … Day 2 / Section # 3

Looking at the table you made in Section # 2 is a great way to compare the cost of both cars. However, it would also be helpful to see the data on a graph because visual data can be more persuasive and convincing than numerical data. A line graph would do a nice job showing the cost of each car for the first five years. You are going to be making three graphs today. One for each car’s cost and then one combined graph that features both cars’ costs. Remember that the points for your graphs will come from your data tables (in Section # 2). Use your knowledge of inputs and outputs to make ordered pairs from the data. You will have five ordered pairs for each car. Make sure to label your graph (the x-axis, y-axis, and the intervals on x-axis and y-axis). You may use graph paper if you prefer, but you must staple the paper to this page in the assessment packet.

Graph for Car # 1

Cost to Own Car # 1

![Graph for Car # 1](image-url)
Graph for Car # 2

Cost to Own Car # 2
Graph for Both Cars: Please use two different colors on this graph so that it is obvious which points belong to which car. 😊

Cost Comparison for Car # 1 and Car # 2

Car # 1 Color: __________
Car # 2 Color: __________
Self-Evaluation:
Now it is time to think about how you did on Section # 3. Did you prove that you can graph functions correctly? Please use the scoring rubric for Section # 3 (located on the last page of the assessment packet) to score your work.

I think that I deserve a score of ______ on Section # 3 because…
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

Congratulations, you are finished with Day # 2 of the assessment! However, there are a few things you need to do before you can turn in your paper. Please reread Section # 3 and examine each of your graphs. Then (if you have not already done this) check out the scoring rubrics for Section # 3 and use it to score your graphs.

Once you have checked and scored your work, you may turn in your assessment packet to the turn in tray for your hour. When you get your packet tomorrow, you will need to turn back to this page to see my feedback. One of the options below will be checked so that you know what to do tomorrow.

______ Awesome! You are ready to move on to Day 3: Section # 4

______ Oops! It looks like you might have made a silly mistake or two. Please go back and check over your work before continuing on to Day 3: Section # 4. If you can not find your mistake(s) please ask a neighbor to help or raise your hand for my assistance.

______ Wait! There seems to be a problem with Section # 3. Please raise your hand (when you see that Mrs. Sanders is available) so that we can go over your work. It might take a few minutes for me to get to you, so please start looking over your work from Section # 3. Feel free to look ahead to Day # 3. Don’t worry, we will take care of the problem and get you going again in no time! ☺
TIME TO DECIDE … Day 3 / Section # 4

The time has come to compare the cost of the two cars and make a decision about which one to buy. In order to compare the costs, you are going to look back at your function equations (Section # 1), your data table (Section # 2), and your graphs (Section # 3). First, I will ask you a few questions to get you thinking about how the cars’ costs are alike and different. Then you need to write a short paragraph where you compare the cars’ overall costs. After doing the comparison, you will decide what car to buy. Please write your choice on the line provided. Last, you need to write a letter to someone at home (parent, grandparent, older brother/sister, guardian, etc) who is over 18 years old. You will need their help to sign the papers and buy the car. They will only help you buy the car if they think you are making a good choice, so you must use your letter to convince them your car is a good deal. You must support your choice with data from the previous three sections. You may actually cut out any data tables and graphs you need from the assessment packet and paste them onto the letter. (Note: Do not throw anything away. All tables and graphs need to be somewhere – either in the original section or pasted onto the letter – so that I can score them with the rubric!)

Dare to Compare:

How does the cost to buy car # 1 compare to the cost to buy car # 2?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

How does the cost to drive car # 1 compare to the cost to drive car # 2?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

How does the graph for car # 1 look different from the graph of car # 2?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
Summarize:

How does the total cost to own car # 1 compare to the cost to own car # 2? (Talk about what happens to the costs over the next five years. Use your answers from the first three questions.)

_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

Make a Decision:

I have chosen to buy car # _____ , which is a ______________________(make/model of the car).

Write a Letter:

Please use a separate piece of paper to write your letter. Remember you must cut out and glue/staple on any evidence (tables & graphs) you need to prove you have chosen a good car. Staple your completed letter to the back of the assessment packet.

Do not forget to do the self-assessment for section # 4.
Self-Evaluation:
Now it is time to think about how you did on Section # 4. Did you prove that you can compare two functions? Please use the scoring rubric for Section # 4 (located on the last page of the assessment packet) to score your work.

I think that I deserve a score of ______ on Section # 4 because…
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
____________________________________

Congratulations, you are finished with Day # 3 of the assessment! However, there are a few things you need to do before you can turn in your paper. Please reread Section # 4 and examine your work. Then (if you have not already done this) check out the scoring rubrics for Section # 4 and use it to score yourself.

Once you have checked and scored your work, you may turn in your assessment packet to the turn in tray for your hour. When you get your packet back, you will need to turn back to this page to see my feedback. One of the options below will be checked so that you know what to do tomorrow.

______ Awesome! You are done with this assessment. ☺

______ Oops! It looks like you might have made a silly mistake or two. Please go back and check over your work for section # 4. When you have found and corrected your silly mistake(s) you are done with this assessment! ☺. If you can not find your mistake(s) please ask a neighbor to help or raise your hand for my assistance.

______ Wait! There seems to be a problem with Section # 4. Please come in before school, at lunch, or after school in the next few days so that we can go over your work. Don’t worry, we will take care of the problem and you will finish this assessment successfully in no time! ☺
**Performance Task Scoring Rubric**

<table>
<thead>
<tr>
<th>Section # 1</th>
<th>Learning Target: I can write a function in function notation using given information and/or data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>I wrote two correct functions in function notation.</td>
</tr>
<tr>
<td>3</td>
<td>I wrote two correct functions, but they were not in function notation. OR I wrote one correct function, but the second function had a minor error. Both were in function notation.</td>
</tr>
<tr>
<td>1</td>
<td>I wrote two functions both containing minor or major errors. AND I did not write either function in function notation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section # 2</th>
<th>Learning Target: I can evaluate a function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>I evaluated both functions correctly for all of the given input values.</td>
</tr>
<tr>
<td>3</td>
<td>I evaluated one function correctly for all of the given input values, but I incorrectly evaluated the other function. OR I evaluated only some of the input values correctly for both functions.</td>
</tr>
<tr>
<td>1</td>
<td>I evaluated the majority of the given input values incorrectly for both functions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section # 3</th>
<th>Learning Target: I can graph a function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>I graphed five ordered pairs correctly for both functions.</td>
</tr>
<tr>
<td>3</td>
<td>I graphed five ordered pairs correctly for one function, but I graphed most of the ordered pairs incorrectly for the other function. OR I graphed only some of the ordered pairs correctly for both functions.</td>
</tr>
<tr>
<td>1</td>
<td>I graphed most of the ordered pairs incorrectly for both functions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section # 4</th>
<th>Learning Target: I can compare functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>I compared both functions by correctly identifying the differences in starting cost, gasoline cost, and total cost of ownership.</td>
</tr>
<tr>
<td>3</td>
<td>I compared both functions by correctly identifying the differences in two of the following: - starting cost - gasoline cost - total cost of ownership</td>
</tr>
<tr>
<td>1</td>
<td>I compared both functions by correctly identifying the differences in one or none of the following: - starting cost - gasoline cost - total cost of ownership</td>
</tr>
</tbody>
</table>
Activity for Exploring Cubic Functions

Grade Level: 8th     Subject: Algebra
Concept: Cubic Functions
Size of group: Individual or Small Group
Duration: 60 minutes

Goals/Objectives:

I can identify cubic functions from tables, graphs, and equations.
I can represent cubic functions in tables, graphs, and equations.

Algebra GLCE Standards Addressed:

A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inversely proportional relationships (y = k/x); cubics (y = ax³); roots (y = √x ); and exponentials (y = a^x, a > 0); using tables, graphs, and equations.

Materials:

• Graphing calculators (one per student)
• Four Quadrant Graph Paper (two per student)
• Notebook paper (two per student)

PROCEDURE:

• Opening prompt: “What types of functions have we learned about this year so far?”
• Read the directions for the following sheet with student(s) before allowing student(s) to work.

Functions: Linear, Quadratic, and Cubic

Analytical Section

As you know from our work in chapters five and nine, you can identify a function from an equation, table, or graph. We learned that linear functions have common first differences in y-values and quadratic functions have common second differences in y-values. This might lead us to logically conclude that cubic functions have ________________________________.

The following activity allows you to spend some time exploring cubic functions and discovering whether the logical conclusion above is correct.
Directions: Use the equations provided to create a table of ordered pairs and a graph for each parent function. There is one linear, quadratic, and cubic function. Use the standard domain and you may graph on a separate sheet of graph paper. You may check your graphs with a graphing calculator after they are completed.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Quadratic</th>
<th>Cubic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x$</td>
<td>$y = x^2$</td>
<td>$y = x^3$</td>
</tr>
</tbody>
</table>

Next, use the tables above to confirm that the linear function has common first differences and the quadratic function has common second differences. Then use the cubic table to analyze whether the logical assumption you made in the introduction is correct. Write your analysis here:

Now, investigate whether the original logical assumption concerning cubic functions is supported or unsupported by the graphs you made. Write a brief response here:

Creative Section

Directions: Come up with your own original cubic function (make sure it fulfills the requirements of being a function). Write the equation, make a table of ordered pairs,
and graph it on a separate piece of graph paper. Then choose from the two options below.

Equation: \( y = \)  

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Two Options:
1.) On the graph, turn your function into a common living or nonliving thing. The shape of the cubic function must be still recognizable and emphasized in the drawing when you are done. Write a brief rationale about how you decided what to draw.
2.) Imagine that the cubic function is the design of a new roller coaster you are riding. Describe your ride in vivid detail so that someone who didn’t know what a cubic function looks like could visualize what you are experiencing. Then compare it to riding a linear or quadratic coaster.

Practical Section

Directions: Answer the following questions.
1.) Describe a real world problem or situation that could be modeled using a cubic function. (Remember that real world problems often have limited domains.)

2.) Do you think that linear, quadratic, or cubic functions would be used most in the “real-world” to model situations in industries like business, banking, and architecture? Support your answer.

EVALUATION OF LESSON EFFECTIVENESS

Exit Ticket:
1.) How can you identify a cubic function?
2.) Do you think that logic always works in math class? Why or why not? You may reference what you discovered in this activity if you would like.
Activity for Exploring Unknown Functions

Grade Level: 8th
Subject: Algebra
Concept: Unknown functions
Size of group: Individual or Small Group
Duration: 60 minutes

Goals/Objectives:

I can use my knowledge of functions to design a “new” function.
I can analyze a “new” function.

Algebra GLCE Standards Addressed:

A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inversely proportional relationships (y = k/x); cubics (y = ax³); roots (y = √x ); and exponentials (y = aˣ, a > 0); using tables, graphs, and equations.

Materials:

• Graphing calculators (one per student)
• Four Quadrant Graph Paper (two per student)
• Notebook paper (two per student)

PROCEDURE:

• Teacher poses the questions
  o “Do you suppose we have learned about all types of mathematical functions?” “Are there other mathematical operations we could apply to the domain that would result in an equation that is a proper function?”
• Teacher then describes the challenge, which is as follows:
  o Design a new function. Write an equation, make a table of ordered pairs, graph it, and do anything else that will help you learn about it. (Graphing calculators are available.)
  o Compare and contrast the new function to one we have learned about in class. Compare the equations, table, graph, and anything else you can compare.
  o Analyze whether what you designed fits the actual requirements of a function (from Ch. 4).
  o Join together with another individual (or group) and go over your design, your comparison, and your analysis.
  o As a pair (or whole group) judge which design is most likely to be featured in chapter 11 or 12 in our book (which cover the remaining simple functions). Justify your answer.
**EVALUATION OF LESSON EFFECTIVENESS**

- Rubric to be filled out by student.

<table>
<thead>
<tr>
<th>Directions: Rate yourself honestly using the following scale.</th>
<th>Name: ____________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Disagree 1</td>
<td>Disagree 2</td>
</tr>
<tr>
<td>My function contains an input value (x) and a math operation (exponents, radicals, inverse, etc) uncommon to linear or quadratic functions.</td>
<td></td>
</tr>
<tr>
<td>I thought about the characteristics of a function as I was making my design.</td>
<td></td>
</tr>
<tr>
<td>I put forth 100% effort into this activity.</td>
<td></td>
</tr>
</tbody>
</table>

Free Response:
Respond to the applicable question.

If your function is not a “real” function, will you still be proud of your effort? Why or why not?

If your function is a real function but is not original (maybe it’s in our textbook) will you still be proud of your effort? Why or why not?
Activity for Exponential Functions

Grade Level: 8th  Subject: Algebra
Concept: Exponential Functions  Size of group: Individual  Duration: 60 minutes

Goals/Objectives:
I can identify exponential functions in various settings.
I can relate exponential functions to concepts within the real world.

Algebra GLCE Standards Addressed:
A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inversely proportional relationships (y = k/x); cubics (y = ax^3); roots (y = √x ); and exponentials (y = a^x, a > 0); using tables, graphs, and equations.

Materials:
• Computer
• Math notebook
• Graph paper
• Five different colored pencils

PROCEDURE:

Logical: Experiment with exponential equations to see what transformations are possible and how changing the equation will transform the graph. Write at least five exponential equations and graph them on the same graph in different colors. Try to have your five functions be as different as possible.

Verbal: Take the role of a news reporter, commercial writer, playwright, or another type of writer and write a creative piece that describes an exponential function and how it differs from linear and quadratic functions.

Creative: Create a song, rap, limerick, or some sort of writing about exponential functions.

Auditory: Create a voice recording that embodies the shape of an exponential function. Include a second recording that features the inverse. Include a brief explanation of how your recording represents an exponential function.

Motion: Create a dance (it can be modern, classical, theatrical, interpretive, etc.) or series of movements that embody an exponential function and include a brief explanation of how your piece represents an exponential function.
Appendix G:

Linear Functions Unit

Includes:

- Flexible Solving Activity (Flexible Solving)
- Transformation Lesson Plan (Active Learning)
Linear Functions
Flexible Solving Methods Activity

Directions: With a partner, solve the following problems.

1a.) Find the x- and y-intercepts of the following linear equation. Then make each intercept into an ordered pair.

\[-6x + 8y = 24\]

x-intercept: Ordered pair ( , )
y-intercept: Ordered pair ( , )

1b. Use the ordered pairs from part A to find the slope of \[-6x + 8y = 24\] using the slope formula. \[\text{Slope} = \]

1c.) Graph \[-6x + 8y = 24\] using the intercepts from part A.

1d.) Find the slope of the line you graphed in part C. Explain how you found it using the graph.

\[\text{Slope} = \]

How did you find it? 

Does it match the slope from part B? ______ Why or why not? ______
2a.) Find the slope and y-intercept of the following linear equation.
(3 Steps: find the intercepts, calculate slope, write equation in S-I form)

Standard Form Equation: $3x - 4y = 36$

2b.) Find the slope and y-intercept of the same linear equation using a different method. (Solve for $y$)

Standard Form Equation: $3x - 4y = 36$

3a.) Write an equation for the line parallel to $y = 3x + 10$ that contains the point $(4, 8)$. Hint: Figure out the slope of the new line first. Then use the given point and the slope to plug into slope-intercept form and solve for $b$.

3b.) Write an equation for the line parallel to $y = 3x + 10$ that contains the point $(4, 8)$. Alternate method: Graph the original line. Now use the new point and the parallel slope to graph the new line. Write an equation for the new line using the graph.
Lesson Plan Idea for Linear Functions

Emphasizing Active Learning

**Topic:** Transforming Linear Functions

**Unit:** Linear Functions

**Materials** (for each student): whiteboard graph, two different colored pipe cleaners, graphing calculator

**Idea:** To help students learn about linear transformations, students will have two pipe cleaners (lines) to use interactivity on their whiteboard graphs. To begin, there will be a brief discussion about hamburgers and how each student likes to each his/her hamburger (condiments, type of bread, etc). After the discussion, the variety of burger preferences will be compared to linear function transformations. The basic burger (we will decide on what makes the most basic burger) will be compared to the parent function for linear equations ($y = x$). Class will then discuss what components of burgers we changed for personal preference and what components of linear functions could be changed to create a different linear function. We will brainstorm about what changing these components will do to the graph of each line. After the discussion/brainstorming, each student will be instructed to place their white pipe cleaner on the graph as the parent function ($y = x$). The parent equation will be written on the board. We will then make changes to the parent function equation and write a new transformed equation. Students will use their green pipe cleaner to show what they believe will happen with the transformation. After making a pipe cleaner prediction, students will plug both equations into their graphing calculator to compare their prediction with the actual transformation. We will continue to explore transformations by moving around the pipe cleaners and verifying the results. After the activity, we will debrief and discuss what we have learned. We will establish as a class how transformations work and write about it in our notes.
Appendix H:

Quadratic Functions Unit

Includes:

Unit Introduction
Unit Overview

Unit Lesson Plans (Active Learning & Conceptual Understanding)
Learning Objectives Packet (Conceptual Understanding)
Pre-Assessment (Results used for Differentiation)
Anchor Activity (Differentiation)
Independent Study (Differentiation)
Parabola Activity (Differentiation)
Exit Card (Differentiation)

Problem-Based Learning Project (Active Learning & Flexible Solving)
Dice Activity (Active Learning)
Profiler Activity (Differentiation)
Tiered Solving Quadratics Assignment (Differentiation)
RAFT Writing Activity (Differentiation)
Formative Assessment (Active Learning)
Structured Academic Controversy (Active Learning & Conceptual Understanding)
Quadratic TriMind Activity (Differentiation)
**Quadratic Function Unit**

**Introduction**  
This three week unit on the quadratic function will prepare students for working with quadratic applications in future math classes, Physics, engineering, and in the world around them. This unit is intentionally placed toward the end of the school year so that the previously covered Algebra concepts will create the necessary knowledge base. This prior knowledge will be combined with novel concepts to take students from the first step of identifying quadratic functions to the rigorous work of solving and graphing. This unit will encourage students to identify quadratics that surround them in the world and are a part of everyday life. Understanding the common real-world applications of quadratic functions should help students to view them as less abstract and more applicable to their lives. The overall goal of this unit is to make a complex subject matter into something tangible for all students. While the aim is for all students to master the same learning objectives, it is assumed that students will begin this unit with varying levels of readiness, interests, and learning profiles. A pre-assessment will be given early in the unit to assist in understanding student needs. Throughout the unit the content, process, and assessment will be adjusted in an effort to meet the needs and preferences of students. The students will also be closely monitored with formative assessments during the unit to verify that all students are on track to meet the learning objectives.

**Algebra GLCE Standards Addressed**

A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inversely proportional relationships \((y = k/x)\); cubics \((y = ax^3)\); roots \((y = \sqrt{x})\); and exponentials \((y = ax^n, a > 0)\); using tables, graphs, and equations.*

A.PA.08.02 For basic functions, e.g., simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

A.RP.08.05 Relate quadratic functions in factored form and vertex form to their graphs, and vice versa; in particular, note that solutions of a quadratic equation are the x-intercepts of the corresponding quadratic function.

A.RP.08.06 Graph factorable quadratic functions, finding where the graph intersects the x-axis and the coordinates of the vertex; use words “parabola” and “roots”; include functions in vertex form and those with leading coefficient \(-1\), e.g., \(y = x^2 - 36, y = (x - 2)^2 - 9; y = -x^2; y = -(x - 3)^2\).

A.FO.08.08 Factor simple quadratic expressions with integer coefficients, e.g., \(x^2 + 6x + 9, x^2 + 2x - 3, \text{ and } x^2 - 4\); solve simple quadratic equations, e.g., \(x^2 = 16\) or \(x^2 = 5\) (by taking square roots); \(x^2 - x - 6 = 0, x^2 - 2x = 15\) (by factoring); verify solutions by evaluation.

A.FO.08.09 Solve applied problems involving simple quadratic equations.
# Quadratic Function Unit Overview

## 8th Grade Algebra

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Whole Class Elements</th>
<th>Differentiated Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hook Activities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students will participate in a structured foam ball toss event.</td>
<td>Unit Introduction Present learning objectives and distribute target self-evaluation checklist. (See Supplementary Materials.)</td>
<td>10 minutes</td>
</tr>
<tr>
<td>• The class will make graphs and discuss the foam ball toss.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>30 minutes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Lesson 1:</strong> Introduction to Quadratic Unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Class Period 90 minutes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Students will take Pre-Assessment for unit. (See Supplementary Materials.)</td>
<td>20 minutes</td>
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<tr>
<td></td>
<td>Determine teammates for unit.</td>
<td>10 minutes</td>
</tr>
<tr>
<td></td>
<td>Distribute and describe unit Anchor Activity. (See Supplementary Materials.)</td>
<td>20 minutes</td>
</tr>
<tr>
<td></td>
<td>Explain Independent Study to qualifying student(s). (See Supplementary Materials.)</td>
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</tr>
<tr>
<td></td>
<td>Instruction on identifying quadratic functions, parabolas, and components of parabolas. Lesson includes whole class, partner, and individual practice with drawing and labeling</td>
<td></td>
</tr>
<tr>
<td>Lesson 2: Parabolas</td>
<td>Parabolas. 60 minutes</td>
<td></td>
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<td>---------------------</td>
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<tr>
<td>1 Class Period 90 Minutes</td>
<td>Parabola activity with choice component for student preference. Students will describe the parts of a parabola through various methods. (See Supplementary Materials.) 20 minutes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Peer groups to share parabola activity. (See Supplementary Materials.) 10 minutes</td>
<td></td>
</tr>
<tr>
<td>Lesson 3: Axis of Symmetry and Vertex</td>
<td>Instruction on finding the axis of symmetry of a quadratic function/parabola with the formula $x = -\frac{b}{2a}$. 30 minutes</td>
<td></td>
</tr>
<tr>
<td>1 Class Period 90 Minutes</td>
<td>Think-Pair-Share Why is it important to know the axis of symmetry in a parabola? 10 minutes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Instruction on finding the vertex of a quadratic function/parabola by using axis of symmetry. 30 minutes</td>
<td></td>
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<tr>
<td></td>
<td>Think-Pair-Share (new partner) Is there a way to find the vertex without knowing the axis of symmetry? Explain. 10 minutes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exit card with choice component on finding the axis of symmetry and vertex. (See Supplementary Materials.) 10 minutes</td>
<td></td>
</tr>
<tr>
<td>Lesson 4:</td>
<td>Instruction on the six steps to graphing quadratic functions.</td>
<td></td>
</tr>
</tbody>
</table>
| Lesson 2: Graphing Quadratic Functions | Students will practice graphing as a whole class, partners, and individuals.  
60 minutes | Problem-Based Learning and PBL presentations.  
(See Supplementary Materials.)  
120 minutes |
|---|---|---|
| Lesson 5: Transformations | Instruction on quadratic function transformations. The lesson will focus on width, y-intercept, and direction of opening.  
45 minutes | Think Dots with self-evaluation. Concepts from lessons 2-5 are featured.  
(See Supplementary Materials.)  
45 minutes |
30 minutes | Students complete self-evaluations on progress with learning objectives. Mrs. Sanders conferences with individual students.  
(See Supplementary Materials.)  
30 minutes |
| Lesson 7: | Instruction on identifying solutions to quadratic functions and graphing to find solutions using the six steps previously learned. | Anchor Activity as time permits |
### Solve by Graphing

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Class Period</td>
<td>90 minutes</td>
</tr>
<tr>
<td><strong>Solve by Graphing</strong></td>
<td>25 minutes</td>
</tr>
<tr>
<td>Profiler Activity (See Supplementary Materials.)</td>
<td></td>
</tr>
<tr>
<td><strong>Anchor Activity as time permits</strong></td>
<td></td>
</tr>
<tr>
<td>Peer grouping to present products from Profiler activity.</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>

### Lesson 8:
**Solving Quadratic Functions Algebraically and by Using Square Roots**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Class Periods</td>
<td>180 minutes</td>
</tr>
<tr>
<td><strong>Instruction on solving quadratic functions algebraically by factoring.</strong></td>
<td>30 minutes</td>
</tr>
<tr>
<td><strong>Section I: Tiered Assignment</strong></td>
<td></td>
</tr>
<tr>
<td>Practice on solving quadratic functions algebraically. (See Supplementary Materials.)</td>
<td>20 minutes</td>
</tr>
<tr>
<td><strong>Instruction on solving quadratic functions by using square roots.</strong></td>
<td>25 minutes</td>
</tr>
<tr>
<td><strong>Section II: Tiered Assignment</strong></td>
<td></td>
</tr>
<tr>
<td>Practice on solving quadratic functions using square roots. (See Supplementary Materials.)</td>
<td>15 minutes</td>
</tr>
<tr>
<td><strong>Anchor Activity as time permits</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Day 2 of Lesson 8

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section III: Tiered Assignment</strong></td>
<td></td>
</tr>
<tr>
<td>Open-ended questions about solving quadratic functions. (Finish any incomplete sections during this time.) (See Supplementary Materials.)</td>
<td>25 minutes</td>
</tr>
<tr>
<td><strong>Anchor Activity as time permits</strong></td>
<td></td>
</tr>
<tr>
<td>Lesson 9: Quadratic Formula</td>
<td>2 Class Periods 180 minutes</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>Introduction of the quadratic formula with whole class practice to solve various quadratic functions.</td>
<td>40 minutes</td>
</tr>
<tr>
<td>Flexible grouping whiteboard practice on the quadratic formula.</td>
<td>20 minutes</td>
</tr>
<tr>
<td>Formative Assessment Card completed individually on the quadratic formula. Students will be paired according to results of card to go over the problem in partners. (See Supplementary Materials.)</td>
<td>20 minutes</td>
</tr>
</tbody>
</table>

**Think-Pair-Share**
When using the quadratic formula, at what point do you know how many solutions there will be?  
10 minutes

<table>
<thead>
<tr>
<th>Day 2 of Lesson 9</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Structured Academic Controversy over whether the quadratic formula is the best way to solve a quadratic function. (See Supplementary Materials.)</td>
<td>70 minutes</td>
</tr>
<tr>
<td>Students will update their learning objectives self-evaluation checklist. (See Supplementary Materials.)</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>

**RAFT writing activity**  
(See Supplementary Materials.)  
55 minutes

**Students will update their learning objectives self-evaluation checklist.** (See Supplementary Materials.)  
10 minutes

**RAFT writing activity**  
(See Supplementary Materials.)  
55 minutes
<table>
<thead>
<tr>
<th>Lesson 10: The Discriminant</th>
<th>Instruction on the Discriminant and how to find the number of solutions without completing the entire quadratic formula.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Class Period 90 minutes</td>
<td>20 minutes</td>
</tr>
<tr>
<td></td>
<td>TriMind Activity about discriminant. (See Supplementary Materials.) 50 minutes</td>
</tr>
<tr>
<td></td>
<td>Anchor Activity as time permits</td>
</tr>
<tr>
<td></td>
<td>Peer grouping to present TriMind activity results. 20 minutes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 11: Conclusion of Unit &amp; Unit Assessment</th>
<th>Review of unit concepts including Q &amp; A session and unit wrap-up.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Class Period 90 minutes</td>
<td>45 minutes</td>
</tr>
<tr>
<td></td>
<td>Unit 9 Summative Assessment over all unit learning targets. 45 minutes</td>
</tr>
</tbody>
</table>
Detailed Lessons- Quadratic Unit

Differentiated elements are indicated with an asterisk (*).

Lesson 1: Introduction
1 Class Period– 90 minutes

Hook Activities– 30 minutes
This unit will begin with several fun activities to get students thinking about quadratic functions and generate interest and excitement for the unit. The hook activities will begin with a structured foam ball toss event. For this event, the class will go outside and students will pair up and spread out. If the weather is unfavorable, we can do this activity in the gym or the classroom. Pairs of students will pass the foam ball back and forth in as many ways as possible. For example, students might roll the ball once and throw it high in the air another time. Before the activity commences, students will be asked to pay close attention to the different trajectories of their ball. After a few minutes, we will return to the classroom. Once in the classroom, each pair of students will be given a large sheet of paper and a sheet of graph paper. They will be asked to sketch the different trajectories of their ball on the large paper. On the piece of graph paper, they will be asked to graph what they feel is the most common trajectory of a tossed ball. Students will be given a few minutes to complete their sketches and their graph. Once they are finished, the papers will be taped to the front wall of the room. As a class, we will discuss the drawings. Discussion questions will include the following. Is it true that “what goes up must come down”? What is the most common trajectory we see on the sketches and graphs? What is the shape of that trajectory? Is the path of a tossed ball the same as a golf ball being hit? Does it take the same amount of time for a tossed object to go up as to come down?

*Unit Introduction–10 minutes
Students will be given a packet that contains a copy of the learning objectives for the unit. As a class, we will briefly read through these goals. Students will be reminded to not feel overwhelmed at all the unknown vocabulary and concepts because we will be mastering them throughout the unit as an ongoing process. Next, the checklist will be introduced and the format will be explained. Next to each learning objective there is a box for the date mastered and the evidence of mastery. Students will be instructed to review and update these pages individually throughout the unit (there are several specified class times for updating as well). When a student thinks that he/she has mastered a particular learning objective, the current date should be written in the corresponding box. Then some type of evidence of mastery should be recorded in the last column. Evidence of mastery can include feedback and scores from homework, activities, and assessments. Other types of evidence will also be considered if presented by the student. Lastly, the self-evaluation section will be introduced. Students will be evaluating their progress on the learning objectives three times during the unit. The self-evaluations will provide a chance for students to realize
whether they are on track to master the learning objectives and what needs to be done to get on track if necessary. Students will also receive teacher feedback on their progress.

*Pre-Assessment—20 minutes

Students will be given the pre-assessment for the unit. The pre-assessment is based on the fundamental and vital skills and concepts in this unit. The directions will be read aloud and emphasis will be placed on the fact that this is a no stress assessment. Students will be encouraged to do their best without worrying about novel and/or unfamiliar concepts and skills. Students will be instructed to work on the pretest individually and correct their work when finished using the answer keys (there will be multiple copies) located on the front table. The pre-assessments should be turned in to the tray when finished. In an effort to avoid empty time and any classroom management issues from students finishing at different times, students will be allowed to begin the two surveys upon finishing the pre-assessment.

*Determine Teammates—10 minutes

Students will be informed that they will be on a “team” throughout this unit. They will have a 1st, 2nd, 3rd, and home base teammate, which they will work with at various times and for various activities. The teammates diagram will be distributed and students will be informed that they will choose a 1st and home base partner, while 2nd and 3rd base will be assigned to them. At this time, students will be instructed to find and record their 1st and home base partners. The one stipulation is that students must sit near (next to, behind, in front of, etc.) their 1st base partner. Home base will be a free choice. Before students are allowed to move around and find their partners, they will be briefly reminded of how to choose partners without hurting someone’s feelings. Students will then be given four minutes to find those two partners and record their names. As per the directions, students will fill out two duplicate copies of the diagram. Both copies will be turned in and the student copy will handed back the next day with the 2nd and 3rd base partners assigned. The 2nd and 3rd base partners will be assigned based on readiness levels indicated on the pre-assessment and observations and evidence from previous units. 2nd base partners will be students with different readiness levels and 3rd base teammates will be students with similar readiness levels.

*Introduce Anchor Activity Tic-Tac-Toe Board—10 minutes

Distribute the Anchor Activity Tic-Tac-Toe Board to students who have not yet picked one up. Introduce the concept of an anchor activity to the class. Explain to students that they will choose three activities to work on throughout the unit whenever they have extra time. They should choose the three activities that are most appealing to them because all activities are worth the same amount of points. Clarify that at the end of the unit three completed activities are due. If they do not have adequate class time to finish, students must work on the activities at home. After the
directions, each activity will be read. Students will be directed to look at the attached rubric to see how each activity will be scored. Questions will be taken.

*Independent Study—as needed*
The pre-assessments will be examined by the teacher while students work on the surveys. Any student who demonstrates exemplary knowledge will be offered the chance to do the Independent Study project. This project will be explained in detail to any qualifying students after class or at another time when the other students are not directly present. If the project is accepted, the student(s) will present on the last day of the unit.

**Lesson 2: Parabolas**
1 Class Period—90 minutes

**Instruction on Parabolas—60 minutes**
Pose the question “What is usually graphed on a coordinate plane?” Ask for volunteers to share their answer with the class. Briefly discuss the possibilities as a class. At the end of the discussion, explain that in previous units we focused on graphing linear functions. Explain that today we will be looking at a new kind of function. We will be studying quadratic functions. Instead of being linear (a straight line), quadratic functions are parabolas. Show the overhead with several parabolas for a brief moment. Write “the standard form of a quadratic function is \( y = ax^2 + bx + c \)” on the board. Ask students to brainstorm what makes quadratic functions look different than the linear functions they have worked with before. Discuss their ideas as a class. Ask thought-provoking questions like why a quadratic function is a curved shape. Referring to the standard form, point out to students that it appears that there are five variables in a quadratic function, but there are actually only two. The other three are numbers, and sometimes the \( b \) and \( c \) values are not seen in the function at all. Explain in detail each of the variables shown in the standard form of a quadratic function. Students will be given a note sheet with the following information.

**Quadratic Function**—any function that can be written as (standard form) \( y = ax^2 + bx + c \) where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \)

Note: \( b \) and \( c \) can equal zero, which would mean you do not see them.
For example, \( y = x^2 \) is the same as \( y = x^2 + 0x + 0 \)
**\( y = x^2 \) must be there because \( a \neq 0 \) **

Tell students that now we know what a quadratic function looks like, so it is time to play a quick game of “Identify the Quadratic Function.” Five quadratic functions will be displayed on the overhead and students will be asked to identify whether they are quadratic or not. Students will record the functions and their answers in their notes. Students will compare their answers and their reasoning with a student sitting next to
them. After this, we will go through each function one by one. If the students think it is quadratic, they will make a yes sound effect (ex. ding, ding, ding). If the students think it is not quadratic, they will make a no sound effect (ex. buzzer sound). We will discuss the answers as a class.

Pose the question, “Could we determine whether or not a set of ordered pairs was a quadratic function?” Before having students answer, remind them of unit 5 when we examined tables of ordered pairs for a linear relationship. Then ask students to raise their hand if they think we can do the same thing for quadratic functions. Inform them that you can indeed determine whether a set of ordered pairs represents a quadratic function. Explain the process and discuss how it compares to the similar process in unit five. On the note page they were given, it will say the following.

Deciding if a function is quadratic:
- For an equation, does it follow the rules? \( y = ax^2 + bx + c \) and \( a \neq 0 \)
- If you have ordered pairs, look for constant first differences in \( x \), and constant second differences in \( y \) (the difference of the difference).
- If you are looking at a graph, is it a parabola?

As a class, work through several examples to practice determining whether a set of ordered pairs is a quadratic function. Students will then try two examples on their own. When finished, students can compare answers with their 1st base teammate. Then the correct answers will be given to the class and we will go over any questions.

Tell the students that since now we know how to identify quadratic functions, we will learn all about the parabola. Have students say the word parabola five times fast to a neighbor.

On the overhead, several parabolas will be displayed. Students will use their arms to model a parabola opening up and a parabola opening down. Students will be informed that the \( a \) value in a quadratic function determines the direction in which the parabola opens. The following information will be included on their note page.

\[ \text{A parabola opens:} \]
- Upward if \( a > 0 \) (positive number)
- Downward if \( a < 0 \) (negative number)

Four quadratic functions will then be displayed on the board. Students will be asked to examine these functions carefully. For each function, students will be asked to stand up if the parabola opens up or sit down if the parabola opens down.

Next, have students find the following table on the note page.

<table>
<thead>
<tr>
<th>Parts of a parabola</th>
<th>Term</th>
<th>What is it? Where is it located? What does it do?</th>
</tr>
</thead>
</table>


<table>
<thead>
<tr>
<th>maximum/minimum</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>vertex</td>
<td></td>
</tr>
<tr>
<td>axis of symmetry</td>
<td></td>
</tr>
<tr>
<td>domain</td>
<td></td>
</tr>
<tr>
<td>range</td>
<td></td>
</tr>
<tr>
<td>y-intercept</td>
<td></td>
</tr>
<tr>
<td>zeros</td>
<td></td>
</tr>
</tbody>
</table>

Note: Certain students may be provided with a completed copy of these tables if necessary.

On the overhead, several parabolas will again be displayed. Students will sketch these parabolas in their notes. As a class, we will go over each part of the parabola. We will label each part on the overhead parabolas and students will label the parts on their sketches. We will answer the following questions. What is it? Where is it located? What does it do? Students will record the information in the table. When discussing the maximum and minimum (vertex) of a parabola, students will alternately chant max-i-mum or min-i-mum while modeling downward and upward opening parabolas with their arms. When discussing the different number of zeros that a parabola might have, students will turn to a neighbor. One student in the pair will make the x and y axis with their arms while the other partner uses their hand (cupped like a U) to show how the location of a parabola corresponds to how many zeros it will have. Students will do this to model parabolas with no zeros, one zero, and two zeros. Then students will trade jobs and model three more parabolas.

*Parabola Choice Activity—20 minutes*

The parabola activity features tasks focused on exploring the parts of a parabola according to student preference. Students will be given the parabola activity worksheet, and the directions will be read aloud. Students will be instructed to work on the activity of their choice. Students will be informed that although they will work individually on the activity itself, they will be put into groups later in the class period to present their work to each other. After the directions, students will work on the parabola activity.

*Peer Groups—10 minutes*

Students will be divided into groups to present their finished product from the parabola activity to each other. (These groups will be formed on the spot by the teacher.) Before dispersing into groups, directions will be given to the whole class. Each presenter should read the directions to the task he/she completed before presenting the product created for that task. The presentation should last approximately three minutes and include a detailed explanation of what was created. Songs, raps, and skits should be performed.

After the directions are given, students will join their group members somewhere around the room. Once in their groups, the students will decide what order to present.
Lesson 3: Axis of Symmetry and Vertex
1 Class Period—90 minutes

Instruction on Axis of Symmetry—30 minutes
Remind students that yesterday we learned about the features of a parabola. Show an example parabola on the overhead and have student volunteers come to the board and label the features. Once the parabola is correctly and completely labeled, point to the axis of symmetry and ask students the definition of symmetry. After taking a few student definitions, ask students the definition of an axis. After a few volunteered answers, clarify that the axis of symmetry of a parabola is the imaginary line that divides the parabola into two equal halves. Have students record in the journal section of their notes how they think they can find the axis of symmetry on a parabola. Explain that there are two methods of finding the axis of symmetry. One method is faster if you have the graph of the quadratic function to work with, and the other method is more useful if you only have the quadratic function itself.

To teach the first method, display three parabolas (with accompanying quadratic functions) on the board. Have students write their guess of the axis of symmetry in their notes. Then show students how to determine the axis of symmetry using the zero(s) of the parabola. If the function has one zero, the axis of symmetry runs through it. If the function has two zeros, you must take the average to find the axis of symmetry.

To teach the second method, show a quadratic function on the board. Explain that you could graph this function to find the zero(s), or you can use the formula for the axis of symmetry. Introduce the formula \( x = \frac{-b}{2a} \) and have students take notes. Model the formula with several quadratic functions. Have students try using the formula individually on whiteboards. Teacher walks around and assists students during this time.

Think-Pair-Share (Axis of Symmetry)—10 minutes
The following question will be written on the board. Why is it important to know the axis of symmetry in a parabola? Students will be instructed to think about this question and record their ideas in their notes. After adequate time has passed, students will turn as directed to a neighbor sitting near them and share their ideas. Students will record their partner’s ideas along with their own in their notes. If time allows, volunteers will be asked to share their ideas with the class.

Instruction on Vertex—30 minutes
Display the same labeled parabola from earlier in the lesson. Have a student come up and point out the vertex. Direct students to pick up a small object from their desk (ex. an eraser) that will represent a vertex. Students will make two parabolas (opening up and down) with their arms and they will hold the object in between their hands to represent the vertex. Explain that finding the vertex of a parabola on a graph involves
locating the minimum or maximum (depending) and writing the corresponding ordered pair coordinates. Ask students how it might be possible to find the vertex without a graph. Take a few suggestions, and introduce the three steps to discovering the vertex of a parabola. Students will record these steps in their notes.

Step 1: Find the axis of symmetry.
Step 2: Plug the x-value into the original function and evaluate.
Step 3: Write the vertex as an ordered pair.

The process will be modeled several times as the students take notes. Then students will be asked to find the vertex of a given quadratic function with their 1st base teammate. Each pair of students will join another pair to compare answers. Next, students will individually try finding the vertex of two novel quadratic functions. We will go over these answers as a class.

Think-Pair-Share (Vertex)– 10 minutes
The following question will be written on the board. Is there a way to find the vertex without knowing the axis of symmetry? Explain. Students will be instructed to think about this question and record their ideas in their notes. After adequate time has passed, students will turn as directed to a different neighbor sitting near them and share their ideas. Students will record their partner’s ideas along with their own in their notes. If time allows, volunteers will be asked to share their ideas with the class.

* Exit Card– 10 minutes
Students will be given an exit card to complete as their ticket to leave class when the period is over. Students will choose a method (several options will be listed) to demonstrate their understanding of the day’s lesson. Students will present these cards as they are leaving, so that they can be reviewed for misconceptions or problem areas.

Lesson 4: Graphing Quadratic Functions
2 Class Periods– 180 minutes

DAY 1

Instruction on Six Steps– 60 minutes
Pose the question “What do we need to know about a quadratic function in order to graph it?” Discuss this question as a class. Then introduce the six steps to graphing a quadratic function. Students will take notes on the steps as they are introduced. Each step will be explained and demonstrated individually in sequence. The entire process will be demonstrated several times, during which time the students will observe, ask questions, and predict the next step. Before we begin each graph, students will model the direction the parabola will open (based on the “a” value) with their arms. As a class, we will graph several quadratic functions on the board while students graph in their notes. Students will be invited up to the board to complete individual steps. Then
students will graph a given quadratic function with their 1st base teammate. Each pair of students will join another pair to compare graphs. Next, students will individually try using the six steps to graph two quadratic functions. Afterward, they will meet with their home base partners to compare graphs. Finally, we will go over these two graphs as a class.

*Problem Based Learning Activity—30 minutes
The activity will be introduced with the question “Who has been golfing before?” As a class, we will discuss some general things about the game of golf (for the benefit of students who have not had much experience with the game). Then students will be introduced to “The Situation” featured in the problem based learning activity. After this introduction, students should understand that golf balls on hole nine are disappearing in mid-air. Their job is to figure out what is causing this mystery and find a solution. After answering any questions on the problem, we will go over “The Plan” and “What do we do now?” sections. Students will then be placed into groups. These groups will have been previously decided upon by the teacher according to readiness level. Students of similar readiness levels will be grouped together for this activity. Groups will be encouraged to take a look at the rest of the materials in the packet before beginning the activity. Groups will then be free to begin the activity by following the plan of action laid out within the directions. The students’ goal will be to solve the problem posed in “The Situation” and come up with a mathematically supported solution to present to their classmates.

DAY 2

*Problem Based Learning Activity—65 minutes
After addressing any questions, groups will reconvene and continue the activity where they left off the day before. The groups of students with lower readiness levels will receive teacher support and guidance if necessary. Groups will be given adequate time warnings in order to have the activity complete for presentations.

*Problem Based Learning Presentations—25 minutes
Each group of students will present their work on this activity to their classmates. They will describe to their classmates what they believe is causing the golf balls on hole nine to disappear along with their rationale. They will also explain the solution they chose to remedy the problem. Every group member is expected to have an active role in the presentation. Each group’s presentation should last approximately three to five minutes, which will include time for the audience to ask questions about the presentation.

Lesson 5: Transformations
1 Class Period—90 minutes
Instruction on Quadratic Function Transformations—45 minutes

Write the function \( y = x^2 \) on the board. Pose the question “What do we know about this function?” Discuss this question as a class. If the students have not mentioned it, point out that \( a = 1, b = 0, \) and \( c = 0. \) Identify this as the parent function of all quadratic functions. Pose the question “What does the term ‘parent function’ make you think of?” Take ideas and then explain how the parent function is the basic structure for all quadratic functions, and quadratic functions that differ from the parent function have been transformed. Display several pairs/groups of functions on the board.

\[
\begin{align*}
  y &= x^2 \quad \text{and} \quad y = -x^2 \\
  y &= x^2 \quad \text{and} \quad y = \frac{1}{2}x^2 \quad \text{and} \quad y = 2x^2 \\
  y &= x^2 \quad \text{and} \quad y = x^2 + 2 \quad \text{and} \quad y = x^2 - 3
\end{align*}
\]

Working with one pair/group of functions at a time, ask students what value has been changed. Ask students how this change will affect the graph of the parent function. After students have given ideas, show the graph for the pair/group of functions in question to confirm the affect of the change. For each change, have students model the movement with their arms.

\[
\begin{align*}
  y &= x^2 \quad \text{and} \quad y = -x^2 \\
  \text{The ‘}a\text{’ value is changed from a positive to a negative, which changes the direction that the parabola opens. Students model an upward opening for the first function and a downward opening for the second.}
\end{align*}
\]

\[
\begin{align*}
  y &= x^2 \quad \text{and} \quad y = \frac{1}{2}x^2 \quad \text{and} \quad y = 2x^2 \\
  \text{The ‘}a\text{’ value is changed in size, which changes the width of the parabola (they all have an upward opening). Students model a medium width parabola for the parent function, a wider parabola for the second, and a narrower parabola for the third.}
\end{align*}
\]

\[
\begin{align*}
  y &= x^2 \quad \text{and} \quad y = x^2 + 2 \quad \text{and} \quad y = x^2 - 3 \\
  \text{The ‘}c\text{’ value has changed, which caused a vertical translation of the parabola. Students model a steady parabola for the parent function, slide upward two units for the second function, go back to the parent position, and slide downward three units for the third function.}
\end{align*}
\]

After the discussion and arm modeling, students will be given three more pairs/groups of functions to contemplate with their 1st base teammate. After a few minutes, we will discuss our ideas about the transformations as a group. Lastly, students will given three more pairs/groups of functions and asked individually to write a short statement of comparison in their notes for each pair/group based on direction of opening, width, and y-intercept. We will discuss these as a class before moving to the next activity.
*Think Dots Activity—45 minutes*

The Think Dots activity features six different tasks which correspond to the faces of a die. The six tasks focus on concepts from lessons two through five, and the tasks require lower and higher level thought processes. Before beginning the activity, the students will get into partners with their 2nd base teammates. The directions will be read aloud to the entire class, and one die will be distributed per pair. Students will take turns rolling the die and completing the corresponding task while their partner will also have a task to complete for each roll. All work will be recorded in a designated workspace, and students will complete a self-evaluation with their partners upon completion of the activity. Partners will also check their work with the answer key following the self-evaluation.

*Anchor Activity—as time allows*

**Lesson 6: Assessment and Self-Evaluation**

1 Class Period—90 minutes

**Review Lessons Two through Five for Quiz—30 minutes**

A parabola with numbered components will be displayed on the overhead. Students will work with their home base partners to identify and define each component of the parabola. We will go over the answers together as a class. Then several functions and sets of ordered pairs will be displayed. As a class, we will discuss how to identify whether they are quadratic functions and then we will identify each. We will then review the formula for finding the axis of symmetry and how to use the axis of symmetry to find the vertex. Next, we will go over the steps to graphing a quadratic function. Lastly, we will review the ways that quadratic functions can be transformed. Students will do the arm movements for each transformation as they did in the lesson.

**Quiz—30 minutes**

Students will take a quiz over the concepts from lessons two through five. As the students finish, the quizzes will be graded and handed back so that the students can complete the self-evaluation and learning objective checklist. The following learning objectives will be assessed on the quiz.

- I can identify a quadratic function.
- I can identify and define the basic elements of a parabola (minimum or maximum, axis of symmetry, domain, range, y-intercept, zeros, and vertex).
- I can graph quadratic functions.
- I can find the axis of symmetry and vertex of a parabola.
- I can transform quadratic functions by changing a, b, and/or c values.

**Self-Evaluations & Individual Conferences—30 minutes**
Using their quiz and other previous activities, students will update their learning objective checklists and complete self-evaluations on their progress with the learning objectives. During this time, I will conference with individual students about their progress, their self-evaluations, and their checklists.

*Anchor Activity – as time allows

Lesson 7: Solving Quadratic Functions by Graphing
1 Class Period – 90 minutes

Instruction on Solutions & Graphing for Solutions – 25 minutes
Ask students to write the six steps to graphing a quadratic function in their notes from memory. After two or three minutes, allow students to look back in their notes to find and fill in any missing steps. Pose the question “Why did we graph quadratic functions in lesson 4?” Ask for students to volunteer answers and discuss they answers as a class. Explain that there is one primary reason to graph quadratic functions, and that reason is to find the solution(s).

Show a parabola and the corresponding quadratic function on the overhead. Ask students to locate the zeros and record the ordered pairs in their notes. Ask for student volunteer to walk up front and point out the zeros and give the ordered pairs. Have students raise their hands if they see the zeros on the given parabola. Inform students that they are looking at the solutions to that quadratic function. Explain that zeros = solutions. Have students write this prominently in their notes. Challenge students to come up with a unique chant for the phrase zeros = solutions. For example, saying the phrase in a high squeaky voice. Take several volunteers to share their chants, and have the class repeat each chant several times.

Sketch a real life situation of a golf ball trajectory over time. Have students brainstorm and then discuss what the solutions would represent for this situation. Discuss as a class how this example relates to other real life examples and why solutions are important.

Have students hold up (by a show of fingers) the number of zeros for a quadratic function. Ideally, some students will hold up different numbers of fingers because there are multiple correct answers. If this happens, point out that the different answers are all correct because there can be no zeros, one zero, or two zeros. If it does not happen, tell students that they could have held up no fingers, one finger, or two fingers and been correct. Remind students about these possibilities. Now explain that because zeros = solutions there can be no solutions, one solution, or two solutions to quadratic functions.
As a class, go through the six steps to graphing three different quadratic functions (one with no solutions, one solution, and two solutions). Students will sketch each parabola in their notes while I graph it on the board. After each parabola is graphed, students must locate and label the solution(s). We will also discuss how sometimes quadratic functions are written as quadratic equations. I will explain the difference between the two and model how to replace the 0 with a y in order to solve.

*Profiler Activity—55 minutes
The Profiler activity features five different occupations assigned to various tasks involving solving quadratic functions by graphing. Before handing out the activity, students will be given sticky notes. On the overhead five occupations will be displayed in separate columns with brief descriptions.

**Artist**—create a flyer, poster, or brochure  
**Builder**—create a 3-D model  
**Musician**—write and perform a song or rap  
**Auctioneer**—write a script for selling an item at the auction house  
**Engineer**—design a water fountain for a park

Students will be asked to choose the occupation/task that they find most interesting and label it choice #1. Students will be directed to make a second choice on the sticky note as well. Students will then bring their sticky notes to the board and place them in the column of their first choice occupation. The occupation a student chooses with his/her sticky note will be the occupation/task that he/she works on in the Profiler activity. The only exception will be if there is a large number of students who all choose one category or if one category has one single student. If this occurs, then some students will be asked to take their second choice.

At this time, students will be given the Profiler activity sheet and the detailed description sheet that matches their chosen occupation. The situation listed and the general directions that apply to all the occupations will be read aloud. Students will be informed that they have the option to work in partners or groups of three with other students who chose the same occupation. It will also be explained that all students will be put into groups later in the class period to present their work to each other. These groups will be formed by the teacher so that each group has students representing different occupations. An ideal group would be five or six students representing two or more occupations.

After taking questions, students will be dismissed to begin working individually or in the groups described above.

*Anchor Activity— as time allows

*Peer Grouping to Present Profiler Activity—10 minutes
Students will be divided into groups to present their finished product from the Profiler activity to each other. (These groups will have been formed by the teacher so that each group has students representing different occupations. An ideal group would be five or six students representing two or more occupations.) Before dispersing into groups, directions will be given to the whole class. Each presenter or group of presenters should introduce his/her occupation before presenting the product created for that task. The presentation should last approximately three minutes and include a detailed explanation of what was created. The musicians should perform their song during their presentation. After each presentation, group members may ask questions of the presenter(s).

After the directions are given, students will join their group members somewhere around the room. Once in their groups, the students will decide what order to present. They will each present their work and ask appropriate questions of each other’s work.

Lesson 8: Solving Quadratic Functions Algebraically and by using Square Roots
2 Class Periods – 180 minutes

DAY 1

Instruction on Solving Quadratic Functions Algebraically – 30 minutes
Pose the question “If a times b equals zero, what do we know about a and b?” Discuss students’ answers as a class. Introduce the Zero Product Property (If ab = 0 then a = 0 or b = 0). Have students write the property in their notes. Introduce a new problem. (x)(x + 5) = 0 Ask students if we can apply the Zero Product Property. Have students who believe it is possible stand up. Ask one of the standing students about their opinion. Confirm that anytime two factors equal zero, you can use the property. In the case above, you know that one of the factors must equal zero. So either x = 0 or x + 5 = 0. For x + 5 = 0, you solve for x. You get two solutions of x = 0 or x = -5. As a class, practice solving several equations with the Zero Product Property. Have students try several problems independently in their notes, and go over these problems as a class.

Next, ask students if you can solve 0 = x² + 5x + 6 with the Zero Product Property. Have each student come up with an answer and rationale and share it with their 1st base teammate. After teammates have shared, ask for volunteers to share their opinions with the whole class. Discuss these opinions. Inform students that you can use the Zero Product Property, but first you must factor the problem. Remind students of the reverse FOIL method in unit eight where we learned how to factor a polynomial by undoing the FOIL method.

0 = x² + 5x + 6 needs to become
First we label the original equation. $0 = x^2 + 5x + 6$

Then, we ask what two factors make $x^2$? Those go first in the parenthesis. What two numbers multiply to make 6 but add to make 5? Those go last in the parenthesis. Answer: $x$ multiplied by $x = x^2$ and $2$ multiplied by $3 = 6$ and $2 + 3 = 5$

So, $0 = x^2 + 5x + 6$ comes from $0 = (x + 2)(x + 3)$

Now we can use the Zero Product Property to find the solutions. As a class, practice factoring and then solving several quadratic equations. Have students attempt several problems independently in their notes, and go over these problems as a class.

*Section I: Tiered Assignment—20 minutes*

There are three versions of the Practice Makes Perfect—Solving Quadratics assignment. Each version has the same format, the same amount of problems, and focuses on the same concepts and skills. The difference is that each of the versions is uniquely created to match different readiness levels. Version O is the original version of the assignment created for students at grade level readiness. Version A is designed for students above grade level readiness. Version B is for students below grade level readiness. Students will be given the version that best matches their readiness level. Section I of this assignment focuses on solving quadratic functions by factoring and using the Zero Product Property. Students will work individually on each section of this assignment. After completing Section I, students can confer with their 3rd base partners (who will have the same version because both students have similar readiness levels) on their answers.

**Instruction on Solving Quadratic Functions by using Square Roots—25 minutes**

Pose the question “What is the square root of 25?” Have students write the answer down in their notes. Inform students that 5 is not the correct answer. Remind students that there are two square roots to any positive number. The square root of 25 is 5 and -5. Ask students what is the square root of zero? What is the square root of a negative number?

Inform students that we are going to be using our knowledge of square roots to solve quadratic equations. First we are going to refresh our memories by solving a few square roots problems. Solve the following problems as a class.

$81 = x^2$
$20 = x^2$
Have students practice several similar problems in their notes. Go over the answers as a class.

Now display the following problem on the board. \(0 = x^2 - 100\) Inform students that we can solve this problem using square roots, but there is something we must do first. Ask for suggestions. Remind students that when solving an equation we must use reverse order of operations. In this case, we cannot undo the square before undoing the subtraction. Model how to solve this problem and several similar problems. Have students try a few individually in their notes. We will go over them as a class before moving to the next section of the tiered assignment.

*Section II: Tiered Assignment – 15 minutes*
Section II focuses on solving quadratic functions by using square roots. Problems ask students to take the square root of perfect squares, numbers that are not perfect squares, negative numbers, and zero. Students may again confer with their 3rd base partners when finished.

*Anchor Activity – as time allows*

DAY 2

*Section III: Tiered Assignment – 25 minutes*
Section III features open-ended questions about solving quadratic functions. Students may also finish any incomplete sections during this time. When finished, students may again confer with their 3rd base partners. Students may check their answers against the answer key on the front table when they are completely finished with all three sections.

*Anchor Activity – as time allows*

*Student Self-Evaluation on Learning Objectives – 10 minutes*
Students will update their learning objective checklist and complete another self-evaluation. Students will be encouraged to consider all of the activities, assignments, and the assessment that they have completed so far in this unit.

*RAFT Writing Activity – 55 minutes*
Inform students that although they may never have dreamed of becoming a quadratic function, today they will get the chance. Introduce the RAFT writing activity as a creative opportunity to write from a new perspective to demonstrate knowledge about quadratic functions. Give students the RAFT writing activity and rubric. Go over the directions and the five options as a class. Ask for questions, and then let students
begin writing. If time allows, ask for student volunteers to share their RAFT writing with the class.

*Anchor Activity– as time allows

Lesson 9: The Quadratic Formula
2 Class Periods– 180 minutes

DAY 1

Instruction on the Quadratic Formula– 40 minutes
The quadratic formula will be written on the board. Have students record it in their notes. Students will then listen to the quadratic formula song (played from youtube.com through computer speakers). Several (school appropriate) quadratic formula videos from youtube.com will be shown using the LCD projector. The original quadratic formula song will be played two more times and students will be asked to sing along.

Explain that the quadratic formula is a formula that can correctly and efficiently solve every quadratic function. Pose the following question. “In the quadratic formula, where do values for a, b, and c come from?” Answers will be taken from volunteers, and hopefully students will recognize that the a, b, and c values come directly from the quadratic function that you are trying to solve. If not, it will be pointed out.

Next, the process of solving a quadratic function using the quadratic formula will be modeled. Then, as a class students will help in solving several more quadratic functions using the quadratic formula. We will specifically discuss and go over examples of what happens when zero, a negative number, or a number that is not a perfect square is located under the radical sign. Students will write examples in their notes as they are done on the board.

*Flexible Grouping White-board Practice– 20 minutes
This activity will be introduced to students as a chance to get together in groups and practice the quadratic formula with all sorts of different quadratic functions. Students will be assigned to groups to practice using the quadratic formula. The groups will already formed according to readiness levels indicated on the pre-assessment and from work completed previously in the unit. There will be six groups of four students (several groups may have one more or one less based on numbers). There will be two sets of quadratic functions. One set will be for groups of students who are at or above grade level readiness. The other set will be for groups with students who are at or below grade level readiness.
Once in their groups, students will use whiteboards to complete the steps of the quadratic formula with different quadratic functions. They will work through the steps together as a group and check each other’s work. They will arrive at an answer together as a group and write it down in their notes. I will walk around and observe and help groups. I will also check the answers written in their notes and work with groups on any problematic problems. Note: If the whiteboards are not large enough to accommodate the problem, students may do one of two things. First, they may choose to work completely in their notes. Second, they may choose to feature a different step (or two) of the formula on each person’s whiteboard. Students will work on the following quadratic functions.

Quadratic functions for students at or above grade level:

\[
\begin{align*}
y &= 12x^2 - 4x + 1.5 \\
-2x^2 &= 6x - 10 \\
0 &= -5x^2 - 10x + 15
\end{align*}
\]

Quadratic functions for students at or below grade level:

\[
\begin{align*}
y &= 2x^2 - 10x + 8 \\
y &= 2x^2 + 4x + 2 \\
y &= x^2 - 3x - 6
\end{align*}
\]

*Formative Assessment Card on the Quadratic Formula—20 minutes

Students will individually complete the formative assessment card. The card requires students to solve a quadratic function using the quadratic formula. The card also asks students to rate their confidence in their answers and indicate whether they would be willing to help a classmate use the quadratic formula. Students will be given this card, and the directions will be read aloud. When students finish the card they will hand it in so that they can be matched with a partner. Students will be paired according to results of card. Students who had correct answers and indicated that they were willing to help will be paired with students who either had correct answers without confidence or incorrect answers due to a calculation error. Students who have an incorrect answer due to a lack of understanding will work in a small teacher-led group. Once in pairs, students will go over the problem and do another problem together.

Think-Pair-Share on the Quadratic Formula—10 minutes

The following question will be written on the board. When using the quadratic formula, at what point do you know how many solutions there will be? Students will be instructed to think about this question and record their ideas in their notes. After adequate time has passed, students will turn as directed to a neighbor sitting near them and share their ideas. Students will record their partner’s ideas along with their
own in their notes. If time allows, volunteers will be asked to share their ideas with
the class.

*Anchor Activity – as time allows

DAY 2

*Structured Academic Controversy – 70 minutes
Students will participate in a structured academic controversy over whether the
quadratic formula is the best way to solve a quadratic function. Students will be
instructed that today they will participate in a modified debate with their second base
teammates (students of different readiness levels). Pairs of students will be partnered
with other pairs based on readiness level (the groups of four will have been
predetermined). Directions for the activity will be read aloud and thoroughly
explained. After all questions have been answered, the activity will begin. Students
will take a position regarding an issue and try to convince others that their position is
right by presenting evidence. In the end, groups of four will come to a group
consensus.

*Student Self-Evaluation Update on Learning Objectives – 10 minutes
Students will review their progress on the learning Objectives for this unit. They will
provide evidence of mastery for any learning objectives they believe they have
mastered since the last update. They will answer these two questions: How do you
feel about your progress in mastering the unit’s learning objectives? What is your
plan to get on track to meet the remaining learning objectives? Students will turn in
signed and completed updates to receive teacher comments.

*Anchor Activity – 20 minutes
Students are to work independently and quietly on the Anchor Activity.

Lesson 10: The Discriminant
1 Class Period – 90 minutes

Instruction on the Discriminant – 20 minutes
Ask students to sing the quadratic formula song. After the song, write the quadratic
formula on the board and indicate that the portion of the quadratic formula under the
radical sign \((b^2 - 4ac)\) is called the discriminant. Explain that evaluating the
discriminant is no different than evaluating the quadratic formula because you still
use the a, b, and c values from the given quadratic function. Next, explain that the
value of the discriminant indicates the number of solutions for the quadratic function.
If the discriminant is positive, the quadratic function has two real solutions. If the
discriminant is zero, the quadratic function has one real solution. If the discriminant is
negative, the quadratic function has no real solutions. Students will take notes on the
discriminant and how the discriminant value corresponds to the number of solutions. We will evaluate the discriminant for several quadratic functions on the board. Once we have found the value of the discriminant, we will match the value with the correct number of solutions. Students will participate by writing each problem in their notes. They will also respond to teacher-posed questions during the evaluation process, such as “What is the ‘a’ value for this quadratic function?” After several examples, students will learn that they will be exploring the discriminant further through a TriMind activity.

*TriMind Activity—50 minutes

The TriMind activity features tasks focused on exploring the discriminant according to the three different intelligence types. Students will be given the TriMind activity worksheet, and the directions will be read aloud. Students will be instructed to work on the activity that correlates with their intelligence type as decided by a survey earlier in the year. The directions for each of the three activities will be read aloud. Students will be informed that although they will work individually on the activity itself, they will be put into groups later in the class period to present their work to each other. These groups will be formed by the teacher according to intelligence types so that each group has at least one student from each intelligence type if possible. An ideal group would be three students representing all three intelligence types. After the directions, students will work on the TriMind activity.

*Anchor Activity— as time allows

*Peer Grouping to Present TriMind Activity—20 minutes

Students will be divided into groups to present their work from the TriMind activity to each other. (These groups will have been formed by the teacher according to intelligence types so that each group has at least one student from each intelligence type if possible. An ideal group would be three students representing all three intelligence types.) Before dispersing into groups, directions will be given to the whole class. Each presenter should introduce his/her assigned task before showing the work that he/she created for that task. The presentation should last three to five minutes and include a detailed visual and verbal explanation of what was done. After each presentation, group members may ask questions of the presenter.

After the directions are given, students will join their group members somewhere around the room. Once in their groups, the students will decide what order to present. They will each present their work and ask appropriate questions of each other’s work. If time permits, the whole class will come together and volunteers will present their work to the class.

**Lesson 11: Conclusion of Unit & Unit Assessment**

1 Class Period— 90 minutes
Review, Q & A Session, and Unit Wrap-Up—45 minutes
The review session will focus on all of the concepts and skills for the entire unit. Each lesson will be briefly revisited with the most important concepts discussed and/or practiced as a class. Next, there will be a question and answer session. Students will be encouraged to review their learning objectives and ask questions regarding confusing or problematic targets. Students may also ask other questions pertaining to the unit and the upcoming assessment. Lastly, we will do a short unit wrap-up. The wrap-up will include a brief teacher narrated look back at the journey from the hook activities to the final summative assessment. Independent Study presentations will be at this point.

Summative Assessment—45 minutes
The final summative assessment will cover all of the learning objectives for the unit.
Learning Objective Checklist Packet

The Learning Objectives:

As a result of this unit, you should be able to honestly say:

- I can explain the format of a quadratic function and how it differs from other functions.
- I can define the term solutions and explain the potential real-world meaning of solutions.
- I can describe the components of a quadratic function and what they represent.
- I can explain the relationship between the components of the equation and the graphed parabola.
- I can explain why and how changing the components of the equation affects the quadratic function.
- I can explain why a parabola results from graphing a quadratic function.
- I can translate an appropriate real-world situation into a quadratic function.
- I can explain how and why quadratic functions are useful representations of certain real world situations.
- I can explain the various solving methods and identify the situations in which each is most efficient.
- I can explain the difference between a quadratic function and a quadratic equation and the purpose of each.
- I can prove why only the discriminant is needed to determine the number of solutions for a quadratic.

As a result of this unit, you should be able to honestly say:

- I can identify a quadratic function.
- I can identify and define the basic elements of a parabola (minimum or maximum, axis of symmetry, domain, range, y-intercept, zeros, and vertex).
- I can graph quadratic functions.
- I can identify the solutions of a quadratic function.
- I can find the axis of symmetry and vertex of a parabola.
- I can transform quadratic functions by changing a, b, and/or c values.
- I can solve quadratic equations by graphing.
- I can solve quadratic equations by factoring.
- I can solve quadratic equations using square roots.
- I can solve quadratic equations by using the quadratic formula.
- I can determine the number of solutions (of a quadratic equation) by using the discriminant.
<table>
<thead>
<tr>
<th>Learning Objective</th>
<th>Date Mastered</th>
<th>Evidence of Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can identify a quadratic function.</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>I can identify and define the basic elements of a parabola (minimum or maximum, axis of symmetry, domain, range, y-intercept, zeros, and vertex).</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>I can define the term solutions and explain the potential real-world meaning of solutions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can graph quadratic functions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can explain why a parabola results from graphing a quadratic function.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can find the axis of symmetry and vertex of a parabola.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can transform quadratic functions by changing a, b, and/or c values.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can explain the relationship between the components of the equation and the graphed parabola.</td>
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<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>I can explain how and why quadratic functions are useful representations of certain real world situations.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Learning Objective Progress Evaluation

1. Update your learning objective checklist.
2. Honestly answer the following questions.

How do you feel about your progress in mastering the unit’s learning objectives?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

What is your plan to get on track to meet the remaining learning objectives?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

Student Signature ______________

Mrs. Sanders Comments:
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
Pre-assessment: Quadratic Function Unit

Directions: This is a pre-assessment, which means it does not count for a grade. The goal is to find out how much you already know about the unit we are about to learn. Please do the best you can to complete the following problems. If you have no idea, take your best guess and move on. Remember that it is perfectly okay to not know this material because we have not covered it yet! 😊

1. Which of the following is a quadratic function?
   A. \( y = x^2 \)  
   B. \( y = 5x^2 + 2x + 1 \)  
   C. \( y = 8x + 4 \)  
   D. \( y = 6x^3 + 2x + 4 \)  
   E. both A & B  
   F. All of the above

2. Describe the graph of \( y = 2x^2 + 4x + 6 \).

3. Find the axis of symmetry of \( y = 4x^2 + 8x + 3 \) Answer is \( x = \)

4. Use the axis of symmetry from #3 to find the vertex. Answer is ( , )

5. Graph \( y = 3x^2 + 2x - 5 \)
6. Solve for $x$ by using square roots. $y = x^2 - 25$ Answers $x =$ and $x =$

7. Factor $y = x^2 + 7x + 12$
   $y = ( ) ( )$

8. Solve $y = x^2 - 6x + 5$ with the quadratic formula. Answer is $x =$ and $x =$

9. Describe how the following quadratic functions would look different on a graph. $y = x^2$ and $y = 5x^2 + 7$

Match the definition with the term in the box.

<table>
<thead>
<tr>
<th>x-intercept</th>
<th>zeros</th>
<th>discriminant</th>
<th>vertex</th>
<th>domain</th>
<th>maximum range</th>
</tr>
</thead>
<tbody>
<tr>
<td>axis of symmetry</td>
<td>minimum</td>
<td>y-intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. The $y$-values (or output values) of the parabola
11. The highest point on a parabola that opens downward
12. $b^2 - 4ac$
13. The invisible line that divides a parabola into halves
14. The maximum or minimum of a parabola
15. The point(s) where the parabola crosses the $x$-axis
16. The point(s) where the parabola crosses the $y$-axis
17. The $x$-values (or input values) of the parabola
18. The lowest point on a parabola that opens upward
19. Solutions to quadratic functions

20. Please choose your confidence level for the concepts covered in this pre-assessment. Low 1 2 3 4 5 High
1. Which of the following is a quadratic function? E. both A & B

2. Describe the graph of \( y = 2x^2 + 4x + 6 \).
   This shape of this graph is a parabola, which is a curved line that opens upward. Also acceptable is a U shape.

3. Find the axis of symmetry of \( y = 4x^2 + 8x + 3 \) Answer is \( x = -1 \)

4. Use the axis of symmetry from 3a to find the vertex. Answer (-1,-1)

5. Graph \( y = 3x^2 + 2x - 5 \)

6. Solve for \( x \) by using square roots. \( y = x^2 - 25 \) Answers \( x = 5 \) and \( x = -5 \)

7. Factor \( y = x^2 + 7x + 12 \) \( y = (x + 3)(x + 4) \)

8. Solve \( y = x^2 - 6x + 5 \) with the quadratic formula. Answer \( x = 5 \) and \( x = 1 \)

9. Describe how the following quadratic functions would look different on a graph.
   \( y = x^2 \) and \( y = 5x^2 + 7 \)
   \( y = x^2 \) and \( y = 5x^2 + 7 \) both open upward, but \( y = 5x^2 + 7 \) is narrower and translated five units up on the y-axis.
10 - 19 Match the definition with the term in the box.

<table>
<thead>
<tr>
<th>x-intercept</th>
<th>zeros</th>
<th>discriminant</th>
<th>vertex</th>
<th>domain</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>range</td>
<td></td>
<td></td>
<td>axis of symmetry</td>
<td>minimum</td>
<td>y-intercept</td>
</tr>
</tbody>
</table>

10. The y-values (or output values) of the parabola \( \text{range} \)
11. The highest point on a parabola that opens downward \( \text{maximum} \)
12. \( b^2 - 4ac \) \( \text{discriminant} \)
13. The invisible line that divides a parabola into halves \( \text{axis of symmetry} \)

14. The maximum or minimum of a parabola \( \text{vertex} \)
15. The point(s) where the parabola crosses the x-axis \( \text{x-intercept} \)
16. The point(s) where the parabola crosses the y-axis \( \text{y-intercept} \)
17. The x-values (or input values) of the parabola \( \text{domain} \)
18. The lowest point on a parabola that opens upward \( \text{minimum} \)
19. Solutions to quadratic functions \( \text{zeros} \)

20. Please choose your confidence level for the concepts covered in this pre-assessment. All answers are acceptable. 😊
Name ____________

Anchor Activity: Quadratic Functions

Directions: Throughout this unit you will choose three of the activities below to complete. You may work on these activities in class when you have finished your work or outside of class. At the end of the unit, you are expected to turn in your work for the three activities you chose. It’s up to you, so please choose the three activities you think you would enjoy the most.

<table>
<thead>
<tr>
<th>Have a Ball</th>
<th>Do You See What I See?</th>
<th>A Mystery at Sea(World)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find a tennis (or similar) ball. Complete five tasks below and describe the ball’s height over time for each task using a graph or writing about it. Then explain whether each task illustrates a quadratic function.</td>
<td>You are doing a nature scavenger hunt. Find and list the following things. 5 items that have an axis of symmetry 3 items with a vertex point 2 animal/plant movements in the form of a parabola. Describe (using words and sketches) why each item fulfills the requirements of the list.</td>
<td>Your job is to find out as much as you can about the dolphin shows at aquariums. Use an internet search, books, magazines, etc. Once you know all about these shows, tell how dolphin performances can relate to quadratic functions, parabolas, and the quadratic formula. Think about timing and zeros. You can write about it in your own way.</td>
</tr>
<tr>
<td>1. Roll it across the floor. 2. Drop it from a height. 3. Toss it up in the air. 4. Toss it to a partner. 5. Pretend to juggle with it.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Free Space</th>
<th>Lights, Camera, Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you have your own idea that you are interested in doing that explores quadratic functions? Write up a short proposal and submit it with Mrs. Sanders. If it is approved then it can be completed as one of your three activities.</td>
<td>With a partner or a group, create your own quadratic video for YouTube. The video needs to be informational, but it can also be very creative. Tape the performance if possible. Otherwise, perform it live for the class.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Picture This</th>
<th>The Sound of Music</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw three very different parabolas and label all the parts. (You need to use a realistic domain and range.) Then write a letter or make a voice recording explaining each drawing’s details to someone who knows nothing about parabolas.</td>
<td>Write a song or rap about graphing quadratic functions. The lyrics must include information on how to do each of the six steps and what the graph should look like when complete.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Heads Up</th>
<th>What Happened?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate the time it would take a bowling ball to fall from four heights (50ft, 100ft, 150ft, and 200ft). Analyze your results and explain the inconsistent time difference between heights.</td>
<td>Write a story or a newspaper article about the life or journey of a parabola. Be sure to include all of our unit vocabulary. Incorporate the quadratic formula as well. Be creative.</td>
</tr>
</tbody>
</table>
# Unit Project: Rubric for Scoring

The following rubric shows how the work for your three chosen activities will be scored.

<table>
<thead>
<tr>
<th>Have a Ball</th>
<th>Do You See What I See?</th>
<th>A Mystery at Sea(World)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Have a Ball</strong>&lt;br&gt;Includes a sketch or written description of all five events ____/10</td>
<td><strong>Do You See What I See?</strong>&lt;br&gt;Found five items in nature with symmetry ____/5</td>
<td><strong>A Mystery at Sea(World)</strong>&lt;br&gt;Displays evidence of research on dolphin performances ____/9</td>
</tr>
<tr>
<td>Correctly identified which events were in the form of a parabola ____/10</td>
<td>Identified three items in nature with a vertex ____/6</td>
<td>Made reasonable connection to quadratic functions ____/7</td>
</tr>
<tr>
<td>Gave a logical explanation why each event was or was not a quadratic function ____/10</td>
<td>Discovered two parabolic movements in nature ____/4</td>
<td>Made reasonable connection to parabolas ____/7</td>
</tr>
<tr>
<td>**TOTAL ____/30</td>
<td>Included correct explanations of why items were on the list ____/15</td>
<td>Made reasonable connection to the quadratic formula ____/7</td>
</tr>
<tr>
<td>**TOTAL ____/30</td>
<td>**TOTAL ____/30</td>
<td>**TOTAL ____/30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Picture This</th>
<th>Free Space</th>
<th>Lights, Camera, Action</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Picture This</strong>&lt;br&gt;Drew three unique parabolas ____/9</td>
<td><strong>Free Space</strong>&lt;br&gt;To be determined on individual basis.</td>
<td><strong>Lights, Camera, Action</strong>&lt;br&gt;Video includes detailed and relevant information about quadratics ____/15</td>
</tr>
<tr>
<td>Labeled all parts of each parabola ____/9</td>
<td></td>
<td>Uses creativity to interest the viewer ____/15</td>
</tr>
<tr>
<td>Includes a detailed explanation of each drawing ____/12</td>
<td>**TOTAL ____/30</td>
<td>**TOTAL ____/30</td>
</tr>
<tr>
<td>**TOTAL ____/30</td>
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<td>**TOTAL ____/30</td>
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</tbody>
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<th>The Sound of Music</th>
<th>What Happened?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heads Up</strong>&lt;br&gt;Correct calculation and answer (in seconds) for 50ft ____/5&lt;br&gt;100ft ____/5&lt;br&gt;150ft ____/5&lt;br&gt;200ft ____/5&lt;br&gt;Accurate explanation of time differences ____/10</td>
<td><strong>The Sound of Music</strong>&lt;br&gt;Includes each graphing step and a description of the graph ____/14&lt;br&gt;Makes sense ____/8&lt;br&gt;Can be put to a tune or a beat ____/8</td>
<td><strong>What Happened?</strong>&lt;br&gt;Story/article flows well and makes sense ____/10</td>
</tr>
<tr>
<td></td>
<td>**TOTAL ____/30</td>
<td>**TOTAL ____/30</td>
</tr>
<tr>
<td>**TOTAL ____/30</td>
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<tr>
<th>Free Space</th>
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<tr>
<td><strong>Free Space</strong>&lt;br&gt;To be determined on individual basis.</td>
<td><strong>Lights, Camera, Action</strong>&lt;br&gt;Video includes detailed and relevant information about quadratics ____/15</td>
<td><strong>What Happened?</strong>&lt;br&gt;Story/article flows well and makes sense ____/10</td>
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<td></td>
<td>**TOTAL ____/30</td>
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</tr>
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<td>**TOTAL ____/30</td>
<td>**TOTAL ____/30</td>
<td>**TOTAL ____/30</td>
</tr>
</tbody>
</table>

| What Happened? | | |
|----------------|----------------|
| **What Happened?**<br>Story/article flows well and makes sense ____/10<br>Meaningfully includes unit vocabulary words ____/10<br>Includes a description of the quadratic formula ____/10 | **TOTAL ____/30 | **TOTAL ____/30 |
Congratulations! Your pretest results indicate that you would benefit from a novel challenge as opposed to completing the quadratic unit with the class. You are invited to participate in an independent study in place of the quadratic unit. The structure of the independent study is explained below, and you and your parents/guardians must both agree to the guidelines if you choose to complete the independent study. Please note that if you choose to work on the independent study, you may still be required to participate in class activities periodically throughout the unit.

The Challenge:
Students constantly wonder where math applies in the “real-world.” Your challenge is to identify and explore three occupations that use quadratic functions and/or the quadratic formula.

The Task:
After researching three occupations, you will create a report for each occupation explaining how it relates to quadratic functions. Then you will create a presentation for one of the occupations based on your report and findings. You will present this report to your classmates at the end of the unit.

The Timeline:
You will be working on this project for three weeks. You must use your time wisely, as you are expected to turn in and present your project at the end of the unit. You will be following the plan of action and checking in with Mrs. Sanders every day.

Keep in Mind:
- Many people who use quadratic functions, parabolas, and/or the quadratic formula in their jobs have no idea they are doing so because of technology.
- Be creative! Quadratics are everywhere, so it might be more interesting to choose professions that are interesting to you.
- You are expected to use multiple sources of information for your research. Suggestions: internet, encyclopedias and books, interviews, textbook, etc.
- You will be evaluating yourself on this project. Your self-evaluation will be combined with the scoring rubric to determine your overall grade.
Quadratic Function Unit: Independent Study
Guidelines & Contract

I agree to the following terms and conditions:

❖ I will be working independently on this project during class time. If I get behind schedule, I will work on my own time as needed.

❖ I will stay on task during each class period.

❖ I will not distract my classmates from their work.

❖ I will do my best work to ensure that this is a quality project.

❖ I will follow Knapp’s rules of Internet usage when researching for this project.

❖ I understand that I am responsible for creating all of my own original work for this project.

❖ I will follow the plan of action and timeline given for this project, unless otherwise approved by Mrs. Sanders.

❖ I will keep a journal of my progress and check in with Mrs. Sanders every day.

❖ I will complete this project in the three week time period allotted for unit nine.

❖ I will be graded according to a combination of my self-evaluation and the scoring rubric for this project.

Student’s Name ______________ Class Period ______

Student’s Signature ______________ Date _____________

Parent/Guardian Signature ______________ Date _____________
### Quadratic Function Unit: Independent Study

#### Scoring Rubric

**Scoring Criteria:**

<table>
<thead>
<tr>
<th>Description</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Includes list of possible occupations</td>
<td>_____/5</td>
</tr>
<tr>
<td>Initial research was conducted to determine if occupations involve quadratics.</td>
<td>_____/5</td>
</tr>
<tr>
<td>Three occupations were chosen.</td>
<td>_____/5</td>
</tr>
<tr>
<td>Logical research plan created.</td>
<td>_____/5</td>
</tr>
<tr>
<td>Multiple sources were used for research.</td>
<td>_____/10</td>
</tr>
<tr>
<td>Thorough research conducted and recorded on occupation # 1.</td>
<td>_____/15</td>
</tr>
<tr>
<td>Thorough research conducted and recorded on occupation # 2.</td>
<td>_____/15</td>
</tr>
<tr>
<td>Thorough research conducted and recorded on occupation # 3.</td>
<td>_____/15</td>
</tr>
<tr>
<td>Logical report format created.</td>
<td>_____/5</td>
</tr>
<tr>
<td>Report for occupation # 1 is detailed, accurate, and well-organized.</td>
<td>_____/10</td>
</tr>
<tr>
<td>Report for occupation # 2 is detailed, accurate, and well-organized.</td>
<td>_____/10</td>
</tr>
<tr>
<td>Report for occupation # 3 is detailed, accurate, and well-organized.</td>
<td>_____/10</td>
</tr>
<tr>
<td>Presentation on chosen occupation is detailed, accurate, and well-organized.</td>
<td>_____/10</td>
</tr>
<tr>
<td>Score on self-evaluation completed by student.</td>
<td>_____/15</td>
</tr>
</tbody>
</table>
TOTAL SCORE _____/135

Quadratic Function Unit: Independent Study
Self-Evaluation

Please complete this self-evaluation upon the completion of this project.
You will discuss this evaluation with Mrs. Sanders. When both you and
Mrs. Sanders have signed this evaluation, it will become part of your final
grade for this project.

1. I used my time wisely and stayed on task? _____/3
   3- Absolutely
   2- Mostly
   1- Not really

2. I fully completed all elements of the project? _____/5
   5- Yes
   3- Mostly
   1- Not really

3. I honored all aspects of the independent learning contract I signed.
   5- Absolutely _____/5
   4- Mostly
   3- Sometimes
   2- Not really
   1- We rushed through it

4. This project represents my best work. _____/2
   2- Yes
   1- Somewhat
   0- No

TOTAL SCORE_____/15

Things I did well:
________________________________________________________________________________________
________________________________________________________________________________________

Things I could have done better:
________________________________________________________________________________________
________________________________________________________________________________________
Quadratic Function Unit: Independent Study

Plan of Action & Timeline

*Please Note:* This timeline and plan of action may be modified with teacher approval.

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Activities</th>
<th>Date Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1: Days 1-3</td>
<td>List possible occupations involving quadratic function/quadratic formula.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Initial research to determine if occupations involve quadratics.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chose three occupations to focus upon.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Create research plan.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Line up sources to obtain information.</td>
<td></td>
</tr>
<tr>
<td>Week 1: Days 4-5</td>
<td>Research occupation # 1 and record findings.</td>
<td></td>
</tr>
<tr>
<td>Week 2: Days 1-2</td>
<td>Research occupation # 2 and record findings.</td>
<td></td>
</tr>
<tr>
<td>Week 2: Days 3-4</td>
<td>Research occupation # 3 and record findings.</td>
<td></td>
</tr>
<tr>
<td>Week 2: Day 5</td>
<td>Complete any unfinished research on occupations 1, 2, and 3.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Create a report format to record and present your research findings.</td>
<td></td>
</tr>
<tr>
<td>Week 3: Day 1</td>
<td>Create report for occupation # 1.</td>
<td></td>
</tr>
<tr>
<td>Week 3: Day 2</td>
<td>Create report for occupation # 2.</td>
<td></td>
</tr>
<tr>
<td>Week 3: Day 3</td>
<td>Create report for occupation # 3.</td>
<td></td>
</tr>
<tr>
<td>Week 3: Day 4</td>
<td>Prepare a presentation on one of your three chosen occupations.</td>
<td></td>
</tr>
<tr>
<td>Week 3: Day 5</td>
<td>Give a presentation to your classmates on your project.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Submit your three finalized reports to</td>
<td></td>
</tr>
</tbody>
</table>
You will have 20 minutes to complete the activity of your choice individually. You will present your work to a group of your classmates following this activity.

The following components of parabolas must be addressed in every activity: Vertex, Axis of Symmetry, Minimum or Maximum, Zeros, Y-Intercept, Domain, and Range.

Option 1: Draw (on graph paper) and label at least five parabolas. Try to have your five (or more) parabolas be as different as possible. Change and transform them in various ways, but they must still be accurate parabolas with all the required components.

Option 2: Write a letter to a family member or friend describing the interesting new stuff you are learning in math. After reading your letter, the recipient should be able to answer the following questions:
- What is a parabola?
- What are the parts/components of a parabola?
- Where are these parts/components located?

Option 3: Write a song or a rap about the interesting new stuff you are learning in math. After listening to your song/rap, the listener should be able to answer the following questions:
- What is a parabola?
- What are the parts/components of a parabola?
- Where are these parts/components located?

Option 4: Create a skit in which you answer the questions below. You have a lot of freedom to create your skit, but remember that you will be presenting it to receive your grade. Some skit suggestions: Pretend to be a weatherman, use props, act like a teacher, etc.
- What is a parabola?
- What are the parts/components of a parabola?
- Where are these parts/components located?

Option 5: Choose a plant or animal in nature. Compare this natural item to a parabola and its components. Use all the components listed at the top of this page. Include a sketch of your natural item with labels that belong to a parabola.
- For example: A parabola is like a tree. The vertex of a parabola would be like the top needle of an evergreen tree because…
**SCORING CRITERIA:** Work showed accurate understanding of all components of parabolas **___/15**

<table>
<thead>
<tr>
<th>Exit Card: Lesson 3</th>
</tr>
</thead>
</table>

Today we learned how to find the axis of symmetry and the vertex of a parabola. In order to demonstrate your understanding of the lesson, please choose one of the following options to complete. This will be your ticket to leave class today.

Option 1: Write a short paragraph explaining how to find the vertex of a quadratic function. Explain each step in detail.

Option 2: Find the vertex of the following quadratic function. \( y = x^2 + 6x + 9 \)

Option 3: Have a brief conversation with Mrs. Sanders about how to find the vertex of a quadratic function. If you choose this option, you may raise your hand to complete this task while your classmates are working on their exit card. Please keep in mind that if several students choose this option, you may need to wait for a moment during your break if there is a line of students to complete this task.

Option 4: A quadratic function has an axis of symmetry of 2 and a vertex of \((2, 0)\). Work backward to find a quadratic function that would produce these numbers. (There are numerous possibilities.)

*I chose option # ____  

Work Space:*
Problem Based Learning
Golf Course Conundrum

The Situation:
Sandy Pines Golf Course is losing customers. Golfers do not want to golf at their course anymore because of a strange phenomenon on hole number nine. When golfers tee off at hole nine, they watch their golf ball soar into the air towards the green. Amazingly, when many golfers go to find their ball for their next shot, their ball is nowhere to be found on the fairway, the green, or the surrounding rough. It’s like hole nine is the Bermuda Triangle for golf balls.

Since you are a respected researcher, Sandy Pines has hired you to figure out what’s happening on hole nine and fix it. They have already done some testing and determined that the function describing the trajectory of the golf balls that go missing is \( y = -5x^2 + 50x \). They want their customers back and it’s up to you to save their golf course from closing its doors.

The Plan:
In assigned groups of four, you will explore this problem. Each member of your group must choose a role.

- **Facilitator**: Makes sure everyone is involved, staying on task, and moving forward to solve the problem.

- **Time Keeper**: Plans the amount of time for each section of the problem. Keeps the group on schedule to finish in the allotted time.

- **Recorder**: Takes notes on the group's work.
Retriever: This group member can be up and moving around the classroom to get materials or access resources.

Problem Based Learning:
Golf Course Conundrum

Group Members and Roles:
Facilitator-
Time Keeper-
Recorder-
Retriever-

What do we do now?

First things first…
Brainstorm some ideas about the problem by filling in the graphic organizer.
Consider why this problem relates to quadratic functions.

Next…
Follow through on your plans for researching the problem and solution.

Decision Time…
Conclude what has been causing the problem and how the problem can be solved.

Presentation…
Present your solution and rationale to your classmates who will represent the golf course owners.

Finally…
Evaluate yourselves and your group on your work for this activity.

Q: What resources can we use? A: The internet, your textbook, the World Books, the library, interviews, etc.

Q: How much time do we have? A: 90 minutes to work today and tomorrow 30 minutes for all presentations tomorrow

Q: What if my group can’t get along? A: Put aside your differences and work together as a team for this project.

Q: Is this going to count for a grade? A: Yes. There is a rubric attached so that you can see how you will be graded.
Golf Course Conundrum: Rubric for Scoring

This project will be scored as a group.

SCORING CRITERIA 2 = Yes 1 = Somewhat 0 = No

The problem was clearly identified. ____/2

Information the group knew was listed. ____/2

Information the group needed to know was identified and listed. ____/2

A logical plan was formed for what the group was going to do. ____/2

Research was conducted using multiple sources. ____/2

Research was recorded. ____/2

The cause of the problem was identified. ____/2

A logical solution was reached. ____/2

Evidence for the solution was given. ____/2

Presentation included all steps of process. ____/2

Every group member had a meaningful role in the presentation. ____/2
You're on a Roll

Directions: You and a partner will take turns rolling a die and completing the corresponding task. In each box, there is a job for both you and your partner to complete. "You" in each box refers to the person who rolled that number. You each get one do-over roll. Record your answers on the sheet provided. Correct your answers and then evaluate yourselves honestly.

<table>
<thead>
<tr>
<th>Recall</th>
<th>Comprehension</th>
<th>Calculate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>You:</strong> List the six steps for graphing a quadratic function.</td>
<td><strong>You:</strong> How can you tell the difference between a quadratic and a linear function with a table of ordered pairs?</td>
<td><strong>You:</strong> Find the axis of symmetry for the quadratic function $Y = -3x^2 + 6x + 2$.</td>
</tr>
<tr>
<td><strong>Partner:</strong> Write the formula for finding axis of symmetry.</td>
<td><strong>Partner:</strong> How can you tell the difference on a graph?</td>
<td><strong>Partner:</strong> Find the vertex using the axis of symmetry your partner just found.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examine</th>
<th>Evaluate</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>You:</strong> Two quadratic functions (parabolas) are graphed in quadrant I. $y = 2x^2$ and $y = \frac{1}{2}x^2$ Which one would you want to show your bank account balance over time? Why?</td>
<td><strong>You:</strong> Explain whether you agree with the statement “Quadratic Functions are like burgers…they are all the same with different toppings.”</td>
<td><strong>You:</strong> Make a unique design using only parabolas on a graph. Explain how you transformed the different parabolas to create this design.</td>
</tr>
<tr>
<td><strong>Partner:</strong></td>
<td><strong>Partner:</strong> If they were like burgers, what would the toppings be? 😊</td>
<td><strong>Partner:</strong> Did your partner utilize all the different</td>
</tr>
</tbody>
</table>
Directions: Record your work from the activity on this sheet. You and your partner can turn in one paper. Write your initials above each task that you complete so it is clear who is “you” and who is the “partner” for each task.

<table>
<thead>
<tr>
<th>Square One:</th>
<th>Square Two:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initials:</strong></td>
<td><strong>Initials:</strong></td>
</tr>
<tr>
<td><strong>Six Steps:</strong></td>
<td><strong>How can you tell the difference between a quadratic and a linear function with a table of ordered pairs?</strong></td>
</tr>
<tr>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
</tr>
<tr>
<td><strong>Axis of symmetry formula</strong></td>
<td><strong>How can you tell the difference on a graph?</strong></td>
</tr>
</tbody>
</table>

**Notes:**

- For the square one, students need to complete six steps.
- For the square two, students need to compare quadratic and linear functions using a table of ordered pairs and a graph.
- Students should write their initials above each task to indicate who completed it.
**Square Three:**
Initials ____
Axis of symmetry   \( x = \)

Initials ____
Vertex is ( , )

**Square Four:**
Initials ____
Circle your choice   \( y = 2x^2 \) or   \( y = \frac{1}{2}x^2 \)
Explain your choice.

____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________

Initials ____
Do you agree?  Yes or  No
Explain why or why not.

____________________________________________________________________
____________________________________________________________________
____________________________________________________________________

**Square Five:**
Initials ____
Explain whether you agree with the statement "Quadratic Functions are like burgers...they are all the same with different toppings."
Initials ____
If they were like burgers, what would the toppings be?
_______________________________________________________
_______________________________________________________
_______________________________________________________

_Square Six:_   Initials ____
Make a unique design using only parabolas on a graph.

Explain how you transformed the different parabolas to create this design.
_______________________________________________________
_______________________________________________________

Initials ____
Did your partner utilize all the different transformations? Yes or No Explain.
Answer Key for

You’re on a Roll

Square One:
Six Steps
1. Find the axis of symmetry
2. Find the vertex
3. Find the y-intercept
4. Graph what you know
5. Find more points
6. Reflect and Connect

Axis of symmetry formula

\[ x = \frac{-b}{2a} \]

Square Two:
How can you tell the difference between a quadratic and a linear function with a table of ordered pairs?
The ordered pairs of a quadratic function have a constant change in x-values, but they have constant second differences in y-values. A linear function has constant first differences in both x-values and y-values.

How can you tell the difference on a graph?
On a graph, a linear function is a straight line. A quadratic function is a parabola.

Square Three:
Axis of symmetry \[ x = 1 \]
Vertex is (1, 5)

**Square Four:**
You should have chosen \( y = 2x^2 \)

\( y = 2x^2 \) will be narrower. A narrower graph means that the y-values are increasing faster. Since the y-values represent your money, you want them to increase faster. The faster the better because you will end up with more money.

Do you agree?

*If your partner chose \( y = 2x^2 \) you should have agreed.*

**Square Five:**
Explain whether you agree with the statement “Quadratic Functions are like burgers...they are all the same with different toppings.”

Opinions may vary, but technically this is true because \( y = x^2 \) is the parent function for all other quadratic functions.

If they were like burgers, what would the toppings be?

*The toppings would be the ways to change the burger, which would include direction of opening, width, and y-intercept.*

**Square Six:**
Make a unique design using only parabolas on a graph.

*Designs will vary.*

Explain how you transformed the different parabolas to create this design.

*You should have mentioned how you changed direction of opening, width, and y-intercept.*

Did your partner utilize all the different transformations?
The answer should be yes if, and only if, your partner made parabolas that opened up and down, had different widths, and had different y-intercepts.

Profiler Activity: Solving a Quadratic Function

_The Situation:_ Fast forward your life fifteen years. You have been given a new task at work that reminds you of something from your past. You close your eyes and think about where and when you have seen this type of task before. Wait a minute... this task reminds you of Mrs. Sanders’ Algebra class and quadratic functions. Go figure!

_Directions:_ You will complete the job for the occupation you chose (via sticky note). You may work alone or in groups of two or three with classmates who also chose your occupation. The goal is to fully complete each task and prove that you are a master of your craft (your job) and of solving quadratic functions. Complete details for each task are found on the following pages. Each task is worth twenty points. Please see the attached rubric for scoring criteria. Enjoy!

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engineer</strong></td>
<td>Design a water fountain for a local park. You will need to provide a sketch of the design and the corresponding quadratic function for the builders.</td>
</tr>
<tr>
<td><strong>Musician</strong></td>
<td>Create a song or rap about finding the solution(s) to quadratic functions by graphing using the six-steps.</td>
</tr>
<tr>
<td><strong>Builder</strong></td>
<td>Build a three-dimensional model of a parabola. Someone viewing your model should be able to see which parts of the parabola correspond to which of the six-steps.</td>
</tr>
<tr>
<td><strong>Artist</strong></td>
<td>You are selling the &quot;secret&quot; to solving quadratic functions. Create a flyer, poster, or brochure about your product and why someone would want to buy it.</td>
</tr>
<tr>
<td><strong>Auctioneer</strong></td>
<td>You are selling the “secret” to solving quadratic...</td>
</tr>
</tbody>
</table>
functions in the auction house. You are the auctioneer that will describe the product and take bids.

Profiler Activity: Solving a Quadratic Function

**Engineer**

Design a water fountain for a local park. You will need to provide a sketch of the design and the corresponding quadratic function for the builders.

**Specific details of your job:**
The city wants you to design a water fountain for their park, but they have a few requirements that your fountain must meet. The spray of water will form a beautiful parabola. The spray of water from the fountain must reach six feet in height. The spray of water take six seconds to hit the ground. (Please think about these two requirements as they correspond to two key parts of the parabola. Each 6 belongs in a different ordered pair that corresponds to an important point on the parabola. You should automatically know the other value in each ordered pair.)

You should make a sketch of your design before beginning your calculations. Once you have sketched your design, you should know the axis of symmetry, the vertex, and one of the solutions. Work backward through the six-steps of graphing to find a quadratic function that will produce these values. The city needs a detailed sketch and the correct quadratic function to give to the builders.

*Make sure that your quadratic function correctly corresponds to the axis of symmetry, vertex, and zero specified in the problem. You have 55 minutes to complete this job.*
**Profiler Activity: Solving a Quadratic Function**

<table>
<thead>
<tr>
<th><strong>Musician</strong></th>
<th>Create a song or rap about finding the solution(s) to quadratic functions by graphing using the six-steps.</th>
</tr>
</thead>
</table>

**Specific details of your job:**
Your job is to create a song or a rap. The subject of your musical composition is quadratic functions and how to find solutions. You must include details about the solutions of quadratic functions and the six-steps to graphing that will help you find solutions. To make your song/rap interesting you need to include a reason why you want to find the solution(s) to quadratic functions. For example, maybe your sister needs them or maybe you love them. Your song/rap will be more entertaining if you incorporate some type of story line into the lyrics. You must have at least two verses and a chorus.

*Make sure that your song/rap makes sense and includes specific and correct details about solutions, the six-steps to graphing, and the purpose of graphing. Keep in mind that you will be performing the song/rap. You have 55 minutes to complete this job.*

**Scoring Criteria:**
Accurate details about solutions are provided _____/5
Accurate details about the six-steps are provided _____/5
The relationship between graphing and finding
solutions is correctly explained
____/5
 Lynx follow the requirements
____/5

Profiler Activity: Solving a Quadratic Function

<table>
<thead>
<tr>
<th>Builder</th>
<th>Build a three-dimensional model of a parabola. Someone viewing your model should be able to see which parts of the parabola correspond to which of the six-steps.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Specific details of your job:</strong> Your job is to build a three-dimensional model of a parabola. Your model will be put on display at a local school (Knapp Charter Academy). You can be creative in what materials you use to make this three-dimensional model. The materials we have available include playdough, paper, pipe cleaner, straws, felt, and string. Your model must be accurate, built to scale, and each part of the parabola must be labeled. Your model must also show how the different parts of the parabola correspond to each of the six steps. For example, the axis of symmetry found in step 1 should be different somehow (color, material, texture, etc.) than the vertex found in step 2. You must provide a key that shows the relationship you create. For example, if you make the axis of symmetry red your key should show that red = axis of symmetry = step 1.</td>
</tr>
<tr>
<td></td>
<td><em>You might want to start by graphing a quadratic function on a piece of graphing paper. Then you can make the drawing into a three-dimensional model. You have 55 minutes to complete this job.</em></td>
</tr>
</tbody>
</table>

**Scoring Criteria:**
3-D model is accurately built to scale _____/5
Profiler Activity: Solving a Quadratic Function

You are selling the “secret” to solving quadratic functions. Create a flyer, poster or brochure about your product and why someone would want to buy it.

**Specific details of your job:**
A company has discovered a process of six steps that can uncover the mysterious solution(s) to a quadratic function. Following these six steps will enable someone to graph a quadratic function thereby discovering the solution(s). The company has hired you to help them sell the six step process. You are to create an advertising flyer, poster, or a brochure that can be handed out and/or displayed publicly to entice people to buy the product. The flyer/poster/brochure you create must include information about solutions to quadratic functions and about the six steps. You must also include reasons why someone would want to buy these steps and find the solutions to quadratic functions. Lastly, there must be an illustration of some type that relates to a parabola or a quadratic function. Be creative!

*You might want to start by compiling your information and making a sketch. Then you can create your final product. Remember you are trying to convince someone to buy the company’s product. You have 55 minutes to complete this job.*

**Scoring Criteria:**
Accurate details about solutions are provided ____/5
Accurate details about the six-steps are provided ____/5
Profiler Activity: Solving a Quadratic Function

**Auctioneer**

You are selling the "secret" to solving quadratic functions in the auction house. You are the auctioneer that will describe the product and take bids.

**Specific details of your job:**

You are the auctioneer at a local auction house. At tonight's auction (to raise money for charity) someone has donated a "secret" process of six steps that can uncover the solution(s) to a quadratic function. Following these six steps will enable someone to graph a quadratic function thereby discovering the solution(s). Your job is to auction off the six step process for the highest price possible. You must create a script that you will use to introduce and explain the item up for sale. The script you create must include information about solutions to quadratic functions and about the six steps. You must also include reasons why someone would want to buy these steps and find the solutions to quadratic functions. Lastly, the script should include some fictional bids to add to the commentary. Be creative, be funny, and make it interesting!

*You will be turning in a written copy of your script, but you will also perform your script with your best impression of a fast-talking auctioneer. If you are working with a partner or in a group of three, your script should include parts for each person. You have 55 minutes to complete this job.*

**Scoring Criteria:**

- Accurate details about solutions are provided ____/5
- Accurate details about the six-steps are provided ____/5
- Reasons someone would want to buy this item are provided ____/5
### Profiler Activity: Rubric for Scoring

**Student Name(s):** __________, __________  **TOTAL SCORE ____/20**

| **Engineer** | An accurate detailed sketch is provided ____/10  
|             | A correct quadratic formula is provided ____/10  
|             | **TOTAL SCORE ____/20** |
| **Musician** | Accurate details about solutions are provided ____/5  
|             | Accurate details about the six-steps are provided ____/5  
|             | The relationship between graphing and finding solutions is correctly explained ____/5  
|             | Lyrics follow the requirements ____/5  
|             | **TOTAL SCORE ____/20** |
| **Builder** | 3-D model is accurately built to scale ____/5  
|             | 3-D model is labeled correctly ____/5  
|             | Parts of the parabola are displayed differently to represent the six steps of graphing ____/5  
|             | An accurate key is provided ____/5  
|             | **TOTAL SCORE ____/20** |
| **Artist**  | Accurate details about solutions are provided ____/5  
|             | Accurate details about the six-steps are provided ____/5  
|             | Reasons someone would want to solve quadratic functions are provided ____/5  
|             | Accurate illustration is included ____/5  
|             | **TOTAL SCORE ____/20** |
| **Auctioneer** | Accurate details about solutions are provided ____/5  
|              | Accurate details about the six-steps are provided ____/5  
|              | Reasons someone would want to buy this item are provided ____/5  
|              | Script is lively and contains parts for each group member ____/5  
|              | **TOTAL SCORE ____/20** |
Practice Makes Perfect- Solving Quadratics

Directions: There are three sections to this assignment. Section I focuses on solving quadratic equations algebraically. Section II focuses on using square roots to solve quadratic equations. Section III features a few open-ended questions. Please show your work for each problem. Use the back of the page if needed.

SECTION I: Solving quadratic functions algebraically.

1. Explain the Zero Product Property.

The problems in # 2 & 3 are already factored for you! Use the Zero Product Property to solve for x. Then find the original quadratic equation that the factors came from by using the distributive property and/or FOIL.

2. \((x)(x + 3) = 0\)  The original quadratic equation is \(0 = \)

3. \((2x + 6)(x - 4) = 0\)  The original quadratic equation is \(0 = \)

For problems # 4 – 6, factor the quadratic equations with reverse FOIL and use the zero product property to solve.

4. \(0 = x^2 - 9x + 14\)  5. \(0 = x^2 + 8x - 9\)
SECTION II: Solving quadratic functions using square roots.
The quadratic equations in # 1 - 4 are ready to solve. Solve them by using square roots. Round if necessary. Your answer should be \( x = \) and \( x = \) OR \( x = \pm \)
Be careful because one of these is unique and will have a different solution.

1. \( x^2 = 64 \) 
2. \( x^2 = -40 \)

3. \( 0 = x^2 \) 
4. \( 50 = x^2 \)

For problems # 5 - 7, solve for \( x \) by using square roots. Remember that you must isolate \( x^2 \) before you can take the square root of both sides.

5. \( 0 = 2x^2 - 50 \) 
6. \( 0 = 4x^2 - 16 \) 
7. \( 0 = 2x^2 + 32 \)
SECTION III: Think about it!

1. What is different about the quadratic equations in Section I (algebraic solving) versus Section II (square root solving)? Hint: Think about the a, b, and c values.

2. You can not solve the following quadratic equation by using square roots. Explain why not. (If you do not believe me, try it for yourself first.) $0 = x^2 + 4x - 25$

3. Create your own quadratic equation and solve it (by graphing, algebraically, or using by using square roots). If you want, you may simply modify an equation from section I or II.
Practice Makes Perfect- Solving Quadratics

Directions: There are three sections to this assignment. Section I focuses on solving quadratic equations algebraically. Section II focuses on using square roots to solve quadratic equations. Section III features a few open-ended questions. Please show your work for each problem. Use the back of the page if needed.

SECTION I: Solving quadratic functions algebraically.

1. Explain the Zero Product Property.

The problems in # 2 & 3 are already factored for you! Use the Zero Product Property to solve for x. Then find the original quadratic equation that the factors came from by using the distributive property and/or FOIL.

2. \((x)(2x - 8) = 0\) The original quadratic equation is \(0 =\)

3. \((3x + 9)(4x - 2) = 0\) The original quadratic equation is \(0 =\)

For problems # 4 – 6, factor the quadratic equations with reverse FOIL and use the zero product property to solve.

4. \(0 = x^2 - 5x - 24\)

5. \(0 = 2x^2 + 14x + 24\)
0 = (   ) (   )  0 = (   ) (   )

6. $0 = 12x^2 + 6x - 6$
   $0 = (   ) (   )$

SECTION II: Solving quadratic functions using square roots.
The quadratic equations in # 1 - 4 are ready to solve. Solve them by using square roots. Round if necessary. Your answer should be $x = \quad$ and $x =$
OR $x = \pm$
1. $x^2 = 121$
2. $x^2 = -91$
3. $0 = x^2$
4. $70 = x^2$

For problems # 5 - 7, solve for $x$ by using square roots. Remember that you must isolate $x^2$ before you can take the square root of both sides.
5. $0 = 4x^2 - 49$
6. $0 = 5x^2 - 50$
7. $0 = 3x^2 + 27$
SECTION III: Think about it!

1. What is different about the quadratic equations in Section I (algebraic solving) versus Section II (square root solving)?

2. You cannot solve the following quadratic equation by using square roots. Explain why not. (If you do not believe me, try it for yourself first.😊) \(0 = x^2 + 4x - 25\)

3. Create your own quadratic equation and solve it (by graphing, algebraically, or using by using square roots). Optional: Create one to solve each way.
Practice Makes Perfect - Solving Quadratics

Directions: There are three sections to this assignment. Section I focuses on solving quadratic equations algebraically. Section II focuses on using square roots to solve quadratic equations. Section III features a few open-ended questions. Please show your work for each problem. Use the back of the page if needed.

SECTION I: Solving quadratic functions algebraically.

1. Explain the Zero Product Property.

Problems # 2 & 3 are already factored for you! Use the Zero Product Property to solve for x. Then find the original quadratic equation that the factors came from by using the distributive property and/or FOIL.

2. \((x)(x - 4) = 0\)  
   Distribute to get the original quadratic equation. \(0 = \)

3. \((x - 3)(x + 5) = 0\)  
   Use FOIL to get the original quadratic equation. \(0 = \)

For problems # 4 – 6, factor the quadratic equations with reverse FOIL and then use the zero product property to solve.
SECTION II: Solving quadratic functions using square roots.

The quadratic equations in # 1 - 4 are ready to solve. Solve them by using square roots. Round if necessary. Your answer should be $x =$ and

$\text{OR } x = \pm$

Be careful because one of these is unique and the answer will be no solutions.

1. $x^2 = 9$

2. $x^2 = -16$

3. $0 = x^2$

4. $20 = x^2$

For problems # 5 - 7, solve for $x$ by using square roots. Remember that you must isolate $x^2$ before you can take the square root of both sides.

5. $0 = x^2 - 25$

6. $0 = 2x^2 - 8$

7. $0 = x^2 + 16$
SECTION III: Think about it!

1. What is different about the quadratic equations in Section I (algebraic solving) versus Section II (square root solving)? Hint: Think about the a, b, and c values.

2. You can not solve the following quadratic equation by using square roots. Explain why not. (If you do not believe me, try it for yourself first.😊) 0 = x² + 4x - 25

3. Create your own quadratic equation and solve it (by graphing, algebraically, or using by using square roots). If you want, you may use an equation from section I or II and just change the numbers.
Section I:
1. $x = 0$ or $x = -3$  \hspace{1em} \text{original equation } 0 = x^2 + 3x

2. $x = 3$ or $x = 4$  \hspace{1em} \text{original equation } 0 = 2x^2 - 14x + 24

3. $(x - 7)(x - 2)$
   \hspace{1em} $x = 7$ or $x = 2$

4. $(x + 9)(x - 1)$
   \hspace{1em} $x = -9$ or $x = 1$

5. $(2x + 2)(x + 5)$
   \hspace{1em} $x = -1$ or $x = -5$

Section II:
1. $x = ± 8$

2. no solution

3. $x = 0$

4. $x ≈ ± 7.1$

5. $x = ± 5$

6. $x = ± 4$

7. no solution

Section III:
1. If you are solving using square roots, the b value in the quadratic equation must equal zero.
2. You cannot solve because there are two x values in the equation that you cannot combine.
3. Answers may vary.

Practice Makes Perfect—Answer Key

Section I:
1. \( x = 0 \) or \( x = 4 \)  
   original equation \( 0 = 2x^2 - 8x \)
2. \( x = -3 \) or \( x = \frac{1}{2} \)  
   original equation \( 0 = 12x^2 + 30x - 18 \)
3. \( (x + 3)(x - 8) \)
   \( x = -3 \) or \( x = 8 \)
4. \( (2x + 6)(x + 4) \)
   \( x = -3 \) or \( x = -4 \)
5. \( (4x - 2)(3x + 3) \)
   \( x = \frac{1}{2} \) or \( x = -1 \)

Section II:
1. \( x = \pm 11 \)
2. no solution
3. \( x = 0 \)
4. \( x \approx \pm 8.4 \)
5. \( x = \pm 7/2 \)
6. \( x \approx \pm 3.2 \)
7. no solution

Section III:
1. If you are solving using square roots, the b value in the quadratic equation must equal zero.
2. You cannot solve because there are two x values in the equation that you cannot combine.
3. Answers may vary.

Version B

Practice Makes Perfect- Answer Key

Section I:
1. \(x = 0\) or \(x = 4\) original equation \(0 = x^2 - 4x\)

2. \(x = 3\) or \(x = -5\) original equation \(0 = x^2 + 2x - 15\)

3. \((x + 2)(x + 5)\)
   \(x = -2\) or \(x = -5\)

4. \((x - 2)(x - 3)\)
   \(x = 2\) or \(x = 3\)

5. \((x + 4)(x - 1)\)
   \(x = -4\) or \(x = 1\)

Section II:
1. \(x = \pm 3\)

2. no solution

3. \(x = 0\)

4. \(x \approx \pm 4.5\)

5. \(x = \pm 5\)

6. \(x = \pm 2\)

7. no solution

Section III:
1. If you are solving using square roots, the b value in the quadratic equation must equal zero.
2. You cannot solve because there are two x values in the equation that you cannot combine.
3. Answers may vary.

RAFT Writing Activity: Quadratic Functions

**Directions:** You are going to create a piece of writing from one of the options in the table below. You will notice that each option has a different role, audience, format, and topic. Each of these sections is explained below.

**Role:** In your writing you will take on the persona of the given character.

**Audience:** Your writing will be directed at a specific group.

**Format:** The type of writing you will create is listed in this section.

**Topic:** The topic section gives you the specific focus for your writing.

<table>
<thead>
<tr>
<th>Role</th>
<th>Audience</th>
<th>Format</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reporter</td>
<td>The Community (of quadratic functions)</td>
<td>News Story</td>
<td>The parent function ( y = x^2 ) has been transformed!</td>
</tr>
<tr>
<td>Quadratic Function</td>
<td>8th Grade Algebra Students</td>
<td>Advice Column</td>
<td>Are you having trouble graphing me?</td>
</tr>
<tr>
<td>Salesman</td>
<td>The Public</td>
<td>Commercial</td>
<td>Need ways to solve that pesky quadratic function?</td>
</tr>
<tr>
<td>Parabola</td>
<td>Quadratic Functions</td>
<td>Resume</td>
<td>I have got the qualifications you are looking for in a shape.</td>
</tr>
</tbody>
</table>
### RAFT Writing Activity: Scoring Criteria

<table>
<thead>
<tr>
<th>Role: Quadratic Functions &amp; Linear Functions</th>
<th>The Function Family Reunion</th>
<th>Debate</th>
<th>Who is more useful for modeling and solving real-life problems?</th>
</tr>
</thead>
</table>

#### Writing Scoring Criteria 15 Points Possible

<table>
<thead>
<tr>
<th>Writing Role: Reporter</th>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format: News Story</td>
<td>Writing contains accurate information about the three ways a quadratic function can transform</td>
</tr>
<tr>
<td></td>
<td>Writing contains at least one example of a quadratic transformation</td>
</tr>
<tr>
<td>Total ___/15</td>
<td>Writing is in the form of a news story</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Writing Role: Quadratic Function</th>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format: Advice Column</td>
<td>Writing contains accurate information about the steps for graphing quadratic functions</td>
</tr>
<tr>
<td></td>
<td>Writing contains at least one example</td>
</tr>
<tr>
<td>Total ___/15</td>
<td>Writing is in the form of an advice column</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Writing Role: Salesman</th>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format: Commercial</td>
<td>Writing contains accurate information about graphing, solving algebraically, and solving with square roots</td>
</tr>
<tr>
<td></td>
<td>Writing contains at least one example</td>
</tr>
<tr>
<td>Total ___/15</td>
<td>Writing is in the form of a commercial</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Writing Role: Parabola</th>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format: Resume</td>
<td>Writing contains accurate information about all of the components of parabolas</td>
</tr>
<tr>
<td></td>
<td>Writing contains at least one example</td>
</tr>
<tr>
<td>Total ___/15</td>
<td>Writing is in the form of a resume</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Writing Role: Quadratic Functions &amp; Linear Functions</th>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format: Debate</td>
<td>Writing contains accurate information about quadratic functions and linear functions</td>
</tr>
<tr>
<td></td>
<td>Writing contains at least one example</td>
</tr>
<tr>
<td></td>
<td>Writing is in the form of a debate</td>
</tr>
</tbody>
</table>
Try It Out!

Name: _________

Question:
Solve the following quadratic function using the quadratic formula.  \( y = x^2 - 5x + 4 \)

Rate yourself:       Would you be willing to help someone learn the quadratic formula?
1 = I am confident about my answer.       Yes [ ] Not at this time [ ]
2 = I am semi-confident about my answer.   
3 = I am unsure about my answer.           

Directions: With your partner, discuss the Try It Out! problem. Go over each step of quadratic formula together, and decide on the correct solution(s) to this problem. When you have decided on an answer, go up to the front table to check your work. Next: As a team solve the following quadratic function using the quadratic formula. Work through the process together while recording the steps and the solution(s) on your own worksheet.  \( y = 2x^2 + 8x + 8 \)
Structured学术争议：

The Quadratic Formula

The mission:
Today you will participate in a modified debate. You will take a position regarding an issue and try to convince others that your position is right by presenting evidence.

The Positions:

**Position A:** The quadratic formula is the best way to solve quadratic functions.

**Position B:** The quadratic formula is not always the best way to solve quadratic functions.

The Process:

Research & Evidence Collection: 15 minutes
- You and your 2nd base teammate will be assigned to one side of an issue.
- You will research your position and collect evidence supporting it.
- You will decide upon the strongest pieces of evidence that “prove” your position is correct.

The great debate: 8 minutes
- You will have 3 minutes to clearly present your position and evidence to a pair of classmates supporting the opposite position.
- The opposing pair of classmates then has 3 minutes to present their position and best evidence.
- At this point, you have 1 minute to address the evidence the other pair presented. Remember: You believe that your position is correct and you are trying to prove it.
♦ The opposing pair now has 1 minute to address the evidence you presented.

The switch: 23 minutes (15 & 8 Respectively)
♦ Now you will complete the entire process again, except you will be taking the opposite position.
♦ Repeat the research and evidence collection steps for the opposing viewpoint.
♦ Repeat the great debate steps.

Common Ground: 15 minutes
♦ You and your partner will work together with your opposing pair of students to discuss all of the research and evidence.
♦ As a group, you will decide what you all agree upon regarding the two positions.
♦ Your group will make a decision about which of the two positions (or you can create a new position) you support.
♦ Your group will decide what evidence best supports the group's decision.

Sharing is caring: Any remaining time
♦ With any time that remains, your group will meet with another group to compare common ground.

Please Remember:
♦ A debate is not an argument. You must always show respect to the opposing pair of classmates.
♦ When you take the opposite position, you must create new evidence. You may not use the evidence that the other pair of students in your group already presented.
♦ Your textbook is a great resource for research.

Checklist for success for you & your partner:
___ We found and listed evidence for Position A.
___ We found and listed evidence for Position B.
___ We both participated in the 3 minute and 1 minute presentations.
___ We contributed our ideas and thoughts to finding common ground.
___ We supported the group’s decision with evidence.
___ We were respectful and had fun.

Good luck and have fun!

Names

Structured Academic Controversy: Work Space

<table>
<thead>
<tr>
<th><strong>Your Position:</strong> The quadratic formula is the best way to solve quadratic functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record all evidence here. (Hint: brainstorm then research)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The strongest pieces of evidence for this position are...</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>List interesting ideas/evidence presented about the opposing side.</th>
</tr>
</thead>
</table>
Structured Academic Controversy: Work Space

**Your Position:** *The quadratic formula is not always the best way to solve quadratic functions.*
Record all evidence here. (Hint: brainstorm then research)

<table>
<thead>
<tr>
<th>The strongest pieces of evidence for this position are...</th>
</tr>
</thead>
</table>

| List interesting ideas/evidence presented about the opposing side. |
Structured Academic Controversy: Common Ground

<table>
<thead>
<tr>
<th>Discuss and list ideas/evidence your group agrees upon.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>State your position (one of the original two or a new position).</th>
</tr>
</thead>
<tbody>
<tr>
<td>We believe that...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The strongest pieces of evidence for this position are...</th>
</tr>
</thead>
</table>
Structured Academic Controversy:
Rubric for Scoring

Student's Names__________________________  Total Score _____/25

Scoring Criteria:
Listed logical and accurate evidence for Position A
_____/5
Listed logical and accurate evidence for Position B
_____/5
Both partners participated in the 3 minute and 1 minute presentations
_____/5
Group consensus on a position was reached
_____/5
Group's position is supported with logical and accurate evidence
_____/5

Total Score _____/25
Learning Objective: *I can determine the number of solutions of a quadratic equation by evaluating the discriminant.*

**Creative Activity:** As you know, when the discriminant is zero the number of solutions is not zero. Wow, that's confusing! Create a mnemonic device (ex. PEMDAS) to help your classmates remember which discriminant answer corresponds to which solutions. Present your mnemonic device (phrase, rhyme, joke, drawing, etc.) in a song, story, skit, poster, or a similar option. Include how to evaluate the discriminant.

**Analytical Activity:** Uncover the reason(s) why the discriminant works for finding the number of solutions to a quadratic function. Explain the reason(s) you discovered, and then create three of your own unique quadratic functions. Each example function must have a different number of solutions. Show that your examples work by using the discriminant to prove the number of solutions for each example function.

**Practical Activity:** Find three real-world situations when it is more helpful to know how many solutions or answers there are than to know the exact solution(s) or answer(s). Now explain why sometimes using the
discriminant to determine the number of solutions of a quadratic function is more useful than taking the time to do the quadratic formula and get the exact solutions.

Name ____________
Date ____________

**SCORING RUBRIC: TriMind Activity**

**Creative Activity:**
Creation matches each discriminant value to the correct number of solutions

____/5

Creation includes an accurate description of how to solve the discriminant

____/5

Contains original mnemonic device for remembering discriminant values and solutions

____/10

Total ____/20

**Analytical Activity:**
Correct explanation of why the discriminant predicts the correct number of solutions

____/8

Contains three unique examples of quadratic functions (There must be one quadratic function that has zero, one, and two solutions.)

____/6

Solved each of the three example quadratic functions correctly using the discriminant to show the number of solutions

____/6

Total ____/20

**Practical Activity:**
Provides three rational real-world examples of when knowing how many answers is more useful than knowing the exact answers

____/9
Presents a thorough explanation containing valid reasons of why using the discriminant is sometimes more appropriate than doing the quadratic formula

__/11
Total ___/20

Appendix I:
Sequences Unit

Includes:

Sequences Assessment (Real World Connections)
Sequences Assessment

Tom has $40 in his bank account and he is going to deposit $1 next month, $2 the following month, $4 the third deposit, $8 the fourth deposit, and so on.

Jill has $60 in her bank account and she is going to deposit $3 next month. The following month she will deposit $4, followed by a $6 third deposit, $9 fourth deposit, and so on.

Marie has $50 in her bank account and starting next month she will deposit $8 into her account each month from now on.

The balance of each account from month to month could be considered terms in a sequence.

Your task is to classify each of the sequences as an arithmetic sequence, geometric sequence, or neither. Give evidence to support your decision. If the sequence is arithmetic or geometric, find the account balance after 2 years (the deposits start month #2). Do not calculate the 2 year balance if the sequence is considered neither arithmetic nor geometric.

1.) Tom’s account balance is arithmetic geometric neither

Evidence: ______________________________________________________________

Tom’s balance after two years would be $ ________.

2.) Jill’s account balance is arithmetic geometric neither

Evidence: ______________________________________________________________

Jill’s balance after two years would be $ ________.
3.) Marie’s account balance is arithmetic geometric neither

Evidence: _____________________________________________________________

Marie’s balance after two years would be $ ________.

4.) Imagine that you are offered to pick one account and keep the money inside. Whose account balance would you choose to keep? Explain why.

I would choose _______’s account because __________________________________

_____________________________________________________________________

_____________________________________________________________________

_____________________________________________________________________

5.) Now imagine that you get your first job and you are going to start your own bank account. Regardless of what you are saving for (a car, college, or a rainy day) you know that putting your money in the bank is the smart way to go. You plan to keep following the saving pattern you start and adding to your account for all four years of high school. So… which account would you start? Think through all aspects of this question and explain your reasoning.

Account #1
Start by putting $40 in your bank account and deposit $1 next month, $2 the following month, $4 the third deposit, $8 the fourth deposit, and so on.

Account #2
Start by putting $60 in your bank account and deposit $3 next month. The following month deposit $4, followed by a $6 third deposit, $9 fourth deposit, and so on.

Account #3
Start by putting $50 in your bank account and starting next month deposit $8 into your account each month from now on.

I would start account # _____ because _______________________________________

_____________________________________________________________________

_____________________________________________________________________

_____________________________________________________________________

__________________________.
Appendix J:
Copyright Permission Forms

Includes:
Emailed Permission to Use Author’s Writing and Created Activities from Grand Valley State University Courses

EDG 641
EDG 637
EDG 640
ED 660
ED 630
From: Jacque Melin <melinj@gvsu.edu>
To: Karri Sanders <karri.sanders@yahoo.com>
Sent: Friday, April 26, 2013 11:23 AM
Subject: RE: ED 693 - Melin (Spr/Sum 2013): Welcome

Hi Karri,

I think your letter is just fine. And, you definitely have my permission to use and modify the materials that you produced for EDG641 and EDG637. Thank you.

Jacque Melin

From: Karri Sanders [mailto:karri.sanders@yahoo.com]
Sent: Thursday, April 25, 2013 9:36 PM
To: Jacque Melin
Subject: Re: ED 693 - Melin (Spr/Sum 2013): Welcome

Good evening! So, I started combing through my old GVSU differentiation work from various classes and I am pleased that there is a lot that I think I can modify to fit with my idea of more interactive and conceptual based learning. Now my task is writing to the five professors (for the six classes) who taught the course in which I created the work. Since my project is yet unnamed and in the early stages (but I don't want to start my project without knowing whether I can use my own items- especially my 660 chapter one) I wondered if you could look over the following permission slip and make sure it is acceptable for my project. I believe in orientation Wednesday you said we could use (without plagiarizing) the sample copyright permission letter verbatim so I have mostly done that with a few changes and omissions. The things I'm worried about are highlighted. Could you let me know if this sounds okay to send to you and the other GVSU professors? Thanks!

April 25th

Jacquelyn Melin <melinj@gvsu.edu>

Dear Jacque:

I am currently enrolled in the Grand Valley State University (GVSU), Advanced Studies in Education Program, and I am writing a Master's Project for the completion of my Master's Degree in Education. My project is untitled as of yet, but focuses on
moving away from traditional teaching of mathematics to more interactive, differentiated, and conceptual based learning. May I receive permission to include in the appendixes of my Master's Project a copy of the following items?

Any and all items I created for your courses EDG 641 (Teaching for Talent Development) and EDG 637 (Assessment: K-12 Models and Practices).

Your electronic reply confirms your ownership of the items described above and your permission for me to use them in original or modified form as needed in my project. My project may be cataloged in the GVSU library and will be available to other students and colleges for circulation.

Sincerely,
Karri Sanders
12434 Ritchie Ave NE, Cedar Springs, MI 49319
616 696 1250
karri.sanders@yahoo.com

From: Dorothy Armstrong <armstrod@gvsu.edu>
To: Karri Sanders <karri.sanders@yahoo.com>
Sent: Monday, April 29, 2013 9:29 AM
Subject: RE: Permission to use EDG 640 in Master's

Dear Karri,

You have my permission to include materials you developed for the course as described below. It sounds like an interesting project. I wish you all success

Dorothy Armstrong, Professor
College of Education

From: Karri Sanders [karri.sanders@yahoo.com]
Sent: Saturday, April 27, 2013 7:14 PM
To: Dorothy Armstrong
Subject: Permission to use EDG 640 in Master's

April 27th

Dorothy C. Armstrong <armstrod@gvsu.edu>

Dear Professor Armstrong:

I am currently enrolled in the Grand Valley State University (GVSU), Advanced Studies in Education Program, and I am writing a Master's Project for the completion of my Master's Degree in Education. My
project is untitled as of yet, but focuses on moving away from traditional teaching of mathematics to more interactive, differentiated, and conceptual based learning. May I receive permission to include in the appendixes of my Master's Project a copy of the following items?

Any and all items I created for your EDG 640, Fundamentals of Talent Development.

Your electronic reply confirms your ownership of the items described above and your permission for me to use them in original or modified form as needed in my project. My project may be cataloged in the GVSU library and will be available to other students and colleges for circulation.

Sincerely,
Karri Sanders
12434 Ritchie Ave NE, Cedar Springs, MI 49319
616 696 1250
karri.sanders@yahoo.com
Eberhard Center

Grand Rapids, MI  49504

From: Karri Sanders [mailto:karri.sanders@yahoo.com]
Sent: Saturday, April 27, 2013 7:09 PM
To: Sherie Williams
Subject: permission to use ED 660 for Master's

April 27th

Dr. Sherie Williams <willishe@gvsu.edu>

Dear Dr. Williams:

I am currently enrolled in the Grand Valley State University (GVSU), Advanced Studies in Education Program, and I am writing a Master's Project for the completion of my Master's Degree in Education. My project is untitled as of yet, but focuses on moving away from traditional teaching of mathematics to more interactive, differentiated, and conceptual based learning. In thinking about my project, I realized that much of what I would like to do is very similar to what I did last summer in ED 660. As a result, I would like to use my work from ED 660 as a springboard for my work on my Master's. May I receive permission to include in my Master's Project a copy of the following items:

Any or all writing I created for your course ED 660, Educational Inquiry & Evaluation.

Your electronic reply confirms your ownership of the items described above and your permission for me to use them in original or modified form as needed in my project. My project may be cataloged in the GVSU library and will be available to other students and colleges for circulation.

Sincerely,

Karri Sanders
12434 Ritchie Ave NE, Cedar Springs, MI 49319
616 696 1250
karri.sanders@yahoo.com

From: Jacque Melin <melinj@gvsu.edu>
To: Karri Sanders <karri.sanders@yahoo.com>
Sent: Monday, April 29, 2013 4:29 PM
Subject: Re: permission to use ED 660 for Master's
I think it is just fine, Karri. Although you will be using parts of you 660 project, you will certainly be adding much more to it. I think what she means is this alone without additions would not work. Carry on as you were planning.
Jacque

Sent from my iPhone

On Apr 29, 2013, at 3:20 PM, "Karri Sanders" <karri.sanders@yahoo.com> wrote:

Jacque, I don't feel good about this email response from Dr. Williams (please see below). I am unsure why I cannot use my own writing verbatim if there are excellent sections that are now applicable to my Master's Project. When I got this email from her I felt almost ill because I put in so much work for my 660 chapter one and feel that she is telling me I have to redo the work or pick a new topic. Please advise...

From: I konecki <koneckil@hotmail.com>
To: Karri Sanders <karri.sanders@yahoo.com>
Sent: Monday, April 29, 2013 7:46 PM
Subject: Re: Permisson to use ED 630 in Master's project

You have my permission to use them if it is ok with your advisor.
Loretta Konecki, PhD
Professor Emeritia

Sent from my iPad

On Apr 27, 2013, at 7:22 PM, "Karri Sanders" <karri.sanders@yahoo.com> wrote:

April 27th

Dr. Loretta Konecki <koneckil@gvsu.edu> <koneckil@hotmail.com>

Dear Dr. Konecki:

I am currently enrolled in the Grand Valley State University (GVSU), Advanced Studies in Education Program, and I am writing a Master's Project for the completion of my Master's Degree in Education. My project is untitled as of yet, but focuses on moving away from traditional teaching of mathematics to more interactive, differentiated, and conceptual based learning. May I receive permission to include in the appendixes of my Master's Project a copy of the following items?
Any and all items I created for your course ED 630, Curriculum Development.

Your electronic reply confirms your ownership of the items described above and your permission for me to use them in original or modified form as needed in my project. My project may be cataloged in the GVSU library and will be available to other students and colleges for circulation.

Sincerely,
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NAME: Karri Lynn Sanders

MAJOR: (Choose only 1)

_____ Adult & Higher Ed  _____ Advanced Content Spec  _____ Cognitive Impairment  _____ CSAL  _____ Early Childhood  _____ ECDD

X  _____ Ed Differentiation  _____ Ed Leadership  _____ Ed Technology  _____ Elementary Ed  _____ Emotional Impairment  _____ Learning Disabilities

_____ Library Media  _____ Middle Level Ed  _____ Reading  _____ Secondary Level Ed  _____ Special Ed Admin  _____ TESOL

TITLE: Breaking Tradition in the Mathematics Classroom: Making Mathematics Real, Relevant, and Personal

PAPER TYPE: Project  SEM/YR COMPLETED: Spring/Summer 2013

SUPERVISOR’S SIGNATURE OF APPROVAL: ________________________________

Using key words or phrases, choose several ERIC descriptors (5 - 7 minimum) to describe the contents of your project. ERIC descriptors can be found online at:
http://www.eric.ed.gov/ERICWebPortal/Home.portal?_nfpb=true&_pageLabel=Thesaurus&_nfls=false

1. Mathematics Failure  6. Flexible Problem Solving
2. Mathematics Teaching and Learning  7. Real World Connections
4. Differentiation
5. Conceptual Understanding
9. Mathematics Achievement
10. Enhanced Curriculum