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Theoretical and Mathematical Constraints of Interactive Regression Models

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Theoretical and mathematical issues related to the study of interaction effects are discussed. Constraints imposed by the theoretical expectation of ordinal or monotone interactions are defined by the general concept of inversion values. The mathematical constraints implied by these values are demonstrated by the derivation of a general formula. Further definitions related to this general formula are discussed for qualitative and quantitative variables. It is argued that interaction effects of substantial magnitude may not be routinely detected in behavioral science because many interactive theories may be implicitly ordinal. Levels of predictability common to behavioral science make such effects mathematically nonexistent, and thus impossible to detect. To have strong ordinal moderation, there must be a strong effect to be moderated.

Numerous theories and applications in the behavioral sciences involve the concepts of interaction or moderation, the action of a third variable (Z) on the relationship between two variables (X and Y) (Saunders, 1956). Alternatively, this can be conceived as the explanation of variance in a dependent variable by the interaction of two independent variables (X and Z) (Stone, 1988; Zedeck, 1971), or as a situation in which the effect of one independent variable (X) on a dependent variable (Y) depends on the level of another independent variable (Z) (McClelland & Judd, 1993). Although differing slightly in language, all of these conceptualizations involve the expectation of a specific mathematical relationship between variables X, Y, and Z.

The assessment of interaction or moderator effects by means of ANOVA or regression procedures is based on a comparison of models formed from two composition rules, an *additive* (A) (or main effects) rule in which Y is a linear, additive function of X and Z; and an *additive-multiplicative* (AM) rule in which Y is a linear function of X, Z, and their multiplicative composite XZ. Respectively, these statistical models are formulated as[1]:

$$Y_{i(A)} = B_{A0} + B_{A1}X_i + B_{A2}Z_i + \varepsilon_{Ai}, \tag{1}$$

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$$Y_{i(AM)} = B_{AM0} + B_{AM1}X_i + B_{AM2}Z_i + B_{AM3}X_iZ_i + \varepsilon_{AMi}. \quad (2)$$

The relevant hypothesis for a moderating or interactive effect concerns either the point estimate of population parameter B_{AM3} (b_{AM3}) in Equation (2), or the incremental predictability of the AM model over the A model. If no interaction effect exists in the population (H_0), B_{AM3} will equal zero, and A or AM models will offer equivalent predictability ($P_A^2 = P_{AM}^2$). These hypotheses can be tested with F-ratios in standard ANOVA methods, or equivalent tests offered by moderated multiple regression (MMR) (Cohen & Cohen, 1983). A population interaction effect is "detected" in MMR if the null hypothesis stated above can be rejected by a *t* test on regression weight b_3, or the mathematically equivalent F test of incremental predictability ($\Delta P^2 = P_{AM}^2 - P_A^2$, estimated by ΔR^2).

Despite support for predicting interaction effects in several domains of behavioral science, their appearance in actual data has lagged far behind theoretical prevalence or expectations based on substantive reasoning. Zedeck (1971) called moderator variables "as elusive as suppressor variables". Cronbach (1987) notes the difficulties in finding interaction effects to be statistically significant. Moderator variables have been characterized as especially difficult to detect in field studies (McClelland & Judd, 1993), where, in the social sciences, they often account for only single-digit proportions of total variance (Chaplin, 1991). Such magnitudes of ΔR^2 rarely correspond to the substantial moderator effects often expected by researchers, thereby creating a dilemma: Researchers typically have considerable rationale for expecting such effects, yet they continually appear to resist detection.

The logic of traditional hypothesis testing offers two possibilities to account for the lack of observed interaction effects, based on the true state of the null hypothesis (H_0) in the population. If H_0 is true ($B_{AM3} = 0$, $\Delta P^2 = 0$), the AM composition rule offers no incremental explanation over the A rule, and the failure to reject the null hypothesis in sample data results in a correct inference. This possibility likely holds little appeal for most researchers seeking interaction effects, as they typically have substantial rationale to expect an effect, and few successful researchers invest resources in pursuit of effects they do not expect to find. If the null hypothesis is false in the population ($B_{AM3} \neq 0$, $\Delta P^2 > 0$), the rejection of an AM rule based on sample data is considered a Type II error. Unlike the previous possibility, some degree of remedy exists, because Type II errors are a direct consequence of insufficient statistical power. To reduce the number of Type II errors in assessing moderator effects, efforts have naturally been directed toward improving the statistical power of moderated regression and other methods used to assess these effects (Cronbach, 1987). In the context of MMR, numerous studies have examined statistical and methodological artifacts that may contribute to increased probabilities of committing Type II errors when searching for moderator effects. Several factors, such as small sample size (Alexander & DeShon, 1994), measurement error (Busemeyer & Jones, 1983), small population effect sizes (Stone-Romero & Anderson, 1994), range restriction (Aguinis & Stone-Romero, 1997), heterogeneity of error variance (Aguinis & Pierce, 1998), and unequal sample sizes across moderator-based subgroups (Stone-Romero, Alliger, & Aguinis, 1994) have all been shown to increase Type II errors and reduce the statistical power of the MMR method. These factors reduce the probability of concluding that an AM rule holds in a sample when it is, in fact, the correct functional form in the population.

The alternatives thus facing a researcher who has not detected an expected interaction effect are either to reject the rationale for expecting the interaction or to increase the statistical power of the test. In addition to studying the previously cited work on power-related artifacts, the careful researcher will have addressed the second alternative by conducting a power analysis prior to the collection of data, determining the sample size corresponding to a given Type I error rate, power level, and estimate of effect size (Cohen, 1988). Believing they have established sufficient statistical power and eliminated the possibility of committing a Type II error, even careful researchers may opt for the first alternative, rejecting their theoretical rationale and concluding that no interaction effect is present.

The purpose of the present article is to provide a third alternative based on the implicit constraints in many theoretical models involving interactions and the mathematical constraints of the relevant statistical tests. It will be shown that two of the factors noted by McClelland and Judd (1993) as accounting for differential power in moderated regression analysis, overall model error and the expectation of ordinal moderation have rather serious effects when considered in combination. It will be suggested that interaction effects of substantial magnitude may not be found in organizational or behavioral research because the aforementioned theoretical and mathematical constraints render them nonexistent in the population in most situations. This is a consideration independent of statistical power, and researchers will obviously be unable to detect such effects.

First, the theoretical constraints operating on interactive models will be discussed, introducing the concept of *inversion* and its meaning across levels of measurement common to interactive models. Second, a general mathematical formula relating several important factors in statistical tests of interaction effects will be derived. Special cases of this formula across levels of measurement will also be discussed. Finally, the potential implications of these constraints will be discussed, both in general and using two examples from recent literature.

Theoretical Constraints of Interactive Models

Although statements such as “the effect of X varies as a function of Z,” or “the X-Y relationship changes across levels of Z,” adequately describe the mathematical aspects of moderation or interaction, they do little to clarify the theoretical or empirical meaning of the effect. First, statements of this type encompass a general class of functional forms, all of which are not necessarily viable representations for a given theory. Second, empirical meaning is dependent on the particular attributes measured by Y, X, and Z, how they are theorized to combine, and the level of measurement used to assess them.

The first issue noted above involves the identification of functions that adequately represent the interactive theory from the universe of additive-multiplicative functions. As an example, consider a hypothetical theory that predicts an interactive relationship between Ability (A) and Motivation (M) in determining Performance (P). The general additive-multiplicative function will have the form:

$$P\left(A_i, M_j\right) = B_0 + B_1 A_i + B_2 M_j + B_3 A_i M_j, \tag{3}$$

where A_i and M_j denote values or levels of Ability and Motivation, respectively, $P(A_i,M_j)$ denotes the level of Performance at A_i and M_j expected by the theory, and B_0 through B_3 denote real numbers, with $B_3 \neq 0$ defining the additive-multiplicative form of the function. The first constraints applied to this function by theory are the relative magnitudes of constants B_1, B_2, and B_3, indicating the degree of change in Performance associated with Ability, Motivation, and their interaction, respectively. For instance, a particular theory may claim that Ability accounts for more change in Performance than Motivation (i.e., $B_1 > B_2$ if Ability and Motivation have equal variances). In many cases, these constraints will be apparent to the researcher and easily identified and tested when analyzing the data.

A more important and less obvious constraint involves the nature of the individual Ability-Performance and Motivation-Performance relationships. Specifically, an interactive theory should specify whether each of these relationships is thought to be monotone across values of the other component in the interaction (i.e., higher levels of Ability or Motivation are *always expected* to result in higher levels of Performance, independent of each other). While this may at first seem a trivial theoretical assumption, the further constraint it places on the AM model may not be as apparent. Consider a formal statement of monotonicity for both Ability and Motivation in determining Performance:

For any two values of Ability (A_1 and A_2) and any two values of Motivation (M_1 and M_2):

$$P(A_1, M_1) \geq P(A_1, M_2) \rightarrow P(A_2, M_1) \geq P(A_2, M_2), \tag{4}$$

$$P(A_1, M_1) \geq P(A_2, M_1) \rightarrow P(A_1, M_2) \geq P(A_2, M_2). \tag{5}$$

The researcher is simply expecting rank orderings of Performance (P) at a given level of Ability (A) to be the same across all levels of Motivation (M), and the rank orderings at a given level of Motivation to be the same across all levels of Ability. Substitution of expression (3) into (4) yields:

$$\begin{aligned} &B_0 + B_1A_1 + B_2M_1 + B_3A_1M_1 \geq B_0 + B_1A_1 + B_2M_2 + B_3A_1M_2 \rightarrow \\ &\qquad B_0 + B_1A_2 + B_2M_1 + B_3A_2M_1 \geq B_0 + B_1A_2 + B_2M_2 + B_3A_2M_2, \end{aligned} \tag{6}$$

$$B_2M_1 + B_3A_1M_1 \geq B_2M_2 + B_3A_1M_2 \rightarrow B_2M_1 + B_3A_2M_1 \geq B_2M_2 + B_3A_2M_2, \tag{7}$$

$$B_2(M_1 - M_2) \geq B_3A_1(M_2 - M_1) \rightarrow B_2(M_1 - M_2) \geq B_3A_2(M_2 - M_1), \tag{8}$$

$$-B_3A_1 \geq B_2 \rightarrow -B_3A_2 \geq B_2, \tag{9}$$

$$\text{if } B_3 > 0: A_1 \leq \frac{-B_2}{B_3} \rightarrow A_2 \leq \frac{-B_2}{B_3}, \tag{10}$$

$$\text{if } B_3 < 0{:}A_1 \geq \frac{-B_2}{B_3} \rightarrow A_2 \geq \frac{-B_2}{B_3}. \tag{11}$$

Applying the same procedure to Equation (5) gives:

$$\text{if } B_3 > 0{:}\ M_1 \leq \frac{-B_1}{B_3} \rightarrow M_2 \leq \frac{-B_1}{B_3}, \tag{12}$$

$$\text{if } B_3 < 0{:}M_1 \geq \frac{-B_1}{B_3} \rightarrow M_2 \geq \frac{-B_1}{B_3}. \tag{13}$$

Equations (10) through (13) simply state that for the Ability-Performance and Motivation-Performance relationships to maintain rank orders, any two levels of Ability or Motivation must both be ≥ or both be ≤ constants defined by $-B_2/B_3$ and $-B_1/B_3$, respectively. These constants will subsequently be referred to as the *inversion values* for X and Z, denoted by x_i and z_i. Readers familiar with Aiken & West's (1991) discussion of "crossing points" of regression lines with continuous predictors (pp. 23-24) will recognize these constants as their X_{cross} and Z_{cross}. Note, however, that A_i and M_j may represent a variety of operationalizations of Ability and Motivation, and the constraints are equally applicable to all cases. For example, A_1 and A_2 may represent two values of Ability obtained from a quasi-continuous measure (e.g., test score), whereas M_2 and M_1 may denote conditions of experimental manipulation and control, respectively.

In ANOVA contexts, the constraints defined by Equations (10) through (13) distinguish the general classes of "ordinal" and "disordinal" interactions (Lubin, 1961). In the simplest case of a 2 × 2 design, if the numerical codes assigned to A_{HI}, A_{LO}, M_{HI}, and M_{LO} are theorized to meet the requirements of Equations (10) through (13), one is expecting rank orders across rows to be equivalent across columns, and vice versa. When one variable is measured on a continuous scale, the ordinal and disordinal classes are often termed "noncrossing" and "crossing", respectively, although this labeling can be misleading at times. A graph in which regression lines on the continuous variable do not cross across levels of the categorical variable does not necessarily provide evidence of a noncrossing interaction, as it only shows one of two required conditions (i.e., either Equation (4) or (5)). This may occur in "fan-shaped" interactions, when regression lines near the top of the fan have a positive slope and those near the bottom of the fan have a negative slope. Slope inversion across levels of the categorical variable violates the rank ordering requirement, and is evidence of a crossing interaction.

Although the interaction of two continuous variables is more difficult to visualize than those involving categorical variables, the requirements described earlier are easily applied. If a researcher is predicting an ordinal interaction between two continuous measures of Ability and Motivation in determining Performance, the researcher is simply expecting Equations (10) through (13) to hold for *any* theoretical quantity of Ability or Motivation. Special cases for continuous ordinal and ratio scales are worthy of note. When the variables involved are measured on ordinal scales, an ordinal interactive model is indistinguishable from an additive model, as transformations exist that freely convert between the two representations (Busemeyer & Jones, 1983). Thus, ordinal interactions are undetectable when constructs are measured on ordinal scales. In the physical sciences, interactions between continuous quantities are likely to involve ratio scales, which have lower bounds of zero and undefined upper bounds. Here, the expectation of an ordinal interaction entails x_i and z_i both ≤ 0, and because X and Z are ≥ 0 by scale definition, purely multiplicative combinations of physical quantities are necessarily ordinal in form.

Once a researcher has developed both theory and measurement for an interaction study, inversion values x_i and z_i defined by parameters B_1, B_2, and B_3 become relevant to the testing of the interaction effect. The next section will discuss the general constraints operating on the population incremental predictability (ΔP^2) of the additive-multiplicative model over the additive model.

Mathematical Constraints of Interactive Models

Given predictor intercorrelations $\rho_{x,z}$, $\rho_{x,xz}$, and $\rho_{z,xz}$, their standard deviations σ_x, σ_z, and σ_{xz}, and most importantly, inversion values x_i and z_i, the relationship between P_A^2 and ΔP^2 is completely determined:

$$P_A^2 = \frac{\Delta P^2 \begin{pmatrix} 2\rho_{x,z}^3 x_i z_i \sigma_x \sigma_z + (\rho_{x,z}^2 - 1)(x_i^2 \sigma_z^2 + z_i^2 \sigma_x^2 - 2\sigma_{xz}(\rho_{x,xz} z_i \sigma_x + \rho_{z,xz} x_i \sigma_z)) \\ + 2\rho_{x,z}(\sigma_{xz}^2 \rho_{x,xz}\rho_{z,xz} - x_i z_i \sigma_x \sigma_z) - \sigma_{xz}^2(\rho_{x,xz}^2 + \rho_{z,xz}^2) \end{pmatrix}}{\sigma_{xz}^2(\rho_{x,xz}^2 + \rho_{z,xz}^2 + \rho_{x,z}^2 - 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz} - 1)}. \tag{14}$$

The complete derivation of Equation (14) can be found in Appendix A. Although its form is complex, it clearly illustrates the system of variables determining the relationship between additive model predictability (P_A^2) and incremental predictability of the additive-multiplicative model (ΔP^2). To the extent that the researcher expects constraints on x_i and z_i based on theory, he or she can expect constraints placed on the relationship between P_A^2 and ΔP^2 in actual data drawn from a population in which the theory is correct. Due to the complex nature of Equation (14), it is impossible to isolate the direct effects of any variables on the relationship between P_A^2 and ΔP^2. We can, at this stage, simplify the influence of σ_x, σ_z, and σ_{xz} by assuming that variables X and Z have zero expectation (E(X) = E(Z) = 0), and unit variance (V(X) = V(Z) = 1). Cohen (1978) aptly demonstrated that the linear transformations needed to create these rescalings have no effect on tests of interaction effects. Per a special case of Bohrnstedt & Goldberger's (1969) Equation (5), under these rescalings, σ_{xz} becomes:

$$\sigma_{xz} = \sqrt{E(x^2 z^2) - \rho_{x,z}^2}. \tag{15}$$

Per a special case of Bohrnstedt and Goldberger's (1969) Equation (12) defining $\sigma_{x,xz}$ and $\sigma_{z,xz}$:

$$\rho_{x,xz} = \frac{\sigma_{x,xz}}{\sigma_x \sigma_{xz}} = \frac{E(x^2 z)}{\sqrt{E(x^2 z^2) - \rho_{x,z}^2}}, \tag{16}$$

$$\rho_{z,xz} = \frac{\sigma_{z,xz}}{\sigma_z \sigma_{xz}} = \frac{E(xz^2)}{\sqrt{E(x^2z^2) - \rho_{x,z}^2}}. \tag{17}$$

Thus, it is clear that three expectations of products of powers, $E(x^2z)$, $E(xz^2)$, and $E(x^2z^2)$ play a critical role in determining the constraints implied by Equation (14). These elements of expressions (16) and (17) are handled differently for various combinations of continuous and dichotomous predictors, and specific formulas for dealing with cases involving dichotomies can be found in Appendix B. The examples used in the following section make several assumptions to simplify Equation (14), as well as the expressions from Appendix B.

The Case of Two Dichotomous Variables

Dichotomous X and Z variables may represent experimental manipulations in a simple 2 × 2 ANOVA design, observed continuous variables that have been collapsed into two categories, or observed dichotomies such as gender or subsets of ethnic categorizations. These three classes of dichotomies differ in the extent to which the investigator has control over the distributions of X and Z. In the case of an experiment, the ideal is a balanced design achieved through the control of conditions. Collapsed continuous distributions can be split in a balanced manner as well, though these variables throw away information, and the resulting split may not be representative of the distribution's original form. Control is relinquished entirely with observed dichotomies, as one gains representativeness for a particular setting only by accepting the true distribution in the setting.

To simplify the use of an example, let us assume a researcher is using a balanced 2 × 2 ANOVA design ($\rho_{x,z} = 0, p_x = p_z = .5$), $X_{low} = Z_{low} = -1$, $X_{high} = Z_{high} = 1$, thereby simplifying $E(x^2z) = E(xz^2) = 0$ and $E(x^2z^2) = 1$. Equation (14) now simplifies considerably to:

$$P_A^2 = \Delta P^2(x_i^2 + z_i^2). \tag{18}$$

Suppose this researcher is studying the interactive relationship between Stress (High/v Low) and Task Complexity (High/Low) in determining Accuracy on Task. Our theory suggests that both high levels of Stress and high levels of Complexity will decrease Accuracy, and that the combination (High, High) will decrease it over and above the additive effects of either component. Recalling the previous discussion regarding inversion, we have no reason to expect rank orders of Accuracy to change across levels of either experimental factor. Thus, we expect that X_{high} and X_{low} will both either be ≥ or ≤ x_i, and that Z_{high} and Z_{low} will both either be ≥ or ≤ z_i. Because X and Z variables are coded –1/1 in this case, x_i and z_i must either be ≥ 1 or ≤ –1, and because they are squared in expression (18), it will not matter whether both are in the ≥ or ≤ direction. The following constraint now applies to (18):

$$\Delta P^2 \le \frac{P_A^2}{2}. \tag{19}$$

Note that the combined P^2 for main and interaction effects (P_{AM}^2) cannot be greater than unity, thus applying an additional constraint to Equation (19) ($P_A^2 + \Delta P^2 \le 1.0$). Substitution of this constraint into (19) shows that ΔP^2 cannot be greater than 0.33, and

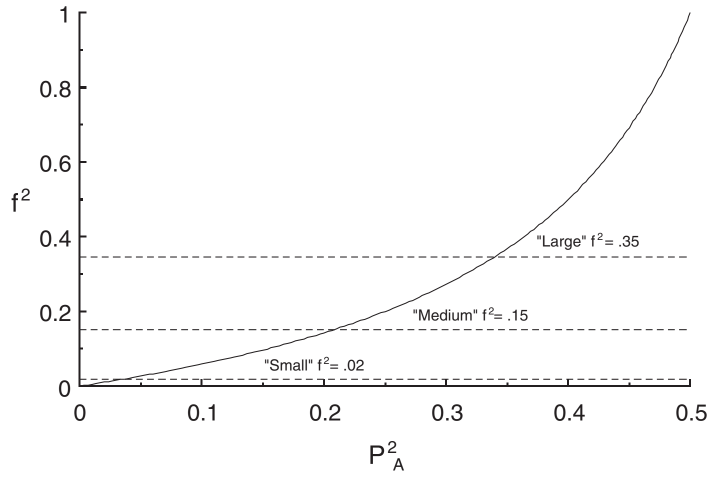

Figure 1: Constraints Placed on Effect Size (f^2) of Ordinal Interaction by Additive Model Predictability (P_A^2) in 2 × 2 ANOVA

this maximum will occur at $P_A^2 = 0.67$. The implications of these constraints may be better understood by presenting Cohen's (1988) formula for effect size,

$$f^2 = \frac{\Delta P^2}{1-(P_A^2+\Delta P^2)}, \tag{20}$$

and by substitution obtaining:

$$f^2 \le \frac{P_A^2}{(2-3P_A^2)}. \tag{21}$$

This constraint is shown graphically in Figure 1. The area underneath the curve represents combinations of f^2 and P_A^2 that can possibly exist for ordinal interactions in a 2×2 ANOVA. The curve asymptotes at $P_A^2 = .67$, the limiting value in Equation (19). Hashed lines designate Cohen's (1988) rule of thumb effect sizes of "small," "medium," and "large." Values of P_A^2 of at least .038, .207, and .341 are required for effect sizes of .02, .15, and .35 to even exist. This is not necessarily a problem for the researcher, as the experimental design has the advantage of lower overall model error (McClelland & Judd, 1993), and values of P_A^2 between .04 and .35 may not be unreasonable to expect. Thus, the experimental researcher whose theory predicts an ordinal interaction of substantial effect size may indeed find it, as it can exist in the population at reasonable levels of main effect predictability (P_A^2). The discussion will now turn to continuous variables, where we will see that the bar is raised considerably higher.

The Case of Two Continuous Variables

Unlike the previous case of dichotomous variables, no simple formulas exist for $E(x^2z)$, $E(xz^2)$, and $E(x^2z^2)$, as these terms define properties of the joint distribution of X and Z. Typically, the standard assumption of bivariate normality in regression is made, resulting in:

$$E(x^2z) = E(xz^2) = 0, \tag{22}$$

$$E(x^2z^2) = 1 + 2\rho^2_{x,z}. \tag{23}$$

Equation (14) now simplifies to:

$$P_A^2 = \frac{\Delta P^2(x_i^2 + z_i^2 + 2x_i z_i \rho_{x,z})}{\rho^2_{x,z} + 1}. \tag{24}$$

Note that (24) is similar to (18), except that we have not assumed $\rho_{x,z} = 0$, an unlikely value unless X and Z are orthogonal component or factor scores. The influence of $\rho_{x,z}$ is subtler than it may first appear, as it adds a term to the numerator with x_i and z_i, in addition to the existing squared terms that were present in the dichotomous case. In the case of nonzero correlation between X and Z, the *signs* of x_i and z_i now have influence. Inversion values x_i and z_i will have different signs when X and Z have different directions of rank ordering on Y. For example, we might have a theory that predicts Motivation and Task Complexity will interact to determine Performance, with higher levels of Motivation resulting in higher Performance, but higher levels of Task Complexity resulting in lower Performance.

For continuous variables, an additional benefit of rescaling X and Z to zero expectation and unit variance is that we can more easily set values of x_i and z_i based on measurement and theory. If our measurement places no restrictions on the range of X or Z, and assumptions of normality hold, permissible values of X or Z can be thought to lie between +C and –C, C being a constant defining the “edge” of the distribution. If X and Z are measured on Likert-type response scales, the Z-value of the most extreme categories may be used to represent this edge. Unfortunately, the normal distribution has infinite tails, so further examples in this article will assume that standardized continuous variables have “real” ranges from –2 *SD* to +2 *SD* units, capturing approximately 95% of the area under the theoretical normal curve.

Returning to a model used in an earlier discussion, consider a researcher studying the interactive relationship between Ability and Motivation in determining Performance. Our theory predicts the pattern shown in expressions (4) and (5), (i.e., the rank orders of Performance on Ability and Motivation will each remain the same across levels of the other variable). Because X and Z have ranges of –2 to +2, and we expect both to have positive relationships with Performance, x_i and z_i must both be ≤ -2. Application of (20) yields:

$$f^2 \leq \frac{-P_A^2(1+\rho^2_{x,z})}{P_A^2(\rho^2_{x,z} + 8\rho_{x,z} + 9) - 8(\rho_{x,z} + 1)}. \tag{25}$$

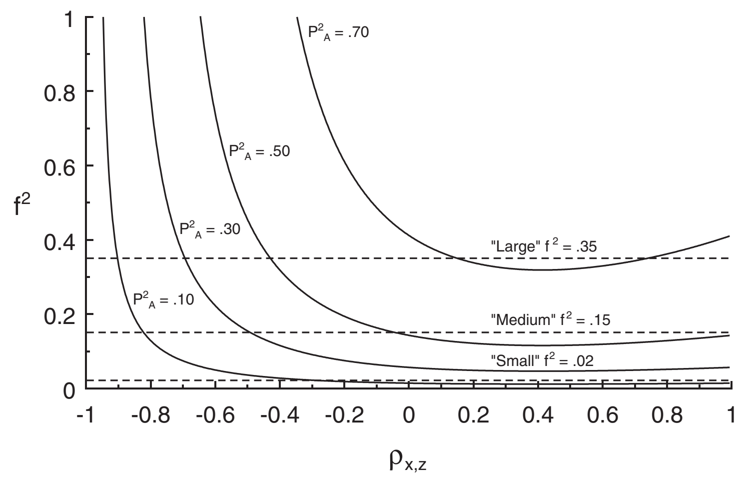

Figure 2: Effect Size (f^2) of Ordinal Interaction by $\rho_{x,z}$ Across Levels of P_A^2: Inversion Values of Same Sign

An f^2 by $\rho_{x,z}$ plot of Equation (25) across four levels of P_A^2 (.1 to .7, stepping by .2) is shown in Figure 2. Curves in Figure 2 represent the boundaries of possible combinations of f^2 and $\rho_{x,z}$ which can exist for ordinal interactions at levels of P_A^2 defining each curve.[2] As was the case with dichotomous X and Z in the previously discussed ANOVA design, higher levels of P_A^2 are associated with greater effect sizes of ordinal interaction, but the strength of this association now varies as a function of $\rho_{x,z}$. The researcher predicting an ordinal interaction with both components having positive or negative relationships will be able to detect a wider range of interaction effect sizes (f^2) for a given P_A^2 as $\rho_{x,z}$ decreases from roughly 0.4, and the range dramatically increases as $\rho_{x,z}$ becomes negative.[3] Moving x_i and z_i values farther from zero (e.g., (–3)) would push the curves in Figure 2 farther downward in the plot.

Figure 2 also illustrates a more troublesome issue for theories involving ordinal interactions of continuous variables. For the levels of $\rho_{x,z}$ near zero, often considered ideal in additive models, the existence of "medium" (f^2 =.15) interaction effect sizes will only occur at levels of P_A^2 of .50 or greater. A P_A^2 of .30 will only allow ordinal interactions of effect sizes up to approximately f^2 =.06 when $\rho_{x,z}$ is near zero. Thus, the much-lamented difficulties in detecting substantial interaction effects may simply be due to the fact that many theories implicitly predict ordinal interactions, and these effect sizes are *mathematically* impossible at levels of P_A^2 common to behavioral science. Further discussion on this issue will be presented later in this article.

The Case of Dichotomous and Continuous Variables

When X is a dichotomy and Z is a continuous variable, several assumptions can be applied to the expressions in Appendix B, depending on the nature of dichotomy X. If X is randomly assigned, we can assume variance differences between groups (ΔV) and $\rho_{x,z}$ are zero. If X is a factor in a balanced ANOVA design, we can also assume $p_x = .5$.

Although they are useful when presenting examples, these assumptions leave out a large proportion of the situations in which interactions between dichotomous and continuous variables are examined. Notable among these is the assessment of test bias in human resource settings, in which the interaction between a variable denoting Minority/Majority group classification and a predictor in determining a criterion is evidence of bias (Cleary, 1968). In these situations, p_x is typically far from 0.5, and ΔV and $\rho_{x,z}$ cannot be assumed zero.

For the sake of example, if we assume X is a balanced factor, and subjects are randomly assigned to each condition, Equation (14) reduces to (18), identical to the standard ANOVA case. As a substantive context, consider a researcher who is studying the interactive effect of the presence of an Incentive (Present/Not Present) and Ability (continuous) on Performance. We expect the presence of an Incentive to result in better Performance, and we expect this increase to be greater at higher levels of Ability. Similarly, we expect higher levels of Ability to result in better Performance in both Incentive conditions. These expectations define an ordinal, or noncrossing interaction. A difference exists, however, in the treatment of x_i and z_i, as they now represent different types of measurement. Applying the bounds used in the earlier two examples, a noncrossing interaction between X and Z requires x_i to be ≤ -1 or ≥ 1, and z_i to be ≤ -2 or ≥ 2. Applying these limits, and converting to f^2 through (20) yields the following constraint:

$$f^2 \leq \frac{P_A^2}{5 - 6P_A^2}. \tag{26}$$

This constraint is shown graphically in Figure 3. Comparing the constraint curve to that in Figure 1 for the 2×2 ANOVA design, it can be seen that for a given level of additive model predictability (P_A^2), the 2×2 ANOVA design is less constraining on interaction effect sizes that are possible in the population. The dichotomous/continuous case thus lies between the pure dichotomous and pure continuous cases in this regard.

Summary and Discussion

The goal of this article was to derive and illustrate the mathematical constraints applied to interactive regression models by conceptual constraints implied in theory or application. It has been demonstrated that the key variables operating in these constraints are the predictability of an additive model in the population (P_A^2), theoretical inversion values (x_i and z_i), component intercorrelation ($\rho_{x,z}$), and three expectations, $E(x^2z)$, $E(xz^2)$, and $E(x^2z^2)$. Definitions of these expectations were derived for combinations of standardized dichotomous and continuous variables. The issues raised in this article apply equally to higher order interaction effects. Considering that even a 3-way interaction involving X, Z, and W will create inversion values x_i, z_i, and w_i, and several new correlations and expectations of higher powers, the mathematical foundations quickly become less tractable.

It is important to note that the limitations described in this article are distinct from “detection” or statistical power issues, as they involve the existence of interaction effects in the population. If a researcher predicts an ordinal interaction effect, and bases a power analysis on a medium effect size, he or she may not realize that the additive effects of the variables in question mathematically prevent them from finding an

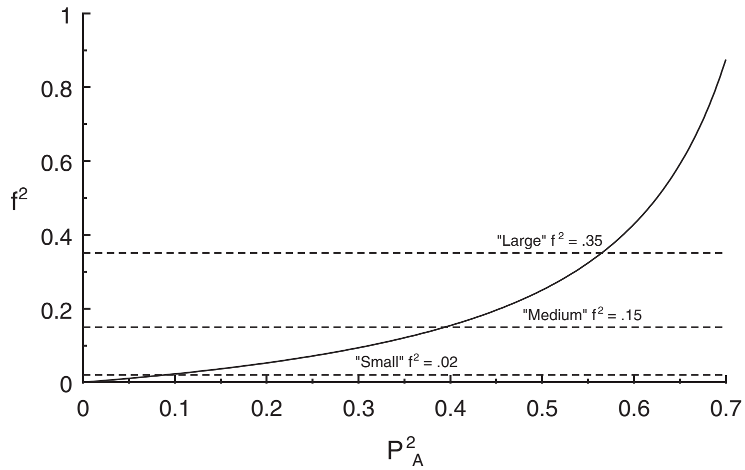

Figure 3: Constraints Placed on Effect Size (f^2) by Additive Model Predictability (P_A^2) in Dichotomous × Continuous Ordinal Interaction

effect size of medium magnitude in the population. Thus, their original power analysis was based on an effect size that could not exist. Prior to conducting a power analysis, a researcher can examine the existing literature to estimate the additive effects of the variables in question (as well as their intercorrelation, if a field study is being conducted). Using Equation (14) in combination with specific formulas for the different types of variables, the researcher can estimate the effect size ceiling in the population, and use the information to determine a minimum sample size needed to detect the interaction.

In their recent article comparing experimental and observational designs, McClelland and Judd (1993) note that higher overall model error and a tendency to expect only ordinal interactions work against the field researcher, making interaction effects more difficult to detect. The examples and derivations in this article supplement these observations, demonstrating that these factors, in combination, place even further constraints on the detection of such effects. These additional constraints are especially severe when the interaction involves continuous measures. One approach to the problem, noted above, is for researchers to identify these limits and work within the constraints of the population. An equally effective, and perhaps ultimately more fruitful approach is to work on increasing P_A^2 so that a wider range of ordinal interaction effect size is mathematically possible. This may involve theoretical refinements, more specific measurement instruments, or accounting for/reducing measurement error in X and Z. To put themselves in a good position to find ordinal effects in an interactive model, researchers must first find ways of achieving high levels of predictability in an additive model. Simply stated, to have strong ordinal moderation, there must be a strong effect to be moderated.

Although this article has shown that models with dichotomous measures generally allow larger ordinal interaction effects to exist in the population than the pure continuous case, it is important to consider the benefits and limitations of each model. Cortina and DeShon (1998) note that the weaknesses of observational designs with regard to statistical power might be offset by the greater parametric information they provide.

This is especially relevant if the attributes involved in the interaction are presumed to be underlying quantities instead of constructed dichotomies. For example, experimental manipulations of stress may be used to demonstrate an interaction effect, but if stress is theoretically conceived as a continuous quantity, the researcher may eventually wish to associate a parameter with the quantitative relationship.

The reader should also be cautioned that the relative advantages afforded by ordinal interactions involving dichotomies should not motivate the creation of such situations by median splits or other forms of artificial dichotomization. As noted by McClelland and Judd (1993) and McClelland (1997), the benefits of dichotomization are largely due to increases in variance. The interpretation of interactions that emerge only after artificial dichotomization is questionable, and the further generalization of such results back to the original continuous measures is usually impossible.

Of course, the fundamental question underlying the constraints described here is: *Do we expect an ordinal or disordinal interaction?* It is difficult to provide a general answer to this question, as empirical rationale for the combination of variables can only be given on a theory-by-theory basis. The literature is often of little help, as one will find the words "ordinal" and "disordinal" used primarily to report a resulting pattern of interaction rather than to provide the theoretical basis for its expectation. Such expectations are typically stated only in terms of the general additive-multiplicative model, (e.g., "X and Z are expected to interact in predicting Y"), saying nothing of the form of the interaction. We can, however, consider a few examples from the literature where the expectation of ordinal interaction effects may have contributed to constraints in effect size.

Donovan and Radosevich (1998), in a meta-analysis of the moderating effect of Goal Commitment on the relationship between Goal Difficulty and Performance, noted that the moderation accounted for less than 3% of total variability. Although Donovan and Radosevich discuss several potential reasons for this surprising finding, ordinality of the proposed interaction could be the primary culprit. Consider that early in their article, Donovan and Radosevich state that "goal commitment is proposed to be a moderator of the relationship between performance goals and task performance such that higher levels of goal commitment lead to a stronger relationship between performance goals and subsequent performance" (p. 308). Provided that this statement is intended to apply to all levels of Goal Commitment and Goal Difficulty, and the main effect of each on Performance is not expected to reverse across these levels, the moderation is expected to be ordinal. The miniscule effect sizes reported in Donovan and Radosevich may merely reflect a mathematical constraint imposed by lower levels of Performance predictability in the population.

O'Neill and Mone (1998) examined the moderating effects of Equity Sensitivity on relationships between Self-Efficacy and three workplace attitudes, finding such effects for Job Satisfaction and Intent to Leave ($\Delta R^2 = .03$ in both cases), but not for Organizational Commitment ($\Delta R^2 = .00$). In all three cases, O'Neill and Mone hypothesize moderation by predicting a relationship between Equity Sensitivity and the positivity/negativity of the relationship between Self-Efficacy and the respective workplace attitude. For this moderation to be ordinal, the relationship between Equity Sensitivity and the workplace attitudes should not change direction across levels of Self-Efficacy, and vice versa. O'Neill and Mone's graphical presentation of the significant moderator effects suggest that they are nearly ordinal in form, with inversion values very close to maximal scores on the Self-Efficacy scale. If the form of moderation was expected to be ordinal across all three workplace attitude measures, then the lack of moderation detected for Organizational Commitment may simply have been due to the lower model predictability prior to entry of the interaction term. Prior to entry of this term, the model R^2 values for Job Satisfaction, Organizational Commitment, and Intent to Leave were 0.28, 0.20, and 0.37, respectively. As in the previous example, the lack of moderating effect may have had more to do with the predictability of the criterion, in this case Organizational Commitment, than any true lack of such an effect.

Although it is not absolutely certain that ordinality of moderation was a theoretical expectation of the researchers in the two examples discussed above, the constraints described in this article offer a potential explanation for both findings. The extent to which the lack of moderating effect is due to mathematical constraints varies directly with the expectation of ordinal form, which itself likely varies across domain and theory. The expected form of interaction will also depend upon the representation used for a particular construct. Consider two scales of Attitude, one simply assessing strength of opinion in a 1 to 10 range, the other also assessing favorability toward the attitude by using a –10 to +10 scale. A disordinal interaction may make theoretical sense with the second scale, with inversion occurring at the zero point due to the change in direction of favorability. Providing similar justification for the first scale is more difficult, as there is no corresponding change.

Researchers should grapple with both theoretical and measurement issues when deciding whether they expect an ordinal or disordinal interaction. More importantly, ordinality of moderation should be treated as a theoretical expectation, and considered well before the collection of data.

APPENDIX A
Derivation of General Constraints

Interactive effects are generally assessed by determining the incremental predictability (ΔR^2) of an additive-multiplicative regression model (R^2_{AM}) over an additive regression model (R^2_A). McNemar (1962) offers the following general formula for R^2, based on determinants of the predictor-criterion correlation matrix:

$$R^2_{1\cdot 2..m} = 1 - \frac{|\mathbf{C}|}{|\mathbf{C}_{11}|}, \tag{27}$$

where **C** is a correlation matrix with Y in row/column 1 and a predictor set occupying rows/columns 2 . . .m, $|\mathbf{C}|$ is the determinant of this matrix, and $|\mathbf{C}_{11}|$ is the minor of **C** with row/column 1 removed. For the purpose of deriving population R^2 (P^2) formulas for A and AM models using Equation (27), correlation matrix **C** will be defined for a the basic AM model composed of criterion Y, components X and Z, and multiplicative composite XZ:

$$\mathbf{C} = \begin{Bmatrix} 1 & \rho_{y,x} & \rho_{y,z} & \rho_{y,xz} \\ \rho_{y,x} & 1 & \rho_{x,z} & \rho_{x,xz} \\ \rho_{y,z} & \rho_{x,z} & 1 & \rho_{z,xz} \\ \rho_{y,xz} & \rho_{x,xz} & \rho_{z,xz} & 1 \end{Bmatrix}. \tag{28}$$

Thus, for the AM model, P^2 is:

$$P_{AM}^2 = 1 - \frac{|\mathbf{C}|}{|\mathbf{C}_{11}|}. \tag{29}$$

The relevant correlation matrix for an A model can be denoted by $\mathbf{C}_{44}$, as one is simply removing the XZ composite row/column from $\mathbf{C}$ in Equation (28). For the A model, P^2 is:

$$P_A^2 = 1 - \frac{|\mathbf{C}_{44}|}{|\mathbf{C}_{44(11)}|}, \tag{30}$$

where $\mathbf{C}_{44(11)}$ denotes the 2×2 intercorrelation matrix of variables X and Z. Subtraction of (30) from (29) yields the population incremental predictability of the AM model over the A model:

$$\Delta P^2 = P_{AM}^2 - P_A^2 = \frac{|\mathbf{C}_{44}|\,|\mathbf{C}_{11}| - |\mathbf{C}|\,|\mathbf{C}_{44(11)}|}{|\mathbf{C}_{11}|\,|\mathbf{C}_{44(11)}|}. \tag{31}$$

Expansion of determinants in Equations (30) and (31) yield the following formulas for P_A^2 and ΔP^2:

$$P_A^2 = \frac{2\rho_{y,x}\rho_{x,z}\rho_{y,z} - \rho_{y,x}^2 - \rho_{y,z}^2}{(\rho_{x,z} - 1)(\rho_{x,z} + 1)}, \tag{32}$$

$$\Delta P^2 = \frac{(\rho_{y,x}(\rho_{x,xz} - \rho_{x,z}\rho_{z,xz}) + \rho_{y,z}(\rho_{z,xz} - \rho_{x,z}p_{x,xz}) + \rho_{y,xz}(\rho_{x,z}^2 - 1))^2}{(\rho_{x,z} - 1)(\rho_{x,z} + 1)(\rho_{x,z}^2 + \rho_{x,xz}^2 + \rho_{z,xz}^2 - 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz} - 1)}. \tag{33}$$

A linkage between the A and AM models exists in the functional relationship of individual component-criterion correlations and standardized regression coefficients (β) for the AM regression model. Another formula of McNemar's (1962) gives β coefficients from matrix determinants:

$$\beta = (-1)^p \frac{|\mathbf{C}_{1p}|}{|\mathbf{C}_{11}|}, \tag{34}$$

where p is the row/column of the relevant predictor variable. For relevant β weights in the AM model, these expand to:

$$\beta_1 = \frac{|\mathbf{C}_{12}|}{|\mathbf{C}_{11}|} = \frac{\rho_{y,x}(1 - \rho_{z,xz}^2) + \rho_{y,z}(\rho_{x,xz}\rho_{z,xz} - \rho_{x,z}) + \rho_{y,xz}(\rho_{z,xz}\rho_{x.z} - \rho_{x,xz})}{1 - \rho_{x,xz}^2 - \rho_{z,xz}^2 - \rho_{x,z}^2 + 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz}}, \tag{35}$$

$$\beta_2 = \frac{-|\mathbf{C}_{13}|}{|\mathbf{C}_{11}|} = \frac{\rho_{y,z}(1 - \rho_{x,xz}^2) + \rho_{y,x}(\rho_{x,xz}\rho_{z,xz} - \rho_{x,z}) + \rho_{y,xz}(\rho_{x,xz}\,\rho_{x,z} - \rho_{z,xz})}{1 - \rho_{x,xz}^2 - \rho_{z,xz}^2 - \rho_{x,z}^2 + 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz}}, \tag{36}$$

$$\beta_3 = \frac{|\mathbf{C}_{14}|}{|\mathbf{C}_{11}|} = \frac{\rho_{y,xz}(1 - \rho_{x,z}^2) + \rho_{y,x}(\rho_{x,z}\rho_{z,xz} - \rho_{x,xz}) + \rho_{y,z}(\rho_{x,xz}\rho_{x,z} - \rho_{z,xz})}{1 - \rho_{x,xz}^2 - \rho_{z,xz}^2 - \rho_{x,z}^2 + 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz}}. \tag{37}$$

Isolation and simultaneous solution of $\rho_{y,x}$ and $\rho_{y,z}$ in (35) and (36) yields:

$$\rho_{y,x} = \beta_1(1-\rho_{x,xz}^2) + \beta_2(\rho_{x,z} - \rho_{x,xz}\rho_{z,xz}) + \rho_{y,xz}\rho_{x,xz}, \tag{38}$$

$$\rho_{y,z} = \beta_2(1-\rho_{z,xz}^2) + \beta_1(\rho_{x,z} - \rho_{x,xz}\rho_{z,xz}) + \rho_{y,xz}\rho_{z,xz}. \tag{39}$$

Recall that the expressions for inversion values noted earlier in this article were functions of parameters B_1, B_2, and B_3 in the AM function. These can be extended to parameters β_1, β_2, and β_3 above by standardization:

$$x_i = \frac{-B_2}{B_3} = \frac{\beta_2 \, {}^{\sigma_y}\!/_{\sigma_z}}{\beta_3 \, {}^{\sigma_y}\!/_{\sigma_{xz}}} = \frac{-\beta_2\sigma_{xz}}{\beta_3\sigma_z}, \tag{40}$$

$$z_i = \frac{-B_1}{B_3} = \frac{-\beta_1 \, {}^{\sigma_y}\!/_{\sigma_x}}{\beta_3 \, {}^{\sigma_y}\!/_{\sigma_{xz}}} = \frac{-\beta_1\sigma_{xz}}{\beta_3\sigma_x}. \tag{41}$$

β_1 and β_2 can then be expressed as functions of β_3, x_i, and z_i:

$$\beta_1 = \frac{-\beta_3 z_i \sigma_x}{\sigma_{xz}}, \tag{42}$$

$$\beta_2 = \frac{-\beta_3 x_i \sigma_z}{\sigma_{xz}}. \tag{43}$$

Substitution of expressions (38) through (43) into (37) yields a simplified expression for β_3:

$$\beta_3 = \frac{-\rho_{y,xz}\sigma_{xz}}{x_i\sigma_z\rho_{z,xz} + z_i\sigma_x\rho_{x,xz} - \sigma_{xz}}. \tag{44}$$

A linkage can now be made between β_3 and ΔP^2 by way of the only remaining criterion-predictor correlation, $\rho_{y,xz}$. Isolating $\rho_{y,xz}$ in (44) and substitution into (33) yields:

$$\Delta P^2 = \frac{\beta_3^2(\rho_{x,xz}^2 + \rho_{z,xz}^2 + \rho_{x,z}^2 - 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz} - 1)}{\rho_{x,z}^2 - 1}. \tag{45}$$

Substitution of (45) into (32) produces the expression denoted as Equation (14) in the text:

$$P_A^2 = \frac{\Delta P^2 \left(\begin{array}{c} 2\rho_{x,z}^3 x_i z_i \sigma_x \sigma_z + (\rho_{x,z}^2 - 1)(x_i^2\sigma_z^2 + z_i^2\sigma_x^2 - 2\sigma_{xz}(\rho_{x,xz} z_i \sigma_x + \rho_{z,xz} x_i \sigma_z)) \\ + 2\rho_{x,z}(\sigma_{xz}^2 \rho_{x,xz}\rho_{z,xz} - x_i z_i \sigma_x \sigma_z) - \sigma_{xz}^2(\rho_{x,xz}^2 + \rho_{z,xz}^2) \end{array} \right)}{\sigma_{zx}^2(\rho_{x,xz}^2 + \rho_{z,xz}^2 + \rho_{x,z}^2 - 2\rho_{x,z}\rho_{x,xz}\rho_{z,xz} - 1)}. \tag{46}$$

APPENDIX B
Definitions of $E(x^2z)$, $E(xz^2)$, and $E(x^2z^2)$

Case 1: Dichotomous X and Z

Under the assumptions of zero expectation and unit variance, the numerical codes assigned to values of X and Z are defined as:

$$X_{high} = \frac{\sqrt{(1-p_x)p_x}}{p_x}, \quad X_{low} = \frac{\sqrt{(1-p_x)p_x}}{p_x - 1}, \tag{47}$$

$$Z_{high} = \frac{\sqrt{(1-p_z)p_z}}{p_z}, \quad Z_{low} = \frac{\sqrt{(1-p_z)p_z}}{p_z - 1}, \tag{48}$$

where p_x and p_z are the proportions in the "high" category for X and Z, respectively. $E(x^2z)$, $E(xz^2)$, and $E(x^2z^2)$ are defined by proportions p_x and p_z, and the intercorrelation between X and $Z(\rho_{x,z})$:

$$E(x^2z) = \frac{\rho_{x,z}(1-2p_x)}{\sqrt{p_x(1-p_x)}}, \tag{49}$$

$$E(xz^2) = \frac{\rho_{x,z}(1-2p_z)}{\sqrt{p_z(1-p_z)}}, \tag{50}$$

$$E(x^2z^2) = 1 + \frac{\rho_{x,z}(2p_x - 1)(2p_z - 1)}{\sqrt{p_x p_z(1-p_x)(1-p_z)}}. \tag{51}$$

Case 3: Dichotomous X and Continuous Z

If X and Z are dichotomous and continuous variables, respectively, both scaled to zero expectation and unit variance, $E(x^2z)$, X_{high}, and X_{low} equate to the expressions used with two dichotomies (Equations 47, 49). $E(xz^2)$ and $E(x^2z^2)$ now become functions of p_x, $\rho_{x,z}$, and the difference in variances between groups defined by X_{high} and X_{low} ($\Delta V = \sigma_z^2 \mid X = X_{high} - \sigma_z^2 \mid X = X_{low}$):

$$E(xz^2) = \frac{\sqrt{p_x(1-p_x)(\Delta V p_x^2 + 2\rho_{x,z}^2 p_x - \Delta V p_x - \rho_{x,z}^2)}}{p_x(1-p_x)}, \tag{52}$$

$$E(x^2z^2) = \frac{\Delta V p_x(p_x - 1)(2p_x - 1) + \rho_{x,z}^2(2p_x - 1)^2 + p_x(1-p_x)}{p_x(1-p_x)}. \tag{53}$$

Notes

1. B and β will be used throughout to denote unstandardized and standardized population parameters, respectively.

2. Note that the constraint of $P_A^2 + \Delta P^2 \leq 1$ also applies in the continuous case, and asymptotes will exist when the denominator of Equation (25) equals zero.

3. Although the case of x_i and z_i having different signs is not discussed in the text, its plot would simply be Figure 2 flipped over the Y axis, such that the dramatic increase in possible f^2 would occur with $\rho_{x,z} > 0$.

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