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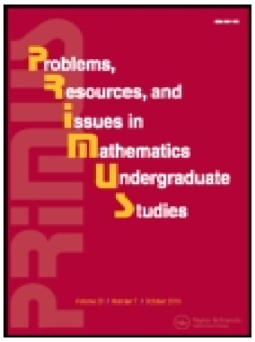
Robert Talbert
Grand Valley State University, talbertr@gvsu.edu



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Abstract: In this paper, we examine the benefits of employing an inverted or "flipped" class design in a Transition-to-Proof course for second-year mathematics majors. The issues concomitant with such courses, particularly student acquisition of "sociomathematical norms" and self-regulated learning strategies, are discussed along with ways that the inverted classroom can address these issues. Finally, results from the redesign of a Transition-to-Poof class at the author's university are given and discussed.

Keywords: Inverted classroom, flipped classroom, proof, transition to proof, self-regulated learning, screencasting, writing.

1. INTRODUCTION

To prepare for proof-based mathematics courses, many college students take courses specifically designed to teach the reading and writing of mathematical proofs, often called *Transition-to-Proof* courses. Although Transition-to-Proof courses vary widely across institutions, they tend to focus on a common instructional objective: to be able to construct a clear, mathematically correct, and convincing proof of a mathematical statement. Students who complete such courses successfully, it is hoped, will be able to focus on the specifics of upper-level, proof-based mathematics courses and not on the process of proof-writing itself.

However, this intended outcome does not always materialize. Despite taking courses that focus on proof, students in subsequent courses often struggle to complete even the most fundamental proof-related tasks. In a four-year study

Address correspondence to Robert Talbert, Department of Mathematics, Grand Valley State University, Allendale, MI 49401-6495, USA. E-mail: talbertr@gvsu.edu

of 61 students in a Transition-to-Proof course, Selden and Selden [8] found that none of those students could take an informal mathematical statement and rewrite it as a logically equivalent formal statement consistently. Similarly, in a study at the University of Georgia, Moore [5] found that despite their training in well-taught Transition-to-Proof and Abstract Algebra courses, students encountered a wide range of difficulties in understanding proofs. In a study that tracked the eye movements of successful undergraduate mathematics students as they read through various proofs, Inglis and Alcock [3] found that the undergraduates focused significantly more on superficial features of proofs and less on the arguments themselves than did professional mathematicians undergoing the same tracking.

Why do students seem to have such a hard time with mathematical proofs, even after performing well in a semester-long course specifically targeted at developing the ability to write them? It appears that this difficulty involves at least three factors.

First, the transition to proof requires a transition away from a primarily computational view of mathematics and toward one that is primarily conceptual. Dreyfuss [1] notes that the struggle for students transitioning to proof centers on coming to terms with mathematics as a system of "intricately related structures." Indeed, in a study in which mathematics students were asked to create concept maps of proof-related ideas, Jones [4] found a positive correlation between the level of interconnectivity among ideas in students' concept maps and students' grades in the class. The computational view manifests itself in the criteria students use to judge whether a proof is valid. Harel and Sowder [2] note that students in Transition-to-Proof courses tend to rely on appeals to authority, "proof ritual," and computational evidence to judge whether a proof is valid and not on the "analytical" scheme typically employed by mathematicians. Finally, Moore [5] points out that students tend to use evidence, rather than a logical argument, as proof of a mathematical statement and will often construct a "proof" simply by stating a tautology.

The second factor is that transitioning to proof requires the acquisition of *self-regulated learning* behaviors that are not always practiced in lower-level mathematics courses. The model of self-regulated learning, formulated by Paul Pintrich [6], holds that self-regulating learners are active participants in the learning process; have the ability to monitor and control aspects of their cognition, motivation, and behaviors related to learning; have criteria against which they can judge whether their current learning status is sufficient or whether more learning needs to take place; and that a learner's self-regulatory activities influence academic achievement. Whereas all four elements of Pintrich's model are inherent in the notion of "independent learning" and can be found in the instructional goals of most Transition-to-Proof courses, courses that precede transition-to-proof that rely mainly on efficient computation of "right answers" do not always stress all, or even any, of these elements in day-to-day practice.

Third, the transition to proof involves internalizing *sociomathematical norms* (Yackel and Cobb [12]), which are "normative aspects of mathematical discussions that are specific to students' mathematical activity." Students with a computational view of mathematics often have little experience with mathematics as it is done by professional mathematicians and hence with appropriate professional norms. As Dreyfuss [1] points out, students "have had little opportunity to learn what are the characteristics of a mathematical explanation," and most college and high school students do not know what a proof is or what it is supposed to achieve.

Thus, Transition-to-Proof courses face a variety of pedagogical challenges, only a subset of which pertain to mathematical content. Although many Transition-to-Proof courses want to teach students "how to think like a mathematician," there is far more to this process than meets the eye, and courses that do not address these difficulties explicitly may fall short of their goals.

2. THE INVERTED CLASSROOM

A traditional lecture-based approach to teaching Transition-to-Proof courses is vulnerable to challenges corresponding to each of the above difficulties.

First, the lecture-based approach places the pedagogical emphasis on the authority of the lecturer and may further entrench students in the use of proof rituals, computation, and external conviction rather than analysis to validate proofs. Second, lecturing may inadvertently create a dependency of the student upon the lecturer, thereby preventing students from developing fully as self-regulating learners. Third, whereas lecture can be effective as a tool for providing perspective on mathematical concepts, the experience of engaging in sociomathematical norms is left up to the student outside of the classroom and not necessarily brought in as an explicit goal of the course.

The traditional course design model also has a practical flaw, namely that students are generally asked to do the most cognitively complex work—the actual construction of proofs—outside the classroom when the instructor is least available. In such an environment, students who encounter difficulties do not have immediate access to expert guidance and may suffer a lack of engagement or even the desire to simply give up.

A course design that provides time and space for students to engage in work that develops not only their mastery of content but also their acquisition of the social norms and learning behaviors of expert mathematicians would allow students and their instructor to address all the areas of difficulty outlined above in a seamless way. The *inverted classroom* provides this kind of design.

In the inverted (or "flipped") classroom, students first encounter new content outside of the classroom, through a combination of reading, video viewing, and guided activity, rather than through an in-person lecture. In class, students use the time that is freed up to work on activities designed to engage them in higher-order thinking tasks, rather than having such tasks relegated to homework. Students can review the lectures as often as they need and according to their own schedules and technology. Since students are working on the most difficult tasks in class, rather than on their own outside of class, the availability of the instructor for questions and coaching is directly, rather than inversely, proportional to the students' needs.

The inverted classroom's history and usage, focusing on its use in linear algebra, is documented in [11]. In this paper, we present a reflection on the use of the inverted classroom in a Transition-to-Proof class, focusing specifically on the redesign of a Transition-to-Proof class at the author's university. This redesign was undertaken specifically to address the systemic issues noted above regarding mathematical practice, self-regulated learning, and sociomathematical norms. The redesigned course will be viewed as a case study, with reflections on successes and items for change to follow.

3. COMMUNICATING IN MATHEMATICS AND PEDAGOGICAL ISSUES

The redesign of the Transition-to-Proof course was done at Grand Valley State University, a public university of approximately 26,000 students in Michigan. Multiple sections of our Transition-to-Proof course, titled *Communicating in Mathematics* (*CiM*) are offered every Fall and Winter semester, with approximately 15–20 students enrolled in each section.

CiM serves three important roles for the Mathematics Department. First, it is a "gateway" for several upper-level mathematics courses, all of which are either required courses for the mathematics degree or among the choices from which students must select at least one. Second, CiM is a designated "Supplemental Writing Skills" course as part of the university's writing-across-the-curriculum program. Courses with this designation must satisfy certain assessment criteria; for example, at least one-third of the course grade must be assessed through student writing, and the writing process must include instructor feedback on multiple revisions of student drafts. Third, the department depends on CiM to establish a baseline of student competency in writing skills and proof-related mathematics (including related content on logic, set, and functions) that will enable instruction in upper-level courses to proceed without having to re-teach those skills.

Since its inception, CiM has been taught in a student-centered way, primarily driven by the textbook [10] used in the course. All sections of CiM expect students to complete reading assignments and "Preview Activities" in the textbook prior to class, and class meetings are typically focused on group activities with some lecture. Students in CiM also complete a "Proof Portfolio" that consists of eight to ten challenging problems whose solutions are proofs. The portfolio is CiM's primary means of fulfilling the requirements for a

Supplemental Writing Skills course; the portfolio typically is worth one-third of the semester grade and involves submissions and revisions of multiple drafts of each proof. Most instructors also give out-of-class homework and in-class timed tests (including a final exam).

Historically, student grades in CiM show a troublesome pattern of high levels of student failure withdrawal. Out of 941 students enrolled in CiM during the 2007–2012 academic years, 246 (26.1%) of these did not complete the course successfully. Of those 246 students, 164 of them (17.4% of all students) earned grades lower than C-, which prevents them from proceeding in the mathematics major; and 82 (8.7% overall) withdrew from the class, usually because of poor grades or because they fell behind in the class early and chose to withdraw. This percentage of failure or withdrawal is the highest such percentage of any course offered through the Mathematics Department, including remedial courses.

Through informal discussions, most CiM instructors identified the main sources of student difficulty in CiM to be precisely those listed in the Introduction to this paper. Namely, students who fail or withdraw from CiM tend to adhere persistently to an authoritative and computational view of mathematics and never move to the "intricately related structures" view; students struggle to adopt self-regulated learning strategies; and students fail to recognize and adopt appropriate sociomathematical norms. Additionally, many students struggled with the heavy out-to-class workload, which consisted of reading, Preview Activities, work on the Proof Portfolio, and work on weekly homework sets.

4. INVERTING THE TRANSITION-TO-PROOF COURSE

As a result of examining CiM student grade data, discussing student difficulties in CiM, and teaching the course myself, I began in the Summer of 2012 to redesign two sections of CiM (which I was scheduled to teach in Fall 2012) using the inverted classroom model. The choice for the inverted classroom was based on my experiences using it previously in computer science [11] and linear algebra [12], and also because the inverted classroom seemed to address all of the major categories of difficulty listed previously.

1. By relocating lectures to online video, large amounts of class time would be freed up for work on activities that could help students move from a computational view of mathematics to a more conceptual, integrated view. Such activities would involve not only the actual construction of proofs but also proof validation [9] and instantiating abstract concepts.
2. By taking on the responsibility for initial acquisition of new concepts on their own (through reading, video, and basic exercises), students would build self-regulated learning skills, especially self-efficacy (the perception of one's own ability to learn independently).

3. By having expanded class time for work on writing proofs and the ability to ask questions directly to the instructor while doing so, the class would take on an environment much more amenable to the transmission of appropriate sociomathematical norms.
4. By shifting the role of homework to being done in class rather than outside of class, the workload for students outside of class would become greatly simplified. Student time outside of class would become focused on just two tasks: preparing for the upcoming class meeting and working on portfolio problems.

As already noted, the existing structure of CiM already possessed a strong flavor of the inverted classroom. Therefore, the shift to a "fully inverted" structure required only a few incremental (though significant) changes.

The most significant change was the replacement of all-class lecture with recorded video. These videos are hosted on YouTube in a playlist located at http://bit.ly/GVSUMathCiM. The playlist includes 107 videos, consisting of over 14 hours of video content and covering the entire range of mathematical content in the course. These videos were made using a combination of LaTeX, *Camtasia* (for screen capture), and *Doceri* (for writing on an iPad screen) throughout July–October 2012. The videos average between 6 and 8 minutes each, and there are typically between three and five videos associated with each section of the textbook.

The videos were made not only to "cover material" but also to model the thinking processes of expert mathematicians. The videos spend significant amounts of time explaining how an expert sets up and navigates a proof and why certain choices are made in the argumentation and writing process. Therefore the videos give students experience with the sociomathematical norms for communication and analytical proof techniques even as new material is being introduced.

With the videos and the textbook in hand, the class was given the following workflow.

1. Prior to a class meeting, students were given an assignment called *Guided Practice*. Each of these consisted of five parts: an overview of the new material; a list of learning objectives explicitly stating tasks that students should master from the new section; print and video resources available to acquire the skills on the Learning Objectives list; practice exercises to lead students through the reading and viewing and instantiate new concepts; and a set of specifications for how to submit their work. A sample Guided Practice assignment can be found at http://bit.ly/1LITKWZ. Each Guided Practice was submitted online, and the students' results were reviewed before class to help me make adjustments to my plans for the day.
2. Upon arrival to class, students took a brief quiz over the reading and viewing. The quizzes motivated the students to do the work outside of class, and the quiz results provided further data for last-minute class plan adjustments.

3. Following the quiz and a brief question-and-answer session, students worked in groups of three or four on a problem of the day, usually involving writing out a single proof. A single problem of the day was equivalent in size and scope to what would have been a homework problem in a non-inverted version of the course. Students were encouraged to complete the problem, in class but were provided with a make-up day once every three to five class days to complete all outstanding problems. A sample class work activity can be found at http://bit.ly/1U0Ka3f.

The course also included two mid-semester exams and a final exam as well as the Proof Portfolio featured in all sections of the class.

5. STUDENT EXPERIENCES IN THE INVERTED CLASSROOM

While working on Guided Practice, students were able to ask questions through a variety of channels. First, they were allowed to work together on Guided Practice assignments as long as the writeup was done using their own ideas. Second, students could come to office hours or send emails with questions. Third, I created an online discussion forum using the web service Piazza (http://www.piazza.com) and placed all students from both sections into it, along with an instructor from another section who requested to be added. Questions that frequently occurred on Piazza or which were unresolved through online discussion were added to the list of items to discuss in class. An unexpected benefit of the Piazza forums was that some students became very active in providing help to others.

During class, students worked in groups of three to four on class work problems. During this vigorous time of work and discussion, I was able to communicate personally with every student, and every student was able to ask questions to me and to their, peers. Different groups worked at sometimes radically different paces, with some students struggling with terminology while others completed the problem of the day within minutes. For, the former groups, I would frequently devise some simple exercises to help them instantiate basic concepts and set the group to work on them while I visited the other groups. For the latter groups, I would make up a new and related problem to prove, or give them a question to investigate. This ability to differentiate instruction spontaneously in class is a major feature of the inverted classroom and becomes especially important in a Transition-to-Proof class where student levels of skill are widely varied.

Near the end of class, all work would stop for a moment while a preview of the next class was given. If students were done with the problem of the day, they would prepare a single group writeup and submit it. If not, then they were instructed to save a copy of their work in progress and either work on it outside of class if they wanted to, or else wait for the next make-up day.

6. EVALUATION OF STUDENT EXPERIENCES IN THE COURSE

In this section, we will examine the results of this redesign from the point of view of data gathered from students as well as my own personal reflections on its outcome.

An attempt was made to gather both qualitative and quantitative data from students to document their experiences in the inverted Transition-to-Proof courses. I administered the Motivated Strategies for Learning Questionnaire (MSLQ) [7], a survey in which subjects are give a list of statements addressing various aspects of self-regulated learning, to students not only in my two inverted sections of the course but to students in other, non-inverted sections of the course. Participants took a shortened version of the MSLQ consisting of only 44 questions rather than the full 88-item survey. This was given as a pre-test in the first week of the semester and as a post-test during the last week of classes. I hypothesized that all students in CiM will undergo significant positive change-in the adoption and practice of self-regulated learning strategies as a result of their experiences with the course, and that students in an inverted classroom will experience more such change as a result of having self-regulation at the core of their course's design.

Unfortunately, the MSLQ data were inconclusive. Only 32 out of approximately 100 students opted into the study, and the only students who completed both the pre- and post-test versions of the MSLQ were from the inverted sections. Most of those who completed the pre-test rated themselves very highly to begin with on several key areas of self-regulation, which left no room for reporting significant growth. (The accuracy of those self-ratings is an issue I did not investigate.) In fact, among the MSLQ items for which a higher self-rating indicates higher skill in self-regulated learning, only two items had an average response lower than five out of seven: "When studying, I copy my notes over to help me remember material" (average response 4.0) and "I work on practice exercises and answer end of chapter questions even when I dont have to" (average response 3.9375).

Students involved in this study also were given the opportunity to respond to four open-ended questions about their experiences. A total of 16 students did so, all of these coming from the two inverted sections of CiM.

The first open-ended Question asked: "How would you describe your overall experience in [the course] this semester?" A total of 15 students responded to this item. Only two of the responses were explicitly negative:

> I felt like I was thrown off the boat and was expected to float. Waves of new material followed by waves of portfolio submissions was a heavy work load. Book was a little helpful, but was very frustrated when asking for help and I couldn't have questions answered.

The final phrase of this comment could refer to one of two things: the perceived inability to ask questions while preparing for class, and the course policy (stated in the syllabus) that explicit hints on proof problems will not be given by the instructor. (More than one student expressed frustration that I would not give direct answers to questions such as "What should I do next?" or "How do I get started?") It is difficult to tell exactly which of these two the student means. However, comment overall illustrates a very important point: Students in an inverted classroom need to have as much support for learning as makes sense for the course, preferably in multiple ways and accessible in multiple places, and this support needs to be advertised to students on a continuous basis. For example, including a blurb on each handout for class stating the ways a student can request help in different contexts (while watching video, while working outside of class but not watching video, etc.) has significantly stemmed this sort of student comment.

Happily, though, most of the students in the course shared a positive outlook on the course format, with 13 of the remaining 14 students explicitly stating their experience in the course was "good," "great," "interesting," or "worth it" (referring to the amount of work required).

The next open-ended question asked: "Do you feel your experiences in CiM have made you better able to learn new content on your own?" Of 16 respondents, 12 indicated "Yes," three indicated "No," and one was unsure. Of the three answering "No," two of these indicated that they felt they could already learn new material on their own and the course did not add to those abilities. Several of the "Yes" comments address the inverted structure of the course:

> I feel that the inverted classroom setting has taught me how to do things more on my own as opposed to just listening to what the professor says.

> Yes, it taught me to teach myself on things that I got stuck on.

> In all I'm sure that the class did help me learn on my own, but because it didn't feel like I was learning it on my own I wasn't really noticing.

> Sort of This course has taught me more how to problem solve on my own because we did so much work outside of the classroom on our own, we didn't have the professor lecturing us, then doing homework at home. We worked through the lecture on our own and it gave us the opportunity to work through our problems first, without instantly being rescued. It was frustrating at times but I guess overall I have benefited from it.

The third open-ended question asked: "How have your attitudes and strategies about learning mathematics have changed over the course of this semester?" Nearly all (11 out of 13) respondents reported some form of positive change as a result of the class; the two students not reporting such a change said that they already engage in the methodology of the inverted classroom prior to the class and therefore the course structure was no change. Two of the respondents credited the inverted class structure with enabling positive changes (even though they were not prompted to do so):

> I would like to take another class in this format, I think it was very helpful. I don't think that before the semester I would have considered this as an option but I think it is a much more viable way of learning topics like the ones in the class.

> I am looking forward to taking math classes of similar styles in the future. I wouldn't say my strategies about learning math have changed very much.

Finally, students were asked: "How have your attitudes and strategies about learning other, non-mathematical subjects changed over the course of this semester?" Unlike the other open-ended items, students on this question generally (8 out of 12 respondents) reported no change in their attitudes and strategies with respect to other, non-mathematical subjects. None of these respondents offered further discussion on why their strategies and attitudes had not changed. The four respondents who did indicate change pointed to improved discipline and time management skills as the main areas in which their CiM experience carried over to other classes. This is a modest indication that students built global self-regulated learning skills, but it more strongly indicates that a single inverted course may not catalyze a global change in student learning strategies all by itself. Instructors who wish to use an inverted classroom as a means of improving *general* study strategies and self-regulated learning behaviors in students will probably need to make a point of addressing the notion of self-regulated learning in class and lead students to think about how their problem-solving skills generalize to other areas of study.

7. PERSONAL EVALUATION OF THE COURSE

The inverted classroom design is a good fit for Transition-to-Proof courses as learning how to write proofs requires time and a safe environment in which to struggle and receive feedback. By removing direct instruction from class meeting times, the class meeting becomes more of this sort of space. It liberates class time for students to work on the tasks that need the most amount of attention, and instructor availability is greatest at the point of greatest student need.

In my experience with the course, most students had some trouble adjusting to the style of the course, and a few students never made the leap from thinking of mathematics as a procedural, computationally-oriented subject to thinking of the subject in terms of "intricately related structures" and nonlinear problem solving. However, this would perhaps be the case in a

Transition-to-Proof course regardless of whether the course is inverted or traditional, merely by virtue of the content and the learning goals of the course. As the student responses to the open-ended questions suggests, most students in the course had a quite positive view of the inverted structure of the course and found it to be worth the effort. Moreover, many of the student responses included language that indicates growth in the discipline; for example, they identified proof as "a different type of math" and "non-computational," which suggests that they are moving incrementally toward sociomathematical norms more like those of expert mathematicians.

After the semester was finished, course grade distributions for the two flipped sections (total of 39 students) were compared with the grade distributions for all sections of the course from Fall 2007 to Summer 2012 (939 students in all who took the course for credit). Grouping plus/minus grades together, we had the percentages in each grade grouping listed in Table 1.

There was a marginal decrease in non-passing grades occurred with the flipped sections as well as a marginal increase in the top grades. However, it is clear that the flipped learning model applied to this course did not, by itself, cause a significant turnaround in course grades when compared with non-flipped sections of the same course taught in the past. It is possible that more noticeable improvements in the number of D/F/W grades may occur in subsequent flipped sections of this class, as instructors using the model learn lessons about how best to manage a flipped classroom and make improvements to the instruction. Clearly, there is more work to be done to take the perceived improvements in self-regulated learning skills emerging from the flipped classroom and translate them into improve performance in actually writing proofs.

Although the inverted classroom has definite advantages for Transition-to-Proof and other courses, instructors interested in adopting this course design should beware of some significant potential pitfalls.

1. The inverted classroom does require a significant up-front investment of time, particularly if an instructor is creating his or her own course materials. For example, in the creation of the course videos for CiM, there was roughly a 6:1 ratio in time spent scripting and producing each video to the running time of the video, so that a 5-minute video usually took half an hour to

Table 1. Grade distributions.

	A (%)	B (%)	C (%)	D/F/W (%)
Fall 2007-Summer 2012	23.1	33.0	17.5	26.2
Flipped Fall 2012	25.6	28.3	20.5	25.6

make. Multiplied by over 100 videos, this represents a large amount of time and effort. This investment can be mitigated by working as part of a team of instructors who are inverting their classrooms, and once made, the videos do not need to be re-made; but it can still be very time-consuming.

2. Although my students were generally quite amenable to the inverted classroom, this does not need to be the case. Many students, having developed a nearly invulnerable sense of learned helplessness through their primary and secondary education that conflates learning with lecture and with high numerical scores on tests, will experience the inverted classroom as a major culture shock and will rebel against it. This possibility is compounded in a Transition-to-Proof class that inherently "changes the rules" about mathematics for many students. In my experience, what separates successful inverted classrooms from unsuccessful ones is communication. Instructors have to explicitly communicate the "why" of the inverted classroom to students, clearly, early and often. They also have to listen to students, provide copious support for their learning, and be willing to accomodate reasonable student requests for change and alterations to the course.
3. Finally, by opening up the class meeting space to a time centered on student learning needs rather than lecture, the instructor will be exposed to the full range of student abilities. This means the instructor must be prepared to work with students at wildly disparate levels of skill and preparation in every class meeting. Hence, there is an additional time cost, namely the one incurred in preparing not only a main activity for the class meeting but also "side" tasks for students who are progressing very quickly and for those progressing very slowly. This kind of improvisational flavor to instruction can be very enjoyable, but it also involves additional preparation.

Helping students emerge as competent, confident, self-regulating learners is hard work, but it is the primary job of higher education. The inverted classroom shows promise of catalyzing progress toward this goal, not only in Transition-to-Proof, courses but in other mathematics courses as well.

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BIOGRAPHICAL SKETCH

Robert Talbert holds a Ph.D. in Mathematics from Vanderbilt University, where he was a Master Teaching Fellow at the Vanderbilt University Center for Teaching. His mathematical interests include cryptography, category theory, and computer science. His interests in mathematics pedagogy include the use of technology to support active-learning environments, particularly through the use of screencasting, classroom response systems, the fusion of math and computer programming, and peer instruction. He blogs on these and other subjects at Casting Out Nines (http://rtalbert.org/blog). Having taught previously at Bethel College (Indiana) and Franklin College, he is currently Associate Professor of Mathematics at Grand Valley State University, where he has been on the faculty since 2011. He lives in Allendale, Michigan with his wife, three children, and three cats.