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Identity, A Transformational Game: Student Understanding of Transformation Geometry and the Application of Classroom Games

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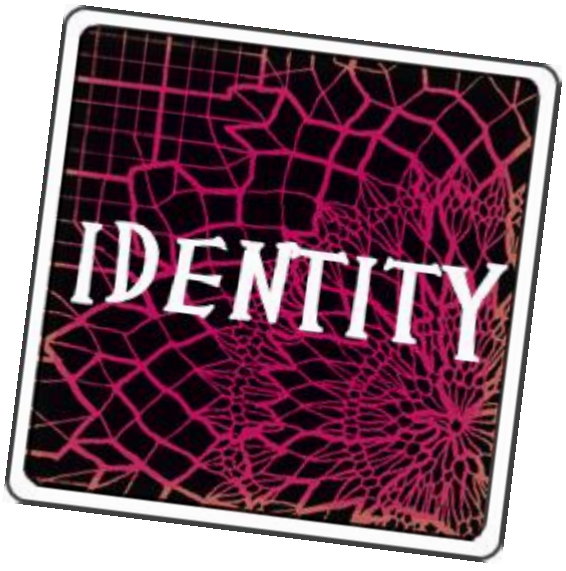
Abstract

In middle school and high school, students become acquainted with transformational geometry, and geometry in the co-ordinate plane. This paper addresses some of the reasons students encounter difficulty involving transformational geometry and ways to help students better understand transformational geometry. The discussion moves into the benefits of implementing games into the math classroom and some essential features of quality math games. This paper culminates with a discussion on the creation and design of my transformational game *Identity*, how *Identity* can help with students' understanding and facility with transformational geometry, and a reflection on the game-creating process.



Identity, A Transformational Game

STUDENT UNDERSTANDING OF TRANSFORMATION GEOMETRY
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In middle school geometry, students begin to learn about transformations, specifically reflections, rotations, and translations. Students are expected to know how the transformations affect a point or a shape that exists in the co-ordinate plane. While students are familiar with reflections, rotations, and translations, making the transition to applying these ideas in the co-ordinate plane can be difficult. Recognizing what leads to or causes difficulty for students is important in order to properly address the factors hindering students' understanding. The use of classroom games can be one such effective way to aid student understanding and engagement. However; not all classroom games are beneficial, and it is crucial to know what features make a game worthwhile for students' learning. Using research on student understanding of transformations and effective features of classroom games, I designed a game on transformational geometry. The game, *Identity,* allows students to learn about and explore rotations, reflections, and translations in the co-ordinate plane while engaging with their classmates and having fun.

Student Understanding of Geometry

The way students encounter geometry in elementary school tends to be quite different from the approach to geometry in middle school that continues into high school. Elementary school geometry often deals with identifying shapes and their properties, and by high school has transitioned into proving and reasoning through complex relationships. Allen and Martinie suggest in "Families Ask: Geometry: More than Just Shapes" that the struggles students face with high school geometry may result due to a lack of transition provided through middle school geometry (2006). Understanding some of the reasons behind this difficult transition and ways to counteract it are important. After all, this situation that students face in middle school will go on to influence their high school and future geometry experiences as well.

Skills and Time. As discussed above, the approach to geometry in middle and high school can differ greatly from the one that students encounter in elementary school. There is also the fact that standards and content, in middle and high school, are generally heavily geared toward algebra and number, not geometry (Jones 2002). This can lead to students lacking, or feeling they lack, the necessary skills to succeed in middle school geometry. In addition, geometry is often the last content covered in classrooms – placed at the ends of the textbooks and thus taught last (Sheats Harkness 2005). Being at the end, geometry can become neglected or not given the necessary time that should be devoted to it so students can successfully build their understanding.

Language. In geometry, as well as other areas of mathematics, language can pose a barrier for students. Hollebrands advocates that teachers use words more familiar to students paired with the new vocabulary (2004). This allows students to perform the action, gain understanding of it, and learn the correct terminology, rather than the new terms deterring them. With regards to transformations, instead of only using "reflection," "rotation," and "translation" when communicating with students, teachers should also use the simpler words "flip," "turn," and "slide," respectively. Students need a common language. Students lacking understanding of terms and definitions are "prevented from truly comprehending" (Sheats Harkness 2005). So, how is a common language of terms and definitions developed in a math classroom?

Discovery Learning. Discovery learning is when students are given the opportunity to explore a concept through a problem or activity before it is formally introduced. Research has proven that students tend to remember things better through doing instead of by memorizing definitions (Groth 2005). When definitions are discovered through an activity, it allows students to more easily put the definition into their own words – making it more memorable and associating it with an example. Learning in this framework also fosters discussion. As students discuss their ideas and findings, they can begin to recognize important concepts and to generalize. Students who are given the chance to explore ideas on their own and with concrete objects can better manage tasks requiring hypothesizing and envisioning. Through discovery learning, students develop the ability to apply the concrete to help understand the abstract (Groth 2005). Giving students more "time to play" and fewer instructions allows them to be "less frustrated and more willing to be creative" (Sheats Harkness 2005). Encouraging students to use "descriptions, demonstrations, and justifications" fosters the development of their "reasoning skills and confidence" (Jones 2002). Discovery learning also gives students a sense of autonomy in the classroom as the learning is put into their hands.

Manipulatives. Geometry is inherently visual. As Jones (2002) puts it, "geometry appeals to our visual, aesthetic, and intuitive senses". Since visuals and construction are at the heart of geometry, it's important for students to be given chances to develop these skills. Using manipulatives allows "students [to] build an understanding of relationships" within geometry (Allen & Martinie 2006). Manipulatives, like *AngLeg*s, pattern blocks, or folding geometric shapes, also give students the chance to explore geometric concepts and ideas for themselves. Having students work with compasses and rulers to create and construct within geometry can show students the connections that exist between art and mathematics (Sheats Harkness 2005). Geometry can help students to "develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof" (Jones 2002). It is an area of mathematics where students can thrive, but we must provide them with the appropriate tools to do so.

Classroom Games

Classrooms are continually changing and improving in order to reach more students and to help students gain deeper understanding. To move away from lecturing, teachers are implementing activities, group work, and discussions. Another way to break up the routine is through the use of classroom games. Math classroom games can provide a way into the mathematics "while by-passing the notation for the time being" that engages students (Devlin 2015). When a quality mathematics game is introduced in the classroom, it can strengthen students' mathematical skills.

Benefits. There are numerous benefits that classroom games can provide students. Classroom games are engaging. Students are eager to participate, which helps to improve their comfort in math (Cavanagh 2008). Math games also hit the "Five Ideas for 21st Century Math Classrooms" (Gasser 2011). These ideas are problem-based instruction, student-led solutions, risk taking, fun, and collaboration time. Each of these ideas is readily addressed through the use of a math game. Students work together to solve problems and collaborate in a fun and meaningful way. It allows students to assess each other's skills, thoughts, and ideas to, in turn, refine their own. Gasser (2011) emphasizes how "cognition and motivation can be welded together using play". Not only do math games offer a way for practice and reinforcement but they also provide a platform for students to access higher level math (Cavanagh 2008). Classroom games are also relevant and accessible for students – especially when the game relates to or is adapted from a game students find familiar. The accessibility of games also extends to students' families. Using games can provide a way for students and their families to engage in mathematics together. Students with lower socio-economic status are less likely to be exposed to math-related activities at home (Cavanagh 2008). Young students from low-income backgrounds who were exposed to an adapted version of *Chutes and Ladders* involving counting vastly increased their understanding of numbers (Cavanagh 2008). Spending time playing a fun math game can help to spark students' interest in math while they're developing math skills.

Essential Features. There are essential features any classroom math game should include to ensure it will truly benefit the students. Key things to avoid when selecting a math game are those with any of the following: confusing math, presenting math as separate from the game mechanics, where students are encouraged to answer quickly, and those reinforcing negative perceptions about math such as it being boring and in need of "sugar-coating" (Devlin 2015). Some math games also may seem like good games – being engaging, fun, or involving interesting stories. However, the games may have been created with a lack of understanding of what math is or how people learn mathematics (Devlin 2015). The purpose of quality math games is to develop students' understanding and mathematical thinking. These games have continuous challenge, flexibility (such as multiple ways to win, and various levels), dynamic, immersive, and empowering (Bruder 2015). Math is at the forefront and tied into the game mechanics – how the game is played, how the pieces or players move, and how the game is won (Devlin 2015).

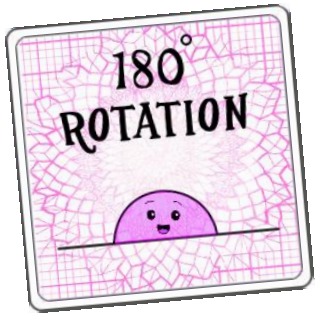
Students are pushed to think critically, to problem solve, and to collaborate or discuss as the game is played. Keeping these ideas in mind will ensure the math game does not only boost enthusiasm but increases cognitive growth as well.

Creating a Game

Watching and engaging with students in geometry, I saw students repeatedly struggling when it came to transformations. The students who struggled appeared to lack a basic understanding of rotations, reflections, and translations of points within the co-ordinate plane. This inspired me to create a game that would allow students to explore these ideas in a fun and engaging manner; to make connections between the definitions and the action the transformation performs; and to recognize the relationships that exist between the three transformations in the co-ordinate plane.

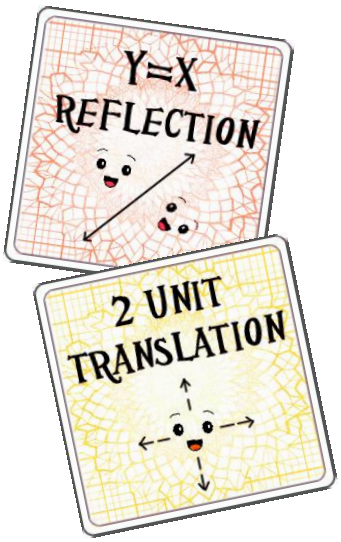
Research. Before developing and diving into game creation, I began with researching key features of quality math games and the subject for which I was interested in creating a game (as discussed in the previous pages). Researching was very influential in the creation of the game as it allowed me to better plan game development, game rules, and game play, and to ensure the game would target the skills and concepts I desired. The research also allowed me to better grasp student understanding of the topic and empowered me to create a game that builds upon their understanding and then expands it.

Development. In developing the game, there were a series of questions I considered: "Would it be a card game?", "Would there be a game board?", "What constitutes a play?", "What would be the goal?", and "How many players would there be?" One of the most important questions, as discussed previously, is: "What should students learn from this game?" My goal was for students to explore and gain a better understanding of three types of transformations, reflections, rotations, and translations, in the co-ordinate plane. I decided a game board would work best, as it could function as the co-ordinate plane where the game play would take place. To incorporate the three transformations, students would play cards that would tell them a transformation to perform on their game piece in the plane. I made the decision to have the player piece be a point. It would be easier for students to see how a point moves in the plane, and then hopefully allow students to transfer this knowledge to transformations on a shape – a collection of points.



The Cards. Determining what the cards would be for each transformation is where more of the research came in. Since students are most familiar with 0°, 90°, 180°, 270°, and 360°, I considered them for the rotation cards (Hollebrands 2004). I decided to remove the 0° and 360° rotation cards since the student's piece would be at its original location after these moves. For each rotation card, I decided to have the

center of rotation be at the origin, for simplicity's sake.

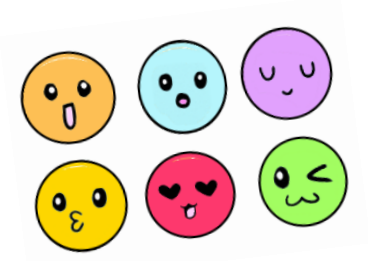


For the reflection cards, my rationale was similar. I considered how the position of the line of reflection contributes to the difficulty of the task – for example, those being horizontal or vertical posing easier tasks (Hollebrands 2004). I also thought of the lines of reflection I have most often dealt with in my mathematics career. I arrived at using the four following lines for my reflection cards: y-axis, x-axis, $y = x$, and $y = -x$.

Determining the translation cards was a bit more challenging since there seemed to be more options: direction (horizontal, vertical, both?), size/magnitude, and how the translation is given (vectors or not). Research allowed me to narrow my selections. In Hollebrands' (2004) study, he found students were comfortable with sliding but not with vectors. Since the goal for my game was to allow students to gain comfort with the transformations, I decided not to use vectors for the translation cards. Instead, I focused on solely having students slide their pieces in the horizontal and vertical directions given a magnitude. The magnitudes would range from 1 unit to 6 units so there would be a variety, and these magnitudes would work well within a co-ordinate plane with both x- and y-axes spanning from -10 to 10.

For both the translation and rotation cards, I decided not to restrict the direction of movement (whether translating horizontally or vertically or rotating clockwise or counterclockwise). This would allow students to be more creative when playing their cards and would require them to determine which direction would most benefit them.

Game Play. The idea for the game play came from the game "Sushi Go" in which players are dealt a hand, select a card, place it face down, pass their remaining cards to the player on their left, flip their card over, and repeat the process until all cards are selected. The card the player keeps from their hand is one they believe will benefit them best in the end and that plays or pairs well with cards they have already selected. I liked the idea of a changing hand, so the cards and moves available would not depend solely on what hand you were dealt; instead, students would have to think about how the card they select would impact the movement of their piece in the plane, as well as considering what cards they may have available when they receive a new hand.



Player Pieces

The goal of the game was built around this type of game play. The dealer would select a point within the co-ordinate plane, this would be the Identity. Students would mark this on their planes (for better exploration, students would have individual boards rather than work from one group board). They would then place their player piece on the Identity. The goal of the game is to collect cards that will keep the piece at or closest to the Identity by the end of the game. After playing

and testing the game a few times, I decided to allow students to rearrange the order of their transformation cards at the end of the game. This would allow them to explore how the order of transformations influences the final location of the image as well as give students the opportunity to see how certain transformations can undo one another.

Reflection. The process of creating a quality game for classroom use can seem like a long process, but it was enjoyable, fun, and a great experience in learning about new and different ways to help students gain understanding. Before completion, the game went through several drafts. I tested the game with students, friends, and family, to get a sense of how it worked with various age groups at various levels of mathematics ability. The game play as described above was great for getting the groups involved. They would laugh and sarcastically complain about bad hands or the final card received. For the transformations selected, sometimes students and those who had been away from math for a while would have questions, but this led to discussion about the transformations and people sharing their strategies for the game. The transformations were accessible – recognizable or familiar – to students. How the cards and pieces looked also changed a lot throughout the process to make the game visually appealing and interesting.

For the Future. The final product is something I am proud of and excited to be able to use with students in the future. However, my creation does not end here. I was able to test the game with a few students, but these students were lower middle school and I would like to test it with 8th grade and high school students and with a larger number of students. This would allow me to further adapt and revise the game. I am also interested in developing variations and/or additions for the game. Currently, I am planning on creating a version with graphics related to historical and cultural uses of transformations – for example, Islamic tilings and Celtic knots. Although not always brought to light in mathematics classrooms, geometry "provides a culturally and historically rich context within which to do mathematics" (Jones 2002). Relating the game to these historical and cultural contexts can show students how geometry has been a part of many different groups of people as well as showcase the relation between mathematics and art. I am also curious about creating additional cards for the game such as adding 45° rotations and vector translations.

http://www.clanbadge.com/images/round_celtic_knot_1.jpg

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Conclusion. Games can be an excellent way for students to practice and explore mathematics. Students can gain mathematical understanding and develop their mathematical thinking. Games can allow students to collaborate and to take risks and can foster discussion. Creating your own game can also be a viable option and a rewarding experience. Developing a game pushes you to think about key

ideas and how students understand the mathematics. Whichever route you may choose, it is important to consider all aspects of a game to ensure that it conveys the concepts you want students to learn and that the math, rather than an afterthought, is at the heart of the game.

NOTE: If you are interested in playing *Identity*, the game materials are given in the following pages.

IDENTITY

Strand

Number & Quantity	
Algebra	
Functions	
Geometry	X
Statistics & Probability	

Why

An exciting game of Transformations!
Can you get back to the Identity?

How

Set-Up

- Shuffle the deck and deal five cards to each player. Place remaining cards to the side. (3-6 players)
- Choose game pieces and pass out co-ordinate planes to each player.
- Player on the dealer's left picks a point on the plane. This is the Identity. Mark the Identity on your co-ordinate plane with a pencil.
- Place your piece on the Identity. This is where every player starts.

Object of the Activity: Explore relationships of transformations in the co-ordinate plane.

How to Play

1. Each player picks one card from their hand that will help keep their piece closest to the Identity. Sit this card face down.
2. When everyone is ready, pass your remaining cards to the person on your left.
3. Flip over your chosen card.
4. REPEAT steps 2 & 3 until all cards have been chosen.
5. Move your piece from the Identity using your cards in any order. The player whose piece is closest to the Identity in the end wins. You must explain your moves to the group to win.

Use your graph and player piece to see the impact of different transformations and how the order of the transformations affect your piece. How will you stay close to the Identity?

What

Materials Needed:

- Deck of Identity cards.
- Player pieces.
- A co-ordinate plane for each player.
- Pencils for each player.

Think About It

- What techniques are you using in the game?
- What transformations can undo another?
- How does the location of the Identity affect play? Try picking a point along one of lines from the reflection cards and one that doesn't lie on one of these lines.

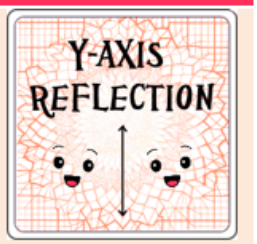
Where

Outside	X
Inside	X
On-line	
On-site	

Variation:

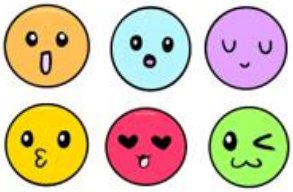
1. **Individual/2-Player:** Pick an Identity point. Shuffle and turn over 3-5 cards. Work together to figure out the order the transformations should be played to end up closest to the Identity.

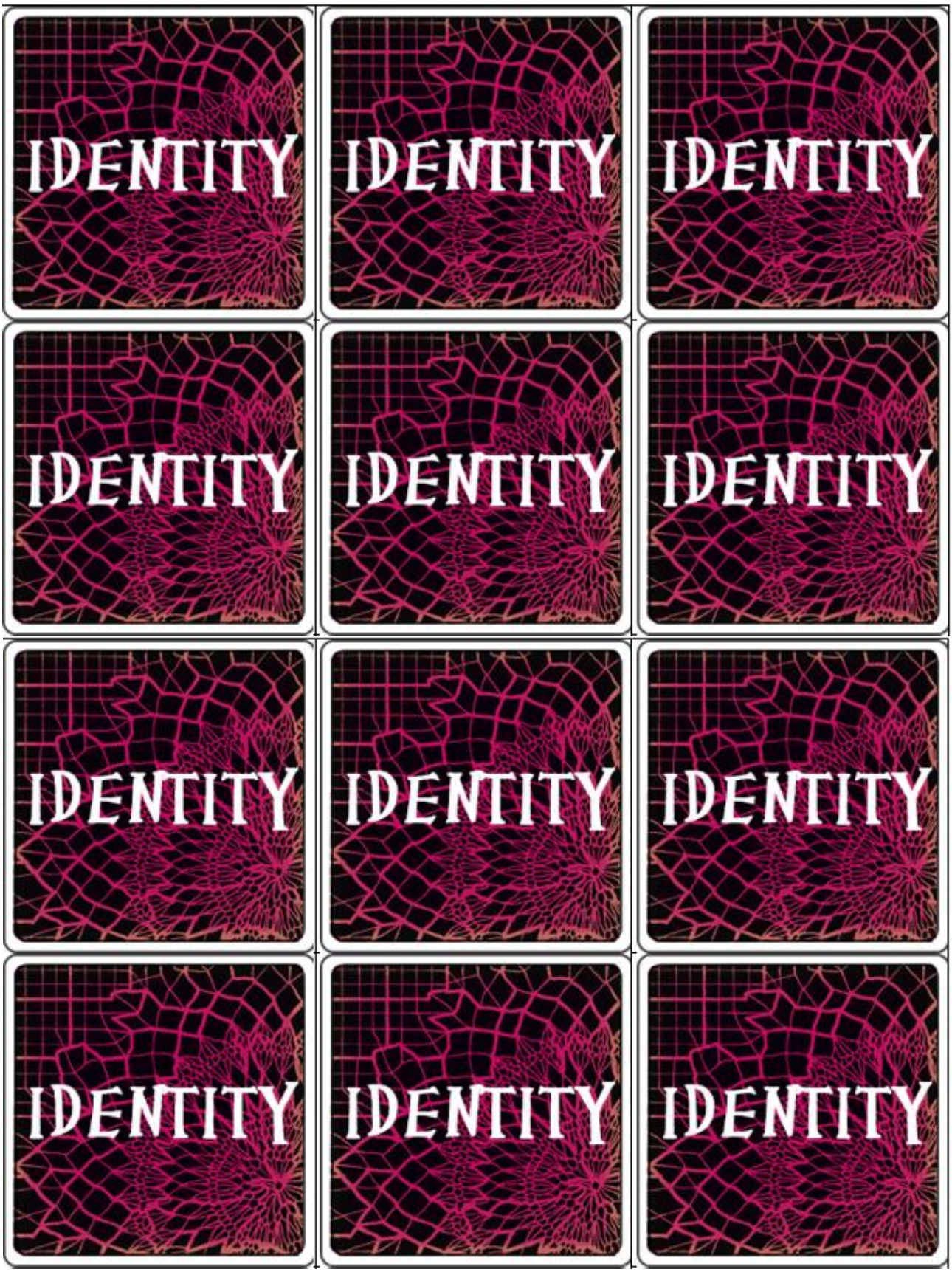
Types of Cards:

Card	# in Deck	Description
	12	**Reflection Cards**: The cards are reflections across the following: • The y-axis • The x-axis • The line y=x • The line y= -x.
	12	**Rotation Cards**: The cards are rotations of the following degrees: • 90 • 180 • 270 For rotation cards, you can move clockwise or counterclockwise.
	24	**Translation Cards**: The cards are translations of 1, 2, 3, 4, 5, and 6 units. For translation cards, you can only move in one direction: up or down, left or right.

Player Pieces:

Extras:

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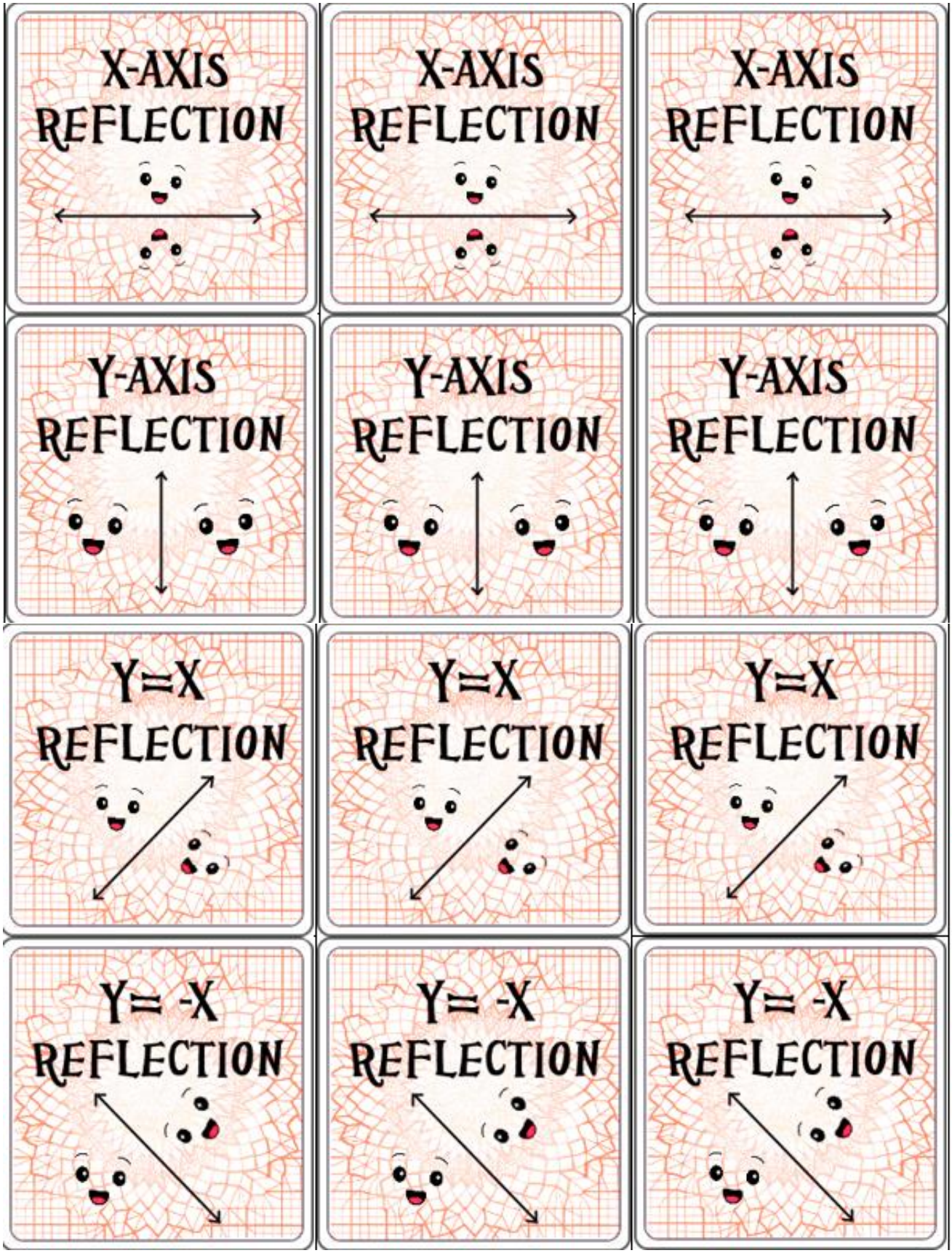
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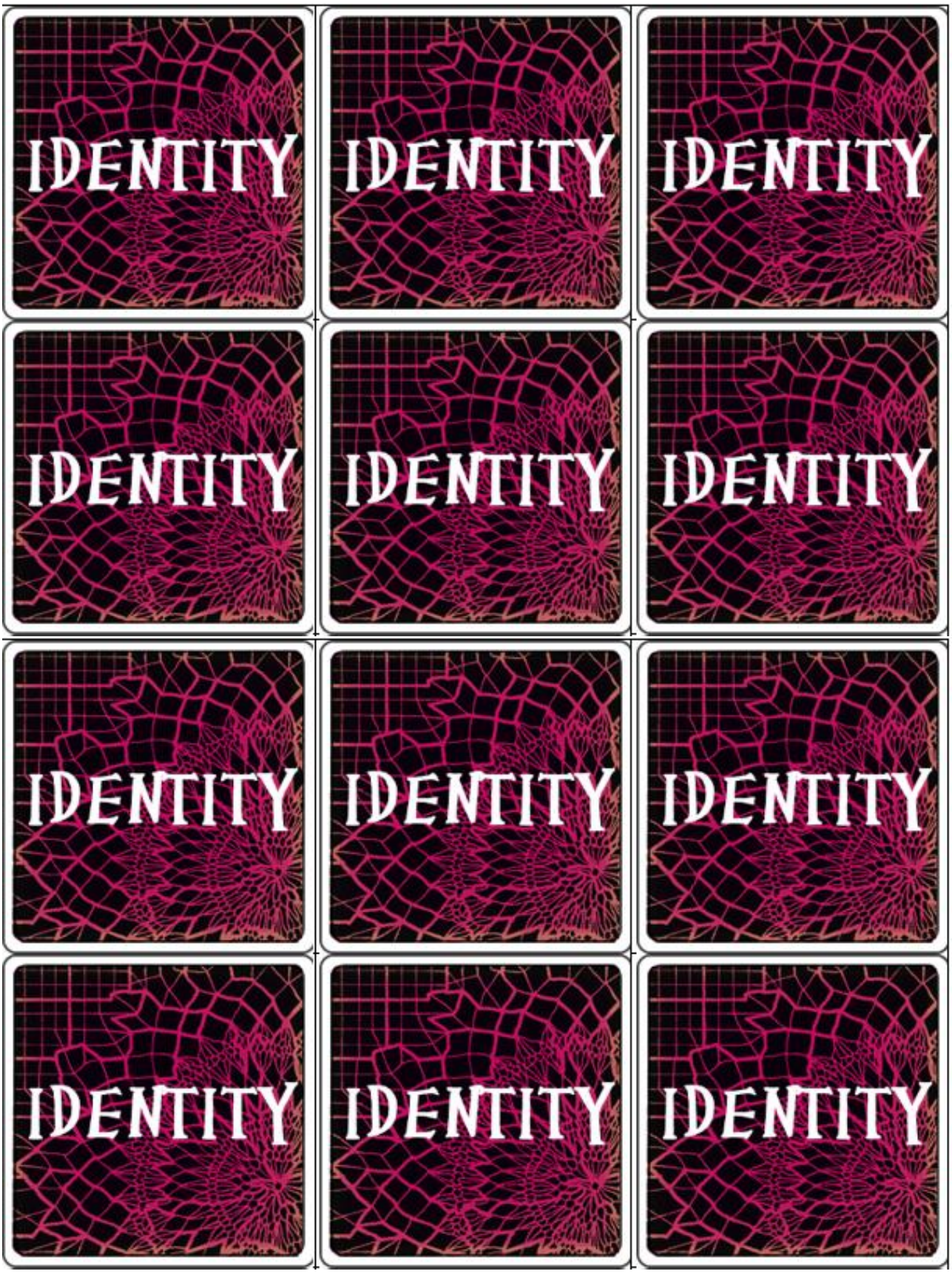
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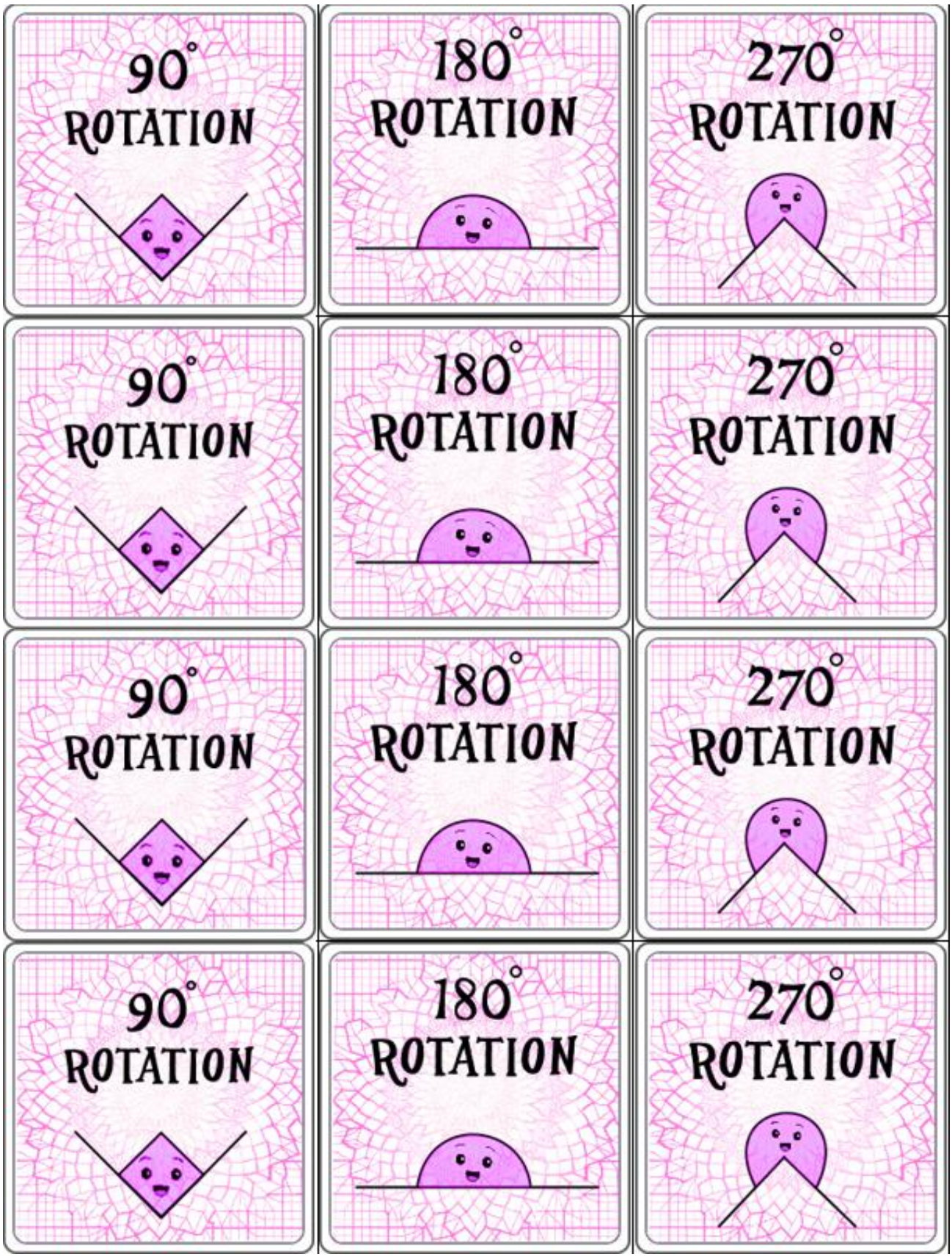
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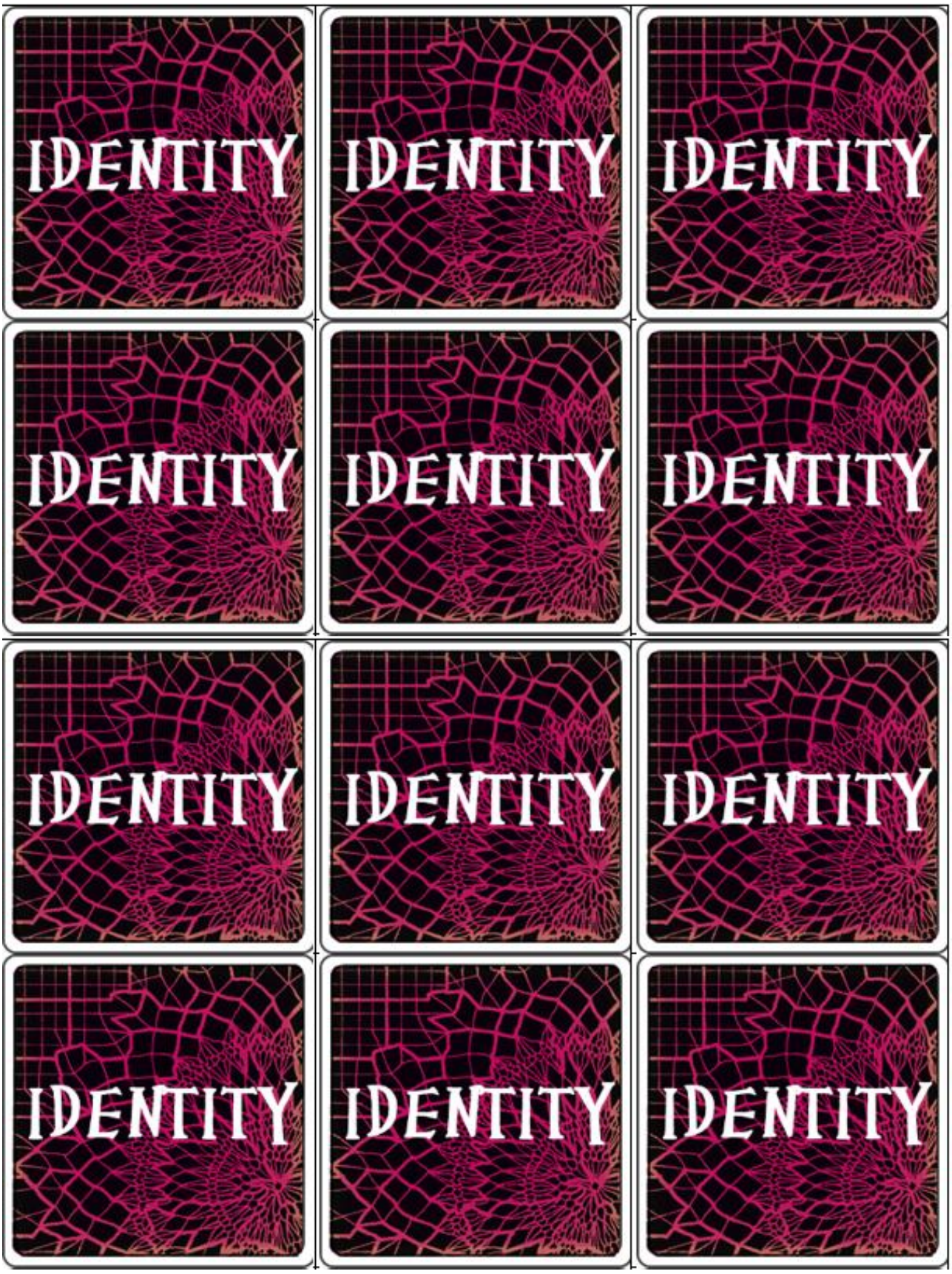
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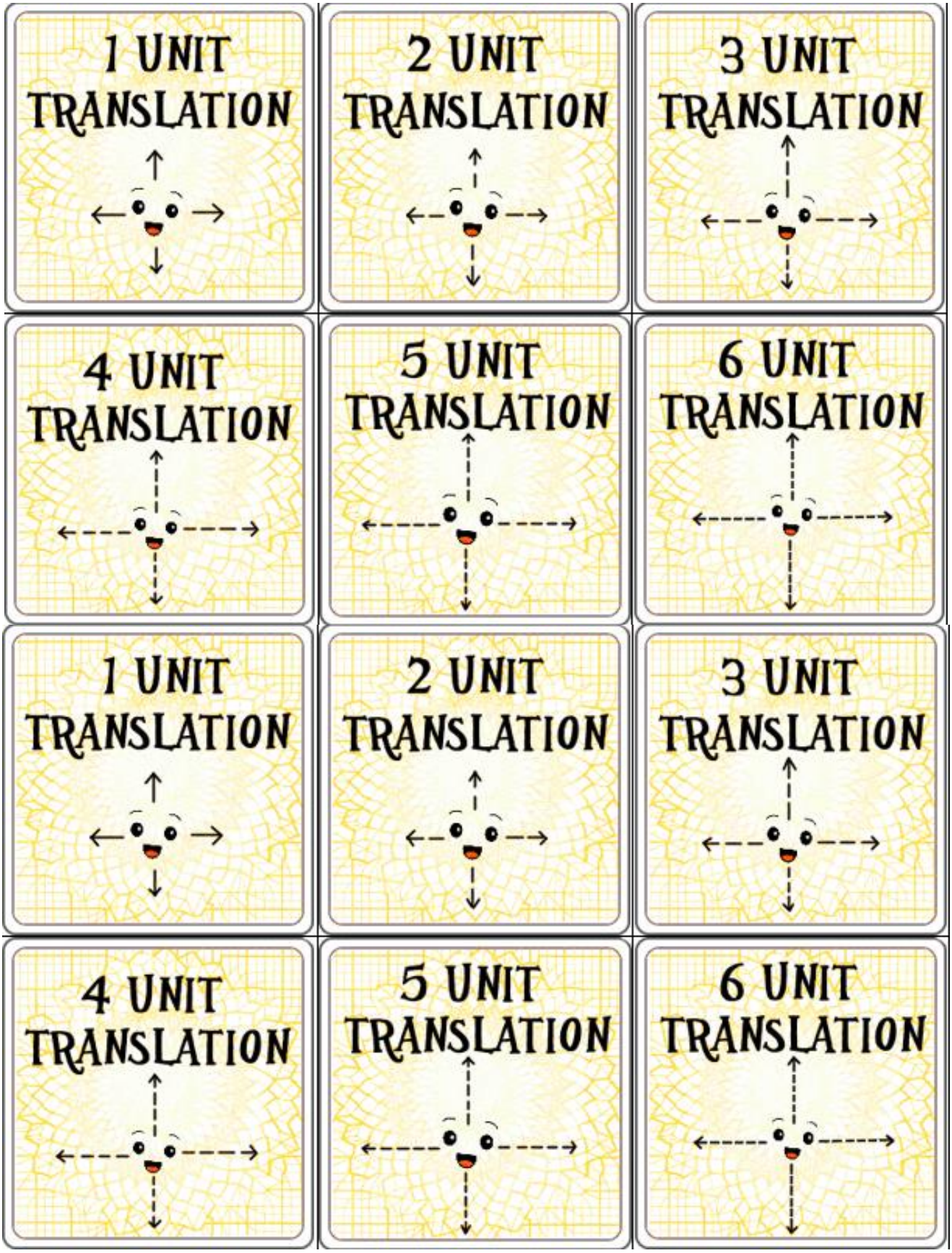
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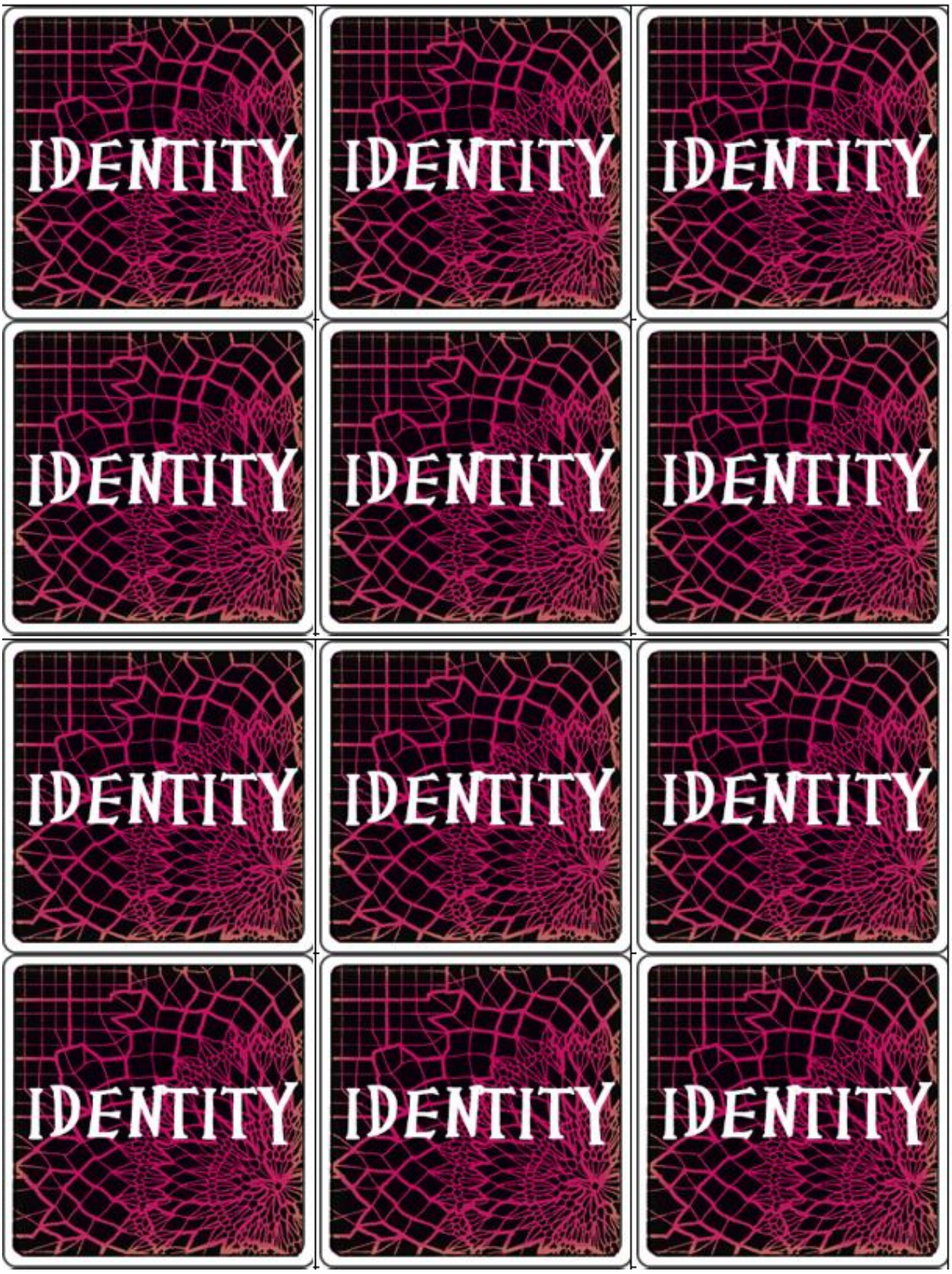
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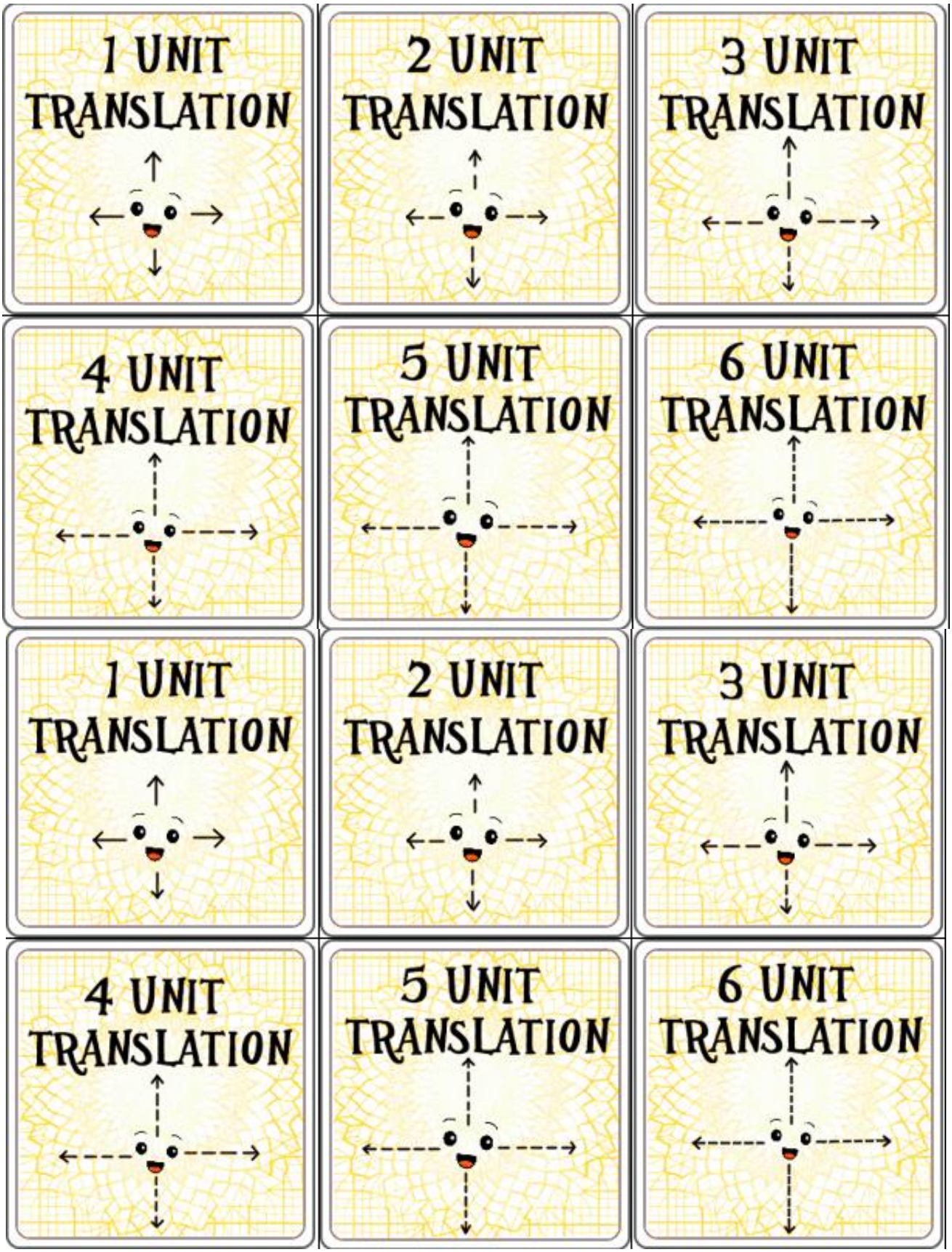
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