Mathematical Modeling and Empirical Validation of a Conceptual Exoskeleton for Astronaut Glove Augmentation

Joseph Kissling
Grand Valley State University

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MATHEMATICAL MODELING AND EMPIRICAL VALIDATION OF A
CONCEPTUAL EXOSKELETON FOR ASTRONAUT GLOVE
AUGMENTATION

Joseph M. Kissling

A Thesis Submitted to the Graduate Faculty of
GRAND VALLEY STATE UNIVERSITY
In
Partial Fulfillment of the Requirements
For the Degree of
Master of Science in Mechanical Engineering

Padnos College of Engineering and Computing

August 2018
To all mankind
ACKNOWLEDGMENTS

Special thanks to my thesis committee for supporting me and providing me with the tools and skills to complete this work. Thank you to Dr. Brent Nowak, who from our very first meeting pushed me to be a better engineer. I cannot put into words how grateful I am for the opportunities to learn and grow my engineering skill set over the years. To Dr. Blake Ashby, for proving me with the programming experience and the education to solve the most challenging parts of my thesis. Thank you Dr. Jeanine Beasley for providing the vital experimental equipment to carry out my work.

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Joseph Kissling
ABSTRACT

Mathematical Modeling and Empirical Validation of a Conceptual Exoskeleton for Astronaut Glove Augmentation

by

Joseph M. Kissling, Master of Science

Grand Valley State University, 2018

Committee Chair: Dr. Brent Nowak
Department: Mechanical Engineering

Space presents numerous difficulties for astronauts conducting their work, not the least of which is the spacesuit that is worn to protect them from space. It has long been known that a spacesuit is difficult to work in, especially the rigid and pressurized gloves that put strain on the astronaut’s hands, frequently leading to injuries. Astronaut gloves inhibit more than 50% of their strength in some cases [1]. NASA and other space agencies have been working to alleviate these problems by attempting to mechanically augment the gloves to reduce the exertions of the astronaut. To date, no augmentation systems have been implemented into spacesuits and prototypes are actively undergoing design and development [2] [3]. Currently existing prototypes are impractical, unconformable, or not effectively augmenting the astronaut as evidenced by the non-implementation of such systems to date.

This work presents a novel conceptual exoskeleton design for astronaut glove augmentation and a mathematical model that is used to predict its performance. In addition, experiments were conducted to validate the math model. The conceptual
exoskeleton is designed to overcome the shortcomings of previous attempts to augment astronaut gloves by using rigid linkages actuated by a single tendon routed through them. This system operates exclusively on the dorsal surface of the hand, limiting the restrictions to the palmar surface of the hand. The mathematical model presents a method to equate the tendon tension to the contact force between the linkages and the object that is being grasped.

Two representative models of the conceptual exoskeleton were built and tested. The experimental fixture, custom designed and fabricated, used a Pliance Pressure Pad to measure the total forces produced by the system. The measured force values were then compared to predictions made by the system to assess the accuracy of the mathematical model. The experimental configurations of the systems were measured using a machine vision system.

The mathematical model was shown to accurately predict the contact forces produced by the representative test rigs. Relationships between the contact forces developed in a grasp and the readings from a Jamar Grip Dynamometer were then used to estimate the magnitude of grip strength that the full exoskeleton could develop [4]. These estimations indicate that the conceptual system would be able to recover up to 124% of the strength that astronauts lose to their gloves.

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NOTATION

Symbols

◦ Degree

\( i \) Is a subscript that denotes segment or joint where:

\[
\begin{align*}
\text{MCP} & \quad \text{if } i = 1 \\
\text{PIP} & \quad \text{if } i = 2 \\
\text{DIP} & \quad \text{if } i = 3
\end{align*}
\]

\( \mathbf{M} = \begin{bmatrix} x \\ y \end{bmatrix} \) Matrices are bold

\( \vec{r} \) Vector

\( \theta_i \) Angle of the joint \( i \)

\( L_i \) Length of the segment \( i \)

\( \text{RF}^{\text{pos}}_k \) Reaction force positions

\( \text{T}A^{\text{pos}}_j \) Tendon attachment Positions

\( \text{LRF}_k \) Reaction force length along segment, where \( k = i \)

\( \text{L}T^{\text{A}}_j \) Length of each tendon attachment point local to its linkage

\( \tau \) Torque

\( \times \) Cross Product

\( \perp \) Perpendicular

\( F \) Force

\( \angle \) Angle between

\( \pi \) Line

\( t \) Tendon subscript

\( \sigma \) Standard Deviation
CHAPTER 1

Introduction

1.1 Introduction

Extravehicular Activities (EVA) pose many challenges for astronauts because space is a harsh environment with numerous environmental threats such as the near vacuum, temperature extremes, solar and cosmic radiation, and high-velocity particles [14]. To protect from these threats astronauts wear a space suit that is made up of multiple layers, each with a particular function. The outermost layers of the suit protect the astronaut from the extremes of temperatures between sunlight and shadow as well as the high-velocity particles found in Earth orbit. The innermost layers are designed to be pressurized with a breathable atmosphere to protect from the near vacuum. Some parts of the suit also have components that provide support for the layers and various seals between suit segments. In places, there can be up to fourteen layers that make up the suit [15]. The entire suit behaves like a miniature spaceship to protect that astronaut, but it also needs to allow the user to work and interact with the world through the gloves.

The National Aeronautics and Space Administration (NASA) has created gloves that allow the astronaut the use of their hands and protects them from the hazards of space by breaking gloves up into three primary layers, each comprised of multiple parts. Starting with the outermost layer, they are the Thermal Micrometeoroid Protection Garment (TMG), the Restraint, and the Bladder. The TMG itself is made up of anywhere between five and seven layers of Mylar and Kevlar designed to insulate the hands from temperature extremes as high as $\pm 200 \, ^\circ C$. Insulation is necessary
because surfaces in low earth orbit may become extremely hot or cold depending on their exposure to the sun and protect from loss of heat to space itself. The outermost layer is a white Ortho-Fabric to reflect away some radiant heat of the sun to prevent overheating [16]. All the layers of material serve to protect from high-velocity orbital debris and micro meteoroids [17]. The TMG is a separate component and not integrated directly into the glove; instead, it can be slid on and off and is held on the Restraint with clasps [18]. The Restraint is beneath the TMG and provides support for the Bladder with customized hard and soft components strategically placed to improve dexterity [19]. Most of its components are made up of fabric with one notable exception being the hand bar. The hand bar is made of 3D printed stainless steel and is set across the Distal Palmar crease to act as an attachment point for other components of the restraint. The innermost layer is the Bladder, which keeps a breathable atmosphere inside of the space suit and maintains pressure. It is made of impermeable urethane that is selectively ridged to improve flexibility. Figure 1.1 shows the different layers of the gloves.

![Fig. 1.1: The three major glove components [5]](image)

The gloves, however, have multiple drawbacks. All the layers make working in
the gloves challenging and sometimes hazardous to the astronaut. Dexterity and tactility are impacted significantly, even with modern gloves and modifications [20] [21]. Most prominent among the drawbacks, however, is the reduction of grip strength. Thompson et al. found that with the current Phase VI glove the grip strength falls to 55% of the nominal value [1]. When the suit is pressurized, it balloons outward and becomes more rigid, reducing the grip strength further to 46% of its nominal value. Table 1.1 shows the full results of the study by Thompson et al. on Phase IV gloves with and without pressure and with and without TMG to show the impact that various configurations have on grip strength.

Table 1.1: The impact of the glove on grip strength from Thompson et al.

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<th>Bare-Hand</th>
<th>Gloved (No TMG)</th>
<th>Gloved (TMG)</th>
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<tr>
<td></td>
<td>0 psi</td>
<td>4.3 psi</td>
<td>0 psi</td>
</tr>
<tr>
<td>Minimum</td>
<td>60</td>
<td>43</td>
<td>33</td>
</tr>
<tr>
<td>Maximum</td>
<td>135</td>
<td>80</td>
<td>73</td>
</tr>
<tr>
<td>Mean</td>
<td>93</td>
<td>59</td>
<td>50</td>
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*Note. All values are reported as pound-force (lbf)*

Working with the gloves can be an exhaustive task, especially when an EVA may last many hours [22]. This, in turn, results in numerous injuries to the hands of astronauts which by far have the highest number of injuries of any part of the body. Figure 1.2 is a summary of an injury study in NASA spaceflight up to the year 2009 conducted by Scheuring et al. [6].
The same study found that of injuries that can be attributed directly to the EVA suit, the hand is the most common place to be injured. Injuries to the hand account for approximately 60% of all injuries incurred in the EVA suit. Figure 1.3 is a summary of the injuries caused by the EVA suit.

Scheuring et al. attributed hand injuries directly to the increased force needed to move pressurized, stiff gloves, and the gloves causing blisters and pain across the astronaut’s joints. Viegas et al. described the types of injuries as fingertip abrasions, traumatic onycholysis which is the separation of the nail from the nail bed, subungual hematoma or bleeding under the nail, compressive neuropathies which is pressure on the nerves of the hand, and displacement or subluxation of the MCP joint [23]. Due to the risk of losing fingernails, there are some reports of astronauts having them
removed before their mission \[24\].

NASA has examined methods to mitigate the injuries to astronaut’s hands with varying degrees of success. One approach that is currently in use is to operate the suit at lower pressures, reducing the severity of the inflation. This method does minimize the impact on the astronaut’s hand and has the added benefit not requiring costly glove redesigns. Additionally, it reduces the strain on the suit itself improving its longevity. This, however, imposes another set of problems particularly the pre-breathe time when the astronauts are conditioned to lower pressures and higher oxygen concentrations. Without this decompression time, up to twenty-four hours in some cases, astronauts risk getting the bends \[25\].

Similarly, recompressing from an EVA requires time though nowhere near as much as decompression. Pre-breathe and recompression causes problems for mission planners because the time it takes up mostly cannot be used for mission work. It also induces challenges in responding to emergencies that may require an EVA.

Another method that is being looked into by NASA and other space agencies is to augment the strength of the astronaut by embedding a robotic system into existing gloves. These systems would in theory work with the astronaut, thereby reducing their exertions, making it easier to work, and lowering the frequency of injury. Augmentation is favored over redesign because it allows using existing glove designs to be adapted rather than brand new developments. During the development of the current Phase VI gloves, the cost was a significant consideration in addition to new designs needing to be more comfortable and simpler to manufacture \[19\]. Space suit development is very costly; the next generation of astronaut suits has already cost NASA almost 200 million dollars in R&D \[26\]. NASA has also awarded $300,000 in competitions for new glove designs \[27\]. As of this writing, no robotic augmentation system or glove redesign has been implemented, and all projects remain prototypes.
1.2 Purpose

This work aims to assess the theoretical performance of a novel conceptual exoskeleton design that may be a practical method of reducing the detrimental effect that gloves have on astronaut grip strength. The theoretical model will be tested against a representative exoskeleton to validate the accuracy of the model. Information gathered from the assessment will be used to drive future improvements and implementation work.

1.3 Hypothesis

The grip strength that astronauts lose to the gloves can be recovered by using an exoskeleton built into the glove that assists with the closing of the hand. This exoskeleton would consist of a set of serial linkages acting on the dorsal aspects of the proximal, intermediate, and distal phalanges of the finger to keep the palmar surface of the hand as clear as possible to avoid contributing to the lack of tactility. Figure 1.4 shows a rendered model of one finger of the system.

![Fig. 1.4: CAD model rendering of the conceptual system.](image)

The conceptual system uses a series of linkages that run along the lateral and dorsal aspect of each finger with hinges that share the same joint axis as the phalanges.
of the finger. For the distal interphalangeal (DIP) and proximal interphalangeal (PIP) joints the system lays directly along the axis of rotation. The metacarpophalangeal (MCP) joint is articulated by a circular slider with the same rotational center as the joint in the finger and would behave as a moving pulley. The system also has a second hinge beneath the MCP slider to accommodate the MCP joint’s second degree of freedom (DOF). Figure 1.5 shows the four primary elements that make up the conceptual system in greater detail.

![Fig. 1.5: Major components of the conceptual system](image)

The system is driven by a tendon that is routed through the linkages in such a way as to facilitate the closing of the hand when the tendon is under tension. Figure 1.6 shows the general routing scheme of the tendon in blue. In this conceptual model, the tendon is attached directly to the linkages, rather than some pulley-like examples as found in Section 2.1.
1.4 Scope of Work

For the scope of this work, the existing conceptual system was modeled, using Mathematica to determine the magnitude of the contact force it is capable of develop-
ing. Representative models, called rigs, of the conceptual system, was then fabricated to validate the mathematical model empirically. The different rigs were subjected to various tendon tensions and orientations; then measured values were compared to mathematical predictions. Lastly, the theoretical performance of the system is used to estimate the grip force it would produce.
2.1 Human Hand Augmentation

The idea of mechanically augmenting the human body for assistance purposes has existed for over a century. Professor H. Wangenstein put forth what is one of the earliest concepts of a machine to assist those who have paraplegia in 1883, describing a pneumatic body frame to replace a wheel chair [28]. In 1890, a patent was filed by Nicholas Yagn for a device to augment the lower body, described as “facilitating walking, running, and jumping.” This early attempt at an exoskeleton did not have an external power source; instead, it relied on the operator to compress the air and store energy in the springs that drove it [29]. This idea was improved upon in 1917 by Leslie Kelley in a patent filed for a similar device that was steam driven and used artificial tendons to facilitate augmentation of the wearer [30]. Figure 2.1 shows examples of what these early systems looked like.

(a) Nicholas Yagn’s exoskeleton sketch adapted from Herr et al. [31]  
(b) Leslie Kelley’s exoskeleton from the patent [30]

Fig. 2.1: Early examples of exoskeletons
In the 1930’s various upper-body exoskeleton-like systems were developed for those affected by Polio. These systems were affixed to tables, wheelchairs, or corsets and were either passively driven or controlled by the feet of the user [32]. The late 1940’s and 1950’s saw the development of electrically powered systems for both upper and lower body [33]. One of the first devices that would be considered a true exoskeleton appeared in the 1960’s as a result of a collaboration between the Army, the Navy, and General Electric [7]. It was called Hardiman and featured a full body mechanical system that augmented the strength of the wearer by a factor of 25. Figure 2.2 shows a concept sketch of the Hardiman Exoskeleton.

![Fig. 2.2: Hardiman concept sketch [7]](image)

Since then numerous other exoskeleton systems have been developed or explored for a variety of uses like military, rehabilitation, and laborer assistance. Figure 2.3 are some modern examples of exoskeletons. Sources [34] [31] [35] contain additional examples of systems and research for the avid reader.
Along with the development of full body exoskeleton systems came the design of systems for specific parts of the body [39]. The late 1980’s and early 1990’s saw the first developments of exoskeletons for the human hand [40]. These early systems were initially part of haptic feedback systems and controllers for remote multi-finger robotic hands [41]. Figure 2.4 shows examples of the first exoskeletons, Figure 2.4a is for hand orientation or range of motion measurements as part of the controller of the UTAH/MIT Dexterous Hand. Figure 2.4b is a force feedback system to serve as both the controller and haptic feedback system for the UTAH hand.

Fig. 2.3: Modern Exoskeleton Examples

Fig. 2.4: Early exoskeleton systems for control
After that came the development of hand exoskeletons for the rehabilitation and assistance of patients who had suffered nerve damage [42] [43]. Figure 2.5a is a device to assist patients that have sustained a C5-6 spinal injury, Figure 2.5b is designed to be used by individuals with quadriplegia.

(a) Myoelectric Hand Orthosis from Benjuya et al. [42]  
(b) SMART Wrist-Hand Orthosis from Makaran et al. [43]

Fig. 2.5: Early Hand Exoskeletons for Rehabilitation

From these early systems, the number and diversity of hand exoskeletons have increased significantly due to technological developments and the complexity of the human hand requiring creative mechanisms to interact with it [9]. Technological advances such as miniaturization of sensors, actuators, and processors; more powerful controllers; and improved manufacturing techniques afforded more options for exoskeleton configurations [44]. The complexity of the human hand is a byproduct of its small size, range of motion, 23 DOF, and coupling of some of the DOF [8]. Figure 2.6 shows the DOF of the human hand, red representing independently controllable joints, blue for those that are coupled with others. The figure shows that the PIP and the DIP joints are 1 DOF while the MCP joint has 2 DOF shown circled in green.
Additionally, the human hand is compliant allowing for some systems to take advantage of the human hand’s inherent adaptability. All the effects on the design are apparent in a review of hand exoskeletons done by Heo et al., in their characterizations of the methods that systems match or bypass the centers of rotation of the joints in the hand or fingers [9]. Figure 2.7 shows these six major classifications.

Fig. 2.6: The 23 DOF of the human hand, modified from Favetto et al. [8]

Fig. 2.7: Heo et al. characterization of exoskeleton systems [9]
Direct matching of joint centers, shown in Figure 2.7a, operates by having a system where its hinges lie directly in line with the joints of the finger. While the finger joints are not pinned like mechanical systems, the human hand is compliant enough to move with the pinned hinge system [45]. This system may be paired with other systems such as tendons or the various systems involving linkages to improve system performance. Figure 2.8a from Chiri et al. is one example of a direct matching of joint centers that uses tendons to transmit the actuation force [46]. The PIP joint and DIP joint centers in this system are matched directly while the MCP joint is matched using a combination hinge and sliding mechanism. Figure 2.8b is another example by Hasegawa et al. that also uses tendons and is meant for the whole hand. This device operates on all the fingers of the hand, but couples the middle, ring, and small finger together while allowing the index finger and thumb to move independently [47]. In this system, all the finger joints have hinges that share their axes of rotation. However, due to the MCP joint being in the interior of the hand, this can only be achieved because the fingers are coupled together allowing the hinges to be placed on the exterior of the hand. Each of these systems is driven by tendons that are routed through them and attached to pulleys to produce torques around the joints.
Linkage for remote center of rotation, shown in Figure 2.7b, uses a series of linkages that act together to form a rotation center that is the same as the joint. Figure 2.9a is an example of an assistive device made by Takagi et al. to aid the elderly with everyday gripping activities [48]. Fang et al. used a system of four-bar linkages, shown in Figure 2.9b, as a haptic feedback system for a robotic hand controller [49].

![Image of assistive device](image1)

![Image of robotic hand controller](image2)

(a) Grip Aid System by Takagi et al. [48]  (b) Master Hand by Fang et al. [49]

Fig. 2.9: Linkage for Remote Center of Rotation

Redundant linkage structure, as shown in Figure 2.7c, is like linkage for remote center of rotation in that it uses a series of linkages to control the position of the finger. However, in this case, the linkages are independently controlled or constrained in their motion to facilitate movement rather than directly controlled by the linkage mechanics. This kind of mechanism is useful for cases when direct control of each of the phalanges is desired [50]. Figure 2.10a is an example of a hand exoskeleton made by Wege et al. to aid with the rehabilitation process of stroke victims or those with hand injuries [51]. Ueki et al. also developed a device to assist in rehabilitation shown in Figure 2.10b, that actively controlled 3 of the 4 DOF of the finger [52].
Tendon-driven mechanisms, as shown in Figure 2.7d, are systems that are driven by artificial tendons affixed to the hands usually by being embedded in gloves. This case is for the systems where there is only a tendon used to actuate the fingers, not the cases where the tendon is paired with another configuration for actuation purposes. Figure 2.11a is the Exo-Glove by In et al. which could augment the strength of a healthy user or assist a patient that has paralysis of the hand to grasp objects [53]. This system has tendons that run along the palmar and dorsal surfaces of the hand to facilitate opening and closing. Figure 2.11b is a system created by a partnership between NASA and GM to assist workers on the factory floor and potentially even astronauts [2]. This device only assisted with the closing of the hand; therefore the tendons were routed on the palmar surface of the hand.
Bending actuators, shown in Figure 2.7e, use a flexible mechanism to facilitate the movement or augmentation of the fingers. Figure 2.12 shows two examples of the many varieties of flexible actuators [54]. Arata et al. developed a mechanism, shown in Figure 2.12a, that uses a series of flat springs that slide past one another to create digit flexion [55]. The system operated by linear actuator pulling or pushing on one of the springs which in turn facilitates the bending motion. Figure 2.12b is a device built by Toya et al. called Power-Assist Glove which uses flexible pneumatic actuators mounted on the dorsal surface of a glove [56]. The Power-Assist Glove is capable of reducing the exertion of the wearer by 1.5 kg [sic].

(a) Sliding Spring Mechanism by Arata et al. [55]  
(b) Power-Assist Glove by Toya et al. [56]

Fig. 2.12: Flexible Mechanisms

Serial linkage attached to a distal segment, shown in Figure 2.7f, uses a series of linkages that are attached to the finger in one place near the fingertip or distal end. This configuration is used in situations when the control of the fingertip is more desired than control of individual phalanges or for continuous passive motion. Figure 2.13a is Hexosys-II by Iqbal et al. to assist the rehabilitation with stroke victims. According to Iqbal et al., at the time of its creation in 2015, it was capable of exerting greater forces than any other rehabilitation system [57]. Ma et al. developed
an exoskeleton intend to be used as part of a haptic feedback system, shown in Figure 2.13b, as a rehabilitation aid [58].

(a) Hexosys-II Prototype by Iqbal et al. [57] 
(b) Haptic Glove Mechanism by Ma et al. [58]

Fig. 2.13: Serial Linkage Attached to a Distal Segment Examples

2.1.1 Exoskeletons for Astronaut Glove Augmentation

This is by no means an exhaustive list of these systems or their capabilities, but a brief overview of the systems to showcase their practicality for use on astronaut gloves. Some system types are more suitable than others for work in space, as Favetto et al. outlined in their paper “Towards a Hand Exoskeleton for a Smart EVA Glove,” in their discussion on constraints for systems [8]. Favetto et al. state that the exoskeleton size and weight are crucial and should have low mass and inertia to facilitate tasks. Therefore, systems that tend to be larger and bulkier such as Serial Linkage Attached to a Distal Segment, Redundant Linkage Structure, and Linkage for Remote Center of Rotation are less favorable. Another critical constraint is ensuring the palmar surface of the hand remains as free as possible because dexterity and tactility are already severely impacted by the gloves. This means that some classes of the tendon driven mechanisms with tendons on the palmar surface of the hand should be avoided. Lastly, Favetto et al. lists the environment of space as being a primary constraint to the type and complexity of the exoskeletons that can be used.
Plasma and shifting magnetic fields from various sources can damage complex controllers. Charged particle bombardment and high-velocity impacts can damage the mechanism, adversely affecting its ability to move [8] [14]. These considerations are apparent in the handful of exoskeletons that have been designed or proposed for astronaut glove augmentation.

Main et al. proposed the first systems intended to augment the strength of an astronaut in the form of two prototypes that incorporated either springs or pneumatic actuators into gloves [10]. In these prototypes, the MCP joints of the fingers were the only joints that were augmented. Figure 2.14 is an example of the pneumatic prototype which performed better than the spring driven system.

Fig. 2.14: Pneumatic glove augmentation from Main et al. [10]

Another early device for astronaut glove augmentation is a device by Shields et al. that used a system of four-bar linkages to augment the joints on a three-fingered hand [11]. Figure 2.15 shows the device on the hand of a test subject.
The device was capable of generating a torque more than twice of what is required to bend the joints of an astronaut glove. The authors note however that the device represented a significant reduction in the number DOF of the hand making it difficult to use. Also, evident in the picture is the fact that the device is large and bulky making it unfavorable for use in spaceflight.

Yamada et al. developed a device, called SkilMate, that augmented the MCP joint of the thumb and first two fingers. SkilMate operates by using an ultrasonic motor to pull a steel belt over pulleys that guide it around a circular track which allows it to rotate about the MCP joint [12]. Figure 2.16 shows the major components of the SkilMate system.

![SkilMate](image)

Fig. 2.16: SkilMate from Yamada et al. [12]
Sorenson et al. developed a similar device that actuates all four of the MCP joints of the hand simultaneously in a redesigned glove that rests in composite digit flexion or closed when the suit is pressurized [13]. That is to say, passive elements are responsible for the closing of the glove, and active ones are used to open the glove. It was successful in reducing the effort required to close the glove by 30% to 40%. The actuator shown in Figure 2.17 is mounted on the dorsal side of the glove and driven by an electric motor and pulley.

Matheson et al. created a prototype device that uses two pneumatic actuators in conjunction with tendons, linkages, and torsion springs to augment the four fingers of the hand [3]. In this configuration the 12 DOF that would typically accompany the four fingers are reduced to three, encompassing the major joints of the finger. One actuator drives the PIP and the DIP joints at the same time while the other drives the MCP joint. This is similar to how the interaction works in the MCP joints and the long flexor works on the PIP and DIP joints during flexion. The pneumatic

Fig. 2.17: Glove augmentation device from Sorenson et al. [13]
actuators open or facilitated extension of the hand and the springs serve to close or facilitate flexion of the hand. Figure 2.18 shows the device in both extended and contracted or flexed forms.

![Fully flexed](image1)

(a) Fully flexed

![Fully extended](image2)

(b) Fully extended

Fig. 2.18: Astronaut glove augmentation device from Matheson *et al.* [3]

In a collaboration between General Motors (GM) and NASA, Roboglove was developed to assist both workers on a terrestrial assembly line and astronauts working in space. These systems feature a series of tendons that run up the palmar surface of the hand and are attached above the DIP joint on the four fingers of the hand. The tendons are driven by linear actuators that are attached to the wrist of the user and respond to input from pressure sensors in the fingers. The ball screw actuators are capable of exerting a force of 23 kg [sic] and have been adapted from Robonaut hand [59]. Figure 2.19 shows a prototype of the Roboglove that has been integrated into an astronaut glove.

![Prototype](image3)

Fig. 2.19: Roboglove prototype from Diftler *et al.* [2]
Roboglove, the most recent attempt at augmenting astronaut gloves, is currently the only system under consideration and active testing. The prototype is capable of restoring approximately 20% of an astronaut’s grip strength; however, it is not comfortable, and the range of augmented motion is limited \[2\] \[60\]. The tendons further interfere with the tactile sensation of the astronauts, which is already severely impacted by the gloves. Astronauts were displeased with the prototype, and one of them found it so uncomfortable that the glove was removed and thrown across the room \[60\]. In summary, there is currently no form of exoskeleton systems used in spacesuits; this work proposes a possible solution.

### 2.2 Mathematical Models

The inherent challenge of this work is determining the contact force produced by the system as a function of the force exerted by the single actuator across its three DOF. This is because the system described is a member of the underactuated mechanisms class, which one of the defining characteristics is the system has fewer actuators, \( n_a \), than DOF, \( n \) \[61\]; or written another way an \( n_a < n \) configuration. There are two other classifications of systems: one is fully actuated which are those of the form \( n_a = n \) and the other is redundantly actuated which is \( n_a > n \) systems. These are not strict definitions of the system types as there are special cases that can change the true classification of the system \[62\], but for this work, these definitions are sufficient. Figure 2.20 provides examples of the three system types on a three bar serial linkage system that is pinned together by revolute joints with one end pinned in place and the other free.
Fig. 2.20: System types where $A_n$ represents an actuator acting on a DOF

In these examples, any given actuator, $A_n$, is intended to apply a torque to a revolute joint directly and only acting in the direction shown by the arrow with no other forces acting on the system. Figure 2.20a is underactuated because there is a single actuator acting on one revolute joint with the others free to move without direct control. Figure 2.20b is an example of the fully actuated case; an actuator directly controls each of the three DOF of the system. Lastly, Figure 2.20c shows a redundantly actuated system with an additional actuator placed to act in the opposite direction of another actuator.

Underactuated mechanisms are ideal for gripping applications because the use of such systems in finger mechanisms allows for the systems to adapt to various objects and reduces the need for complicated mechanisms of control [63]. This also allows for such mechanisms to be generally lighter and simpler than other system types [46]. For example, fully actuated grasping systems, require controllers to deal with singularities, error propagation, and the large numbers of possible configurations [64]. Fully actuated systems also, by definition, require an actuator for each degree of freedom which increases the size, weight, and complexity of the system [65]. Similarly, Redundantly Actuated Systems have an actuator for each DOF plus at least one
additional actuator and also require a sophisticated controller to operate. The other actuator eliminates the problems caused by singularities [66] [67].

Underactuated systems do not require complex mechanisms or controllers because they are self-adapting to the objects that are being grasped. The self-adapting property results in the joint coordinates and the orientation of the system being indirectly controllable [62]. Therefore, the equilibrium conditions become subject to influences other than the geometry that the system is interacting with, presenting a mathematical modeling challenge when using traditional methods to determine the forces that the system develops.

Traditional methods of static analysis involve creating a sum of forces around each object in the system and solving for equilibrium conditions [68]. Equations 2.1 show the general form of this solution approach, where the total sum of forces, \( F \), and Moments, \( M \), acting on an object is zero.

\[
\begin{align*}
\Sigma \vec{F} &= 0 \\
\Sigma \vec{M} &= 0
\end{align*}
\] (2.1)

In an analysis for two dimensions, as would be required for analyzing a planar mechanism, Equations 2.1 are modified to the form shown by Equations 2.2.

\[
\begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma M_z &= 0
\end{align*}
\] (2.2)

Applying these principals to an arbitrary underactuated system shown in Figure 2.21 showcases where they fall short. In the figure, \( F_{cn} \) is a reaction force from some contact point and \( F_{an} \) represents the distributed forces exerted on the system by its single actuator. In this system, the only knowns are the \( F_{an} \) terms and the
geometry of the system.

The forces acting on the first (or left) link are shown in Figure 2.22.

In Figure 2.22 $l_a$ is the length along the segment that $F_{a1}$ is acting, $l_c$ is the length along the segment that $F_{c1}$ is acting, and the overall length of the segment is $l$. The equilibrium equations for the first link are shown by Equations 2.3.

\[
\begin{align*}
\Sigma F_x &= F_{rx_1} - F_{ax_1} - F_{cx_1} - F_{rx_2} = 0 \\
\Sigma F_y &= F_{ry_1} - F_{ay_1} + F_{cy_1} - F_{ry_2} = 0 \\
\Sigma M_1 &= l_a F_{a1\perp} - l_c F_{c1\perp} + l F_{y2\perp} - l F_{x2\perp} = 0
\end{align*}
\]
Using the classical static equilibrium methods yields three equations and five unknowns to solve. The same process was applied to the second linkage in the system, yielding a total of six equations and eight unknowns. In this case the unknowns would be the six joint loads, \(F_{rx_1,...,3}, F_{ry_1,...,3}\) along with the contact forces \(F_{c1}\), and \(F_{c2}\). With more unknown reaction forces than available equations of static equilibrium, the system is statically indeterminate [69]. This class of problem can be solved by applying equations of compatibility by using force-displacement equations that take advantage of the elastic properties of beams. These equations, however, are not general equations and become very specific depending on the geometry and mechanical properties of the system components in question. Therefore, while it is possible to develop a solution using the force-displacement equations, this method is not suitable for finding solutions for general cases. For the general cases, another solution method is required and considered for this work.

Quasi-Static Analysis is a series of methods that can provide solutions for systems where the static analysis method falls short. In such cases the system is subjected to imaginary displacements or velocities and the work or power produced by the various forces in the system is equated to generate solutions. One method available within Quasi-static analysis is the principle of virtual work, which is the work done by a real force acting through a virtual displacement or a virtual force acting through a real displacement [70]. In this process, unknown forces can be calculated by subjecting a system to a virtual displacement and equating the works of the forces involved. The system is in equilibrium if the total work done by the acting forces is zero [71]. Equation 2.4 shows the general form of the principle of virtual work where \(\delta U\) is the virtual work, \(F_n\) are the forces in question and \(\delta_n\) is the virtual displacement.

\[
\delta U = \sum F_n \delta_n = 0
\]  

(2.4)
Chiri et al. used the principle of virtual work as part of the development of their dynamic model of their tendon driven HANDEXOS finger exoskeleton system intended to assist users that suffer from hand spasms. For the model, the Lagrangian Equations of Motion for the finger model use the virtual work principle to determine the relationship between the generalized forces applied to the joints and the generalized forces applied to the links. Equation 2.5 represents their joint-space dynamic model, refer to [46] for full explanation of the terms.

\[
\mathbf{B}(q)\ddot{q} + \mathbf{C}(q, \dot{q})\dot{q} - \mathbf{F}_v \dot{q} + g(q) = \]
\[\tau + \mathbf{K}r^2(q_0 - q) + \mathbf{J}_1^T(q)\mathbf{H}_1(q) + \mathbf{J}_2^T(q)\mathbf{H}_2(q) + \mathbf{J}_3^T(q)\mathbf{H}_3(q) \]

(2.5)

In this model \(\mathbf{J}_i^T(q)\mathbf{H}_i(q)\) terms have been derived from the principle of virtual work where the \(\mathbf{J}_i^T\) is the (6x3) geometric Jacobian matrix and \(\mathbf{H}_i\) is the vector of forces and moments from the resistance forces in each link. The \(\mathbf{K}\) term is a (3x1) vector of spring stiffness coefficients that are built into the system. This model was used to optimize the trajectories of the mechanical finger to improve its design.

Montambault and Gosselin also used the principle of virtual work as part of their analysis of underactuated mechanical grippers'. In their analysis, they begin with developing the kinematic models of their 4 DOF underactuated gripper to create the Jacobian matrix out of the kinematic chains. The principle of virtual work was then used in their static model to determine the grippers output forces. This analysis also included the addition of a passive spring to aid in the force output calculations and assumed no friction between contact points [72].

Prattichizzo et al. used the principle of virtual work in their motion and force control analysis of robotic grasping systems. They note in their study that the statically indeterminate case is frequently encountered in grasping systems and can be remedied by dropping the rigid body assumptions and applying the principle of virtual
work to the resulting deformations from contact compliance [65].

The principle of virtual work is a powerful tool for working around some issues caused by indeterminate static systems; however, it has its limitations. In each of the cases above, there were additional assumptions that were made to facilitate the use of virtual work. In the case of Chiri et al. virtual work was used as a way to convert forces into generalized ones, not as a direct solution scheme. Instead, it was used to augment the Lagrangian solution method. Montambault and Gosselin included an extra passive force, simulating a return spring, within their analysis so that the virtual work could be used. Prattichizzo et al. removed the rigid body assumption so that the system would do work in its deformations. To develop a general solution method for the conceptual system a different type of analysis is needed.

The principle of virtual power, sometimes called minimum power, is another solution method that is available under the quasi-static analysis umbrella and is capable of overcoming the shortcomings of virtual work [73]. Virtual work applies only to quasi-static systems subject to normal forces arising due to contacts among rigid bodies, friction forces and forces independent of velocity [74]. Equation 2.6 shows the general form of virtual power, where $P$ is power, $F_n$ are the forces in question, and $v_n$ is the virtual velocity of the force in question.

$$P = \sum F_n v_n$$  \hspace{1cm} (2.6)

In application, the principle of virtual power is used to equate the input and output powers of the forces acting on the system. Equation 2.7 is an example of the virtual power from Laliberte et al. method as it is commonly used in robotic manipulator analysis [61].

$$t^T \omega_a = f^T v$$  \hspace{1cm} (2.7)
Where $t$ is the input torque vector from the actuator, $\omega_a$ is the corresponding imaginary velocity, $f$ is the vector of contact forces, and $v$ is the vector of the velocity normal to the respective linkages. The contact forces can be solved for by rearranging Equation 2.7 into Equation 2.8.

\[
f^T = v^{-1}t^T\omega_a \tag{2.8}
\]

Laliberte et al. use this formula to develop an expression for the contact forces of a linkage driven 2 DOF robotic finger to assess the risk of ejection during a grasping sequence. Figure 2.23 is the underactuated system Laliberte et al. conducted their analysis on.

Fig. 2.23: 2 DOF Underactuated system from Laliberte et al.

Ha et al. conducted a similar analysis on a 3 DOF finger driven by a series of four-bar linkages and an input torque from an actuator [75]. In this analysis, the velocities of the contact points were calculated by multiplying the virtual rotational velocity times the Jacobian matrix. This information was then used as part of a simulated control system for the robotic finger to assess the accuracy of the model.

Birglen and Gosselin also analyzed a 3 DOF finger using the principle of virtual power. In their analysis, they use screw theory to define the forces as contact wrenches.
and the velocities as the twist of the contact points [76].

Dandash et al. devised a system that is driven by a tendon-cam system which was designed and optimized for isotropic grasping force using the principle of virtual power [62]. In their document, they walk through the full process of performing the virtual power analysis.
CHAPTER 3
Methodology

3.1 Mathematical Model

3.1.1 Overview

The mathematical model was developed in Mathematica™ to take advantage of the software’s ability to model complex systems and create graphics from data and equations [77]. System performance was be determined by calculating the magnitude of the reaction forces that the system would develop during a grasp. Development of the model follows the procedure laid out below.

1. Define Virtual power. Section 3.1.2
2. List Definitions & Assumptions. Section 3.1.3
3. Define Independent Angular Position Variables. Section 3.1.4
4. Define Position Equations for elements in the system. Section 3.1.5, 3.1.6 & 3.1.7
5. Define Forces In the system. Section 3.1.8 & 3.1.9
6. Calculate Velocities. Section 3.1.10
7. Calculate the virtual powers. Section 3.1.11
8. Solve for Reactions. Section 3.1.12
3.1.2 Power Calculation

The principle of virtual power requires calculating and equating the power that the input and output forces of a system would generate, then solving for the unknown forces. Power is defined as the time rate of change of work, shown in Equation 3.1 where $U$ is work and $t$ is time.

$$P = \frac{dU}{dt} \quad (3.1)$$

Where $dU = \vec{F} \cdot d\vec{r}$ therefore, Equation 3.1 can be rewritten as Equation 3.2.

$$P = \frac{\vec{F} \cdot d\vec{r}}{dt} \quad (3.2)$$

Since $\vec{v} = d\vec{r}/dt$ is equal to velocity, $v$, Equation 3.2 becomes Equation 3.3.

$$P = \vec{F} \cdot \vec{v} \quad (3.3)$$

This analysis concerns the virtual power developed by forces acting about the joints of a planar system of linkages, so the velocities and forces are better represented as their angular counterparts. Therefore, $d\vec{r}$ becomes $d\theta$ and the force $F$ becomes a torque, $\tau$. From Equation 3.1 $dU$ now equals $\tau d\theta$, meaning that Equation 3.2 can be rewritten as Equation 3.4.

$$P = \frac{\tau d\theta}{dt} \quad (3.4)$$

Substituting $\dot{\theta}$ for $d\theta/dt$, Equation 3.4 becomes Equation 3.5. This is for planar systems only.

$$P = \tau \dot{\theta} \quad (3.5)$$
To solve for the unknown reaction forces using the principle of virtual power, the system is subjected to virtual rotations (velocities) and the power produced by the reaction forces, $P_r$, is equated to the power produced by the tendon, $P_t$. Equation 3.6 showcases the equivalence.

$$P_t = P_r$$  \hspace{1cm} (3.6)

### 3.1.3 Definitions & Assumptions

The conceptual model will first be simplified into a system of three serial linkages pinned together by three revolute joints. One end is pinned in a fixed support, and the other is free to move. The tendons running along the sides of the system are simplified into one tendon represented as a set of forces acting on the points where it attaches to the system. The contact forces are modeled as frictionless contact points so that the only forces are acting perpendicular to their respective segments. Figure 3.1 shows the simplified system with the revolute joints shown as black circles, tendon forces in blue, and reaction forces in red.

Fig. 3.1: Simplified system representation
The system is parameterized so that the model can represent various configurations. Firstly, the orientation of the system is defined using relative angular positions of each of the three segments as shown in Figure 3.2 using $\theta_i$ where $i$ is either 1, 2, or 3.

Fig. 3.2: Angular Position Variables

Each $\theta$ represents a negative rotation of the joint in the model, directly analogous to the joints in the finger. Therefore, $\theta_1$ is the revolution about the MCP joint or knuckle, $\theta_2$ is the revolution about the PIP joint, and $\theta_3$ is the revolution about the DIP joint.

The other geometric parameters of the system are shown in Figure 3.3, these are dimensions that are set by the design of the system and are independent of the orientation. Lengths of the three segments are defined by the variables shown in Figure 3.3a. $L_{\text{prox}}$ is the length between the MCP joint and the PIP joint, $L_{\text{mid}}$ is the length between the PIP joint and the DIP joint, and $L_{\text{dis}}$ is the length between the DIP joint and the end of the segment. Similarly, the variables that define the tendon
attachment point lengths are shown in Figure 3.3b. The base of each of the arrows is the attachment point. These are local lengths along the segment to which they are attached to. The lengths LTA₁ and LTA₂ are measured from the MCP joint, LTA₃ and LTA₄ from the PIP joint, and LTA₅ from the DIP joint.

![Segment length Variables](image1)

(a) Segment length Variables

![Tendon Attachment length Variables](image2)

(b) Tendon Attachment length Variables

Fig. 3.3: Geometric dimensions

The reaction force contact point lengths are defined in Figure 3.4; these lengths are local to the segment that they are on. LRF₁ is measured from the MCP joint, LRF₂ is measured from the PIP, and LRF₃ is measured from the DIP joint.

![Contact Point Position Variables](image3)

Fig. 3.4: Contact Point Position Variables
Force magnitude variables are shown in Figure 3.5; each reaction force is the total of outside forces acting on each segment. The magnitude of the tendon force, $F_t$, is shown by the blue arrows. The mechanics of the exoskeleton cause the tendon acting about the MCP joint to behave like a massless, frictionless pulley, so its force is always perpendicular to the segment.

![Fig. 3.5: Force Magnitude Variables](image)

Table 3.1 provides a summary of the parameterized inputs to the system. The modifier describes the influences on the variables; Orientation is for parameters that are dependent on the system’s orientation, System is for parameters that are independent of the system orientation and are related to the dimensions of the system.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Modifier</th>
<th>Units</th>
<th>Total Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$ where $i = 1, 2, 3$</td>
<td>Orientation</td>
<td>Degrees</td>
<td>3</td>
</tr>
<tr>
<td>LMCP, LPIP, LDIP</td>
<td>System</td>
<td>Meters</td>
<td>3</td>
</tr>
<tr>
<td>LTA$_j$ where $j = 1, \ldots, 5$</td>
<td>System</td>
<td>Meters</td>
<td>5</td>
</tr>
<tr>
<td>LRF$_k$ where $k = 1, 2, 3$</td>
<td>Orientation</td>
<td>Meters</td>
<td>3</td>
</tr>
<tr>
<td>RF$_i$ where $i = 1, 2, 3$</td>
<td>Orientation</td>
<td>Newtons</td>
<td>3</td>
</tr>
<tr>
<td>$F_t$</td>
<td>System</td>
<td>Newtons</td>
<td>1</td>
</tr>
</tbody>
</table>
The parameters of the system are modeled as ideal. Segments are modeled as rigid bodies that interact without friction at points of contact with other objects. The tendon is modeled as unstretchable, maintains the same tension throughout, and interacts without friction. All other outside forces are neglected.

3.1.4 Angle Position Variables

The angles of the respective joints $\theta_i$ are illustrated in Equation 3.7. These angles are defined with respect to time so that the time derivative of the position can be used to calculate the angular velocity for virtual power calculation.

$$\theta_i = \begin{cases} 
A1 \text{ or } \theta_{\text{MCP}[t]} \quad & i = 1 \\
A2 \text{ or } \theta_{\text{PIP}[t]} \quad & i = 2 \\
A3 \text{ or } \theta_{\text{DIP}[t]} \quad & i = 3
\end{cases} \quad \text{(3.7)}$$

3.1.5 Position Matrices

The orientation of the system is crucial for accurately predicting the magnitude of the output force. Therefore, the position of all known parts of the system must be represented by the model. Positions in two-dimensional space can be expressed as a set of $(x, y)$ coordinates as shown by Figure 3.6.

![Fig. 3.6: Point defined by some length along the x and y axes.](image)
Which may also be written in matrix form as shown in Equation 3.8.

\[
\mathbf{Pos} = \begin{bmatrix} x \\ y \end{bmatrix}
\]  

(3.8)

Another way to represent a point in space is to use the distance between that point and an angle with respect to a reference line, that is to say, the polar coordinate system. For example, Figure 3.7 shows a point defined by its distance from the origin and an angle with respect to the x-axis.

![Diagram of a point defined by length L and angle θ.](Image)

**Fig. 3.7:** Point defined by some length L and an angle θ.

This may be expressed in vector form as a matrix with the magnitude, L, times a unit vector of the form \([\cos(\theta) \quad \sin(\theta)]\). The resulting matrix is shown by Equation 3.9.

\[
\mathbf{Pos} = L \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}
\]  

(3.9)

In this analysis, angles are measured from the y-axis for convenience as illustrated by Figure 3.8. \(\theta\) is also treated as negative.
Therefore, Equation 3.9 requires an adjustment to represent the position of the same point accurately; this adjustment is shown in Equation 3.10.

\[
Pos = L \begin{bmatrix}
\cos(90^\circ - \theta) \\
\sin(90^\circ - \theta)
\end{bmatrix}
\]  

(3.10)

Simplifying by applying trigonometric principals and assuming that \(\theta\) is negative, Equation 3.10 becomes Equation 3.11.

\[
Pos = L \begin{bmatrix}
-\sin(\theta) \\
\cos(\theta)
\end{bmatrix}
\]  

(3.11)

This form is convenient for the expression of the system because the lengths of the segments are known along with the angle of flexion, not the discrete \(x\) and \(y\) positions. Additionally, because the positions of the linkages are dependent on others, the additive nature of vectors makes the expression of the serial linkage position easier. For example, if linkage one is given by Equation 3.12, then the position of linkage two which is connected in series to linkage one, is given by Equation 3.13.
\[ \text{Pos}_1 = L_1 \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} \] (3.12)

\[ \text{Pos}_2 = L_1 \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} + L_2 \begin{bmatrix} -\sin(\theta_1 + \theta_2) \\ \cos(\theta_1 + \theta_2) \end{bmatrix} \] (3.13)

Therefore, for each element \( ele \), in a serial linkage system may be expressed using Equation 3.14.

\[ \text{Pos}_{ele} = \sum_{e=1}^{ele} L_e \begin{bmatrix} -\sin\left(\sum_{e=1}^{e} \theta_e\right) \\ \cos\left(\sum_{e=1}^{e} \theta_e\right) \end{bmatrix} \] (3.14)

As a check on its accuracy, Equation 3.14 is rewritten as Equation 3.15 to generate the positions of linkages in a system analogous to a human finger. Where \( l = 1, 2, 3 \) for the Proximal, Medial, and Distal segments respectively and where \( L_i \) is the length of each segment. These equations are then used in conjunction with Mathematica’s graphics packages to help generate the figures in this section.

\[ \text{Segment}_l = \sum_{i=1}^{l} L_i \begin{bmatrix} -\sin\left(\sum_{i=1}^{i} \theta_i\right) \\ \cos\left(\sum_{i=1}^{i} \theta_i\right) \end{bmatrix} \] (3.15)

The results of using Equation 3.15 to generate expressions are shown by Equations 3.16, 3.17, and 3.18

\[ \text{ProximalSegment} = L_{\text{MCP}} \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} \] (3.16)

\[ \text{MedialSegment} = L_{\text{MCP}} \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} + L_{\text{PIP}} \begin{bmatrix} -\sin(\theta_1 + \theta_2) \\ \cos(\theta_1 + \theta_2) \end{bmatrix} \] (3.17)

\[ \text{DistalSegment} = L_{\text{MCP}} \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} + L_{\text{PIP}} \begin{bmatrix} -\sin(\theta_1 + \theta_2) \\ \cos(\theta_1 + \theta_2) \end{bmatrix} + L_{\text{DIP}} \begin{bmatrix} -\sin(\theta_1 + \theta_2 + \theta_3) \\ \cos(\theta_1 + \theta_2 + \theta_3) \end{bmatrix} \] (3.18)
Figure 3.9 is generated by Mathematica using the information in Equations 3.16, 3.17, and 3.18.

3.1.6 Tendon Attachment Points Position

Tendon attachment points are defined slightly differently to accommodate the cases for tendon routing; however, the general principles still apply. Equation 3.19 shows the equation and rules for defining the tendon attachment points. Where $j$ is the attachment point number and $LTA_j$ is the length along each respective segment.

$$\text{TAPos}_j = \begin{cases} 
LTA_j \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} & \text{if } j = 1, 2 \\
L_1 \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{bmatrix} + LTA_j \begin{bmatrix} -\sin(\sum_{i=1}^{2} \theta_i) \\ \cos(\sum_{i=1}^{2} \theta_i) \end{bmatrix} & \text{if } j = 3, 4 \\
\sum_{i=1}^{2} L_i \begin{bmatrix} -\sin(\sum_{i=1}^{i} \theta_i) \\ \cos(\sum_{i=1}^{i} \theta_i) \end{bmatrix} + LTA_j \begin{bmatrix} -\sin(\sum_{i=1}^{3} \theta_i) \\ \cos(\sum_{i=1}^{3} \theta_i) \end{bmatrix} & \text{if } j = 5 
\end{cases} \quad (3.19)$$
Figure 3.10 is a series of arbitrary attachment points shown in blue, generated using the Mathematica Graphics Package.

![Figure 3.10: Attachment point locations](image)

### 3.1.7 Reaction Force Positions

The reaction forces are defined by Equation 3.20 where $k = 1, 2, 3$, for each reaction on the MCP, PIP, and DIP links respectively. $LRF_k$ is the length along each respective segment.

\[
RF_{pos_k} = \begin{cases} 
LRF_k \begin{bmatrix} -\sin(k\sum_{i=1}^{k} \theta_i) \\ \cos(k\sum_{i=1}^{k} \theta_i) \end{bmatrix} & \text{if } k = 1 \\
\sum_{i=1}^{k-1} L_i \begin{bmatrix} -\sin(k-1\sum_{i=1}^{k-1} \theta_i) \\ \cos(k-1\sum_{i=1}^{k-1} \theta_i) \end{bmatrix} + LRF_k \begin{bmatrix} -\sin(k\sum_{i=1}^{k} \theta_i) \\ \cos(k\sum_{i=1}^{k} \theta_i) \end{bmatrix} & \text{if } k > 1
\end{cases}
\] (3.20)

Figure 3.11 is generated by Equation 3.20 and shows the arbitrary locations of the reaction forces shown in red.
3.1.8 Tendon Forces

The forces developed by the tendon acting on the linkages can be thought of as a series of torques acting about each of the revolute joints. Therefore the singular tendon force can be thought of as three different torque forces as shown in Figure 3.12.

Each of the torques in Figure 3.12 are in turn made up of the contributing torques from the attachment points. $\tau_{\text{MCP}}$ is made up of the contributions from tendon attachment points 1 & 2, $\tau_{\text{PIP}}$ is made up of the contributions from tendon attachment points 3 & 4, and $\tau_{\text{DIP}}$ is only from attachment point 5. Figure 3.13 shows
To convert the tendon forces in the system to torques, the fundamental torque equation, given by Equation 3.21, is used.

\[ \tau = \vec{r} \times \vec{F} \]  \hspace{1cm} (3.21)

Where \( \tau \) is the torque, \( \vec{r} \) is the position vector from the joint center to the line of action of the force, \( \vec{F} \), is acting on and, \( \times \) is the cross product. The cross product operation multiplies the perpendicular component of the force by the length upon which it is acting. So that Equation 3.21 may be rewritten as Equation 3.22.

\[ \tau = r F_\perp \]  \hspace{1cm} (3.22)

If the components of the forces are not directly known, Equation 3.22 may also be expressed as Equation 3.23.

\[ \tau = r F \sin \theta \]  \hspace{1cm} (3.23)
Where $\theta$ is the angle between the line of action of the force and the member that it is acting on. This form is more convenient for this analysis because the magnitude of the force is known and its orientation can be calculated.

Torques are additive, and they are working together such that total torque acting on a member, $\tau_i$, can be expressed by Equation 3.24. Where $m$ is the number of individual torques acting on a segment.

$$
\tau_i = \sum_{a=1}^{m} \tau_a
$$

(3.24)

In the model, the sum of the torques for a given segment is called a Wrench so that Equation 3.24 is rewritten as Equation 3.25.

$$
\text{Wrench}_i = \sum_{a=1}^{m} \tau_a
$$

(3.25)

Therefore, the Wrenches for each of the segments are shown by Equations 3.26, 3.27, and 3.28 respectively. Where $\tau_j$ is the torque contribution from each tendon attachment points.

$$
\text{Wrench}_1 = \tau_1 + \tau_2
$$

(3.26)

$$
\text{Wrench}_2 = \tau_3 + \tau_4
$$

(3.27)

$$
\text{Wrench}_3 = \tau_5
$$

(3.28)

To get the sum of the torques for each segment, its wrench, the moment produced by each tendon force at its attachment point is calculated. That moment is then replaced with an equivalent torque. For each case of the tendon acting on the linkages, Equation 3.23 can be rewritten as the general expression shown in Equation 3.29.

$$
\tau = (\text{LTA})F_i \sin \left( \angle(\text{Segment, Tendon}) \right)
$$

(3.29)
Where the torque equals the tendon force, $F_t$, multiplied by the sine of the angle between the Tendon and the Segment the tendon is attached to. This, in turn, is multiplied by the distance that the attachment point is from the revolute joint, LTA. Figure 3.14 shows the relationship between the angle that the tendon makes with its respective segment and the other attachment points. TApos_1 is perpendicular to the segment because in the conceptual model it is behaving like it is on a pulley, and thus its direction of action does not change with the orientation of the system.

![Fig. 3.14: Tendon attachment points and local angles](image)

Equation 3.30 then gives the calculation of any particular torque.

$$\tau_j = LTA_j F_t \sin \left( \angle \left( \text{Segment}_l, \text{TApos}_{j',j} \right) \right)$$

\[
\begin{cases} 
\angle = 90^\circ & \text{if } j = 1 \\
 l = 1 & \text{if } j = 2 \\
 l = 2 & \text{if } j = 3 \\
 l = 2 & \text{if } j = 4 \\
 l = 3 & \text{if } j = 5 \\
\end{cases} \\
(3.30)
\]

The torque, $\tau_j$, is produced by any given attachment point, $j$, times the tension
of the tendon, $F_t$, times the sine of the angle formed between the tendon and the segment it is acting on. $\overline{TApos}_{j',j}$ is the line of action of the tendon from given attachment point.

Applying Equation 3.30 to the attachment points on segment one, when $j = 1, 2$, results in Equations 3.31 & 3.32.

\[
\tau_1 = LTA_1 F_t \sin(90^\circ) 
\]

\[
\tau_2 = LTA_2 F_t \sin \left( \angle \left( \overline{\text{Segment}_1}, \overline{TApos}_{3,2} \right) \right) \tag{3.32}
\]

The wrench calculation Equation 3.26 becomes Equation 3.33.

\[
\text{Wrench}_1 = F_t \left( LTA_1 + LTA_2 \sin \left( \angle \left( \overline{\text{Segment}_1}, \overline{TApos}_{3,2} \right) \right) \right) \tag{3.33}
\]

The torques acing on segment 2, when $j = 3, 4$, are shown by Equations 3.34 & 3.35.

\[
\tau_3 = LTA_3 F_t \sin \left( \angle \left( \overline{\text{Segment}_2}, \overline{TApos}_{3,2} \right) \right) \tag{3.34}
\]

\[
\tau_4 = LTA_4 F_t \sin \left( \angle \left( \overline{\text{Segment}_2}, \overline{TApos}_{5,4} \right) \right) \tag{3.35}
\]

Equation 3.27 becomes Equation 3.36.

\[
\text{Wrench}_2 = F_t \left( LTA_3 \sin \left( \angle \left( \overline{\text{Segment}_2}, \overline{TApos}_{3,2} \right) \right) + LTA_4 \sin \left( \angle \left( \overline{\text{Segment}_2}, \overline{TApos}_{5,4} \right) \right) \right) \tag{3.36}
\]

For segment three the torque is given by Equation 3.37
\[
\tau_5 = LTA_5 F_t \sin \left( \angle \left( \text{Segment}_3, \overrightarrow{\text{TPos}_{5,4}} \right) \right)
\]  

(3.37)

With the singular torque acting on the system, the wrench becomes Equation 3.38.

\[
\text{Wrench}_3 = F_t \left( LTA_5 \sin \left( \angle \left( \text{Segment}_3, \overrightarrow{\text{TPos}_{5,4}} \right) \right) \right)
\]  

(3.38)

### 3.1.9 Reaction Force Definitions

The reaction forces are assumed to be purely contact forces and are therefore always be acting perpendicular to the segment that they are on, that is to say at 90° to the segment. If \( \theta \) is the orientation of a particular segment, then the direction that the reaction force is acting can be given by Equation 3.39 for a general perpendicular force. Where \( F \) is the matrix containing the force orientation information and \( F \) is the magnitude of the force.

\[
F = F \begin{bmatrix}
- \sin(\theta + 90°) \\
\cos(\theta + 90°)
\end{bmatrix}
\]  

(3.39)

Simplifying with trigonometric properties, Equation 3.39 becomes Equation 3.40.

\[
F = F \begin{bmatrix}
- \cos(\theta) \\
- \sin(\theta)
\end{bmatrix}
\]  

(3.40)

To define the reaction force based on the segment, Equation 3.40 is modified into Equation 3.41.

\[
\text{RF}_i = F_i \begin{bmatrix}
- \cos(\sum_{r=1}^{i} \theta_r) \\
- \sin(\sum_{r=1}^{i} \theta_r)
\end{bmatrix}
\]  

(3.41)
Equation 3.41 is then used to generate the Equations 3.42, 3.43, and 3.44 for the reaction forces on each of the respective segments.

\[
RF_1 = f_1 \begin{bmatrix} -\cos(\theta_1) \\ -\sin(\theta_1) \end{bmatrix} \tag{3.42}
\]

\[
RF_2 = f_2 \begin{bmatrix} -\cos(\theta_1 + \theta_2) \\ -\sin(\theta_1 + \theta_2) \end{bmatrix} \tag{3.43}
\]

\[
RF_3 = f_3 \begin{bmatrix} -\cos(\theta_1 + \theta_2 + \theta_3) \\ -\sin(\theta_1 + \theta_2 + \theta_3) \end{bmatrix} \tag{3.44}
\]

### 3.1.10 Velocity Calculation

The principle of virtual power requires that the forces in the system be subjected to virtual velocities. Since velocity represents a change in position over time, taking the time derivative of a position as shown by Equation 3.45 yields the velocity.

\[
v = \frac{d(pos)}{dt} \tag{3.45}
\]

To calculate the velocity of the reaction forces Equation 3.45 becomes Equation 3.46.

\[
RF_v_k = \frac{d RF_{pos_k}}{dt} \tag{3.46}
\]

### 3.1.11 Power Calculations

With the forces and the velocities known, the power produced by the tendons and reaction forces on the system is calculated. Equation 3.47 shows the power expended by any given reaction force.
Reaction Power

\[ \text{Reaction Power}_i = \mathbf{RF}_i \cdot \mathbf{RFv}_k, \]  
where \( i = k \) \hspace{1cm} (3.47)

The summation gives the total power produced by the reaction forces in Equation 3.48.

\[ \text{ReactionPower} = \sum_{i,k=1}^{3} \mathbf{RF}_i \cdot \mathbf{RFv}_k \] \hspace{1cm} (3.48)

Which yields the expression given by Equation 3.49.

\[ \text{ReactionPower} = (f_1LRF_1 + f_2LRF_2 + f_3LRF_3 + f_3L_2 \cos(\theta_3) + f_2L_1 \cos(\theta_2) + f_3L_1 \cos(\theta_3 + \theta_2)) \dot{\theta}_1 \]  
\[ + (f_2LRF_2 + f_3LRF_3 + f_3L_2 \cos(\theta_2)) \dot{\theta}_2 + f_3LRF_3 \dot{\theta}_3 \] \hspace{1cm} (3.49)

Equation 3.50 shows the power produced by the tendon on any given segment.

\[ \text{WrenchPower}_i = \text{Wrench}_i \cdot \dot{\theta}_i \] \hspace{1cm} (3.50)

Then the total power of all the wrenches and by extension the tendon would be given by Equation 3.51.

\[ \text{WrenchPower} = \sum_{i=1}^{3} \text{Wrench}_i \cdot \dot{\theta}_i \] \hspace{1cm} (3.51)

3.1.12 Solving for Reactions

To solve for the magnitude of the reaction forces, the power expressions are equated to one another as shown in Equation 3.52.

\[ \text{ReactionPower} = \text{WrenchPower} \] \hspace{1cm} (3.52)
The expression for WrenchPower can be rewritten as the product of two matrices, one made up of the wrenches, and one made up of the velocity information. Equation 3.53 is the matrix that contains the wrenches and Equation 3.54 contains the rotational velocity information.

\[
\mathbf{W} = \begin{bmatrix}
\text{Wrench}_1 \\
\text{Wrench}_2 \\
\text{Wrench}_3
\end{bmatrix}
\] (3.53)

\[
\dot{\theta} = \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix}
\] (3.54)

Therefore, Equation 3.52 can be rewritten as Expression 3.55.

\[
\text{ReactionPower} = \mathbf{W} \dot{\theta}
\] (3.55)

Likewise, ReactionPower can be represented as the multiplication of two matrices one containing the magnitudes of the individual reaction forces and one containing the velocities. The matrix containing the velocity information is given by Equation 3.56 and Equation 3.57 contains the reaction force magnitudes.

\[
\mathbf{V} = \begin{bmatrix}
\text{LRF}_1 \dot{\theta}_1 \\
(\text{LMCP} \cos(\theta_2) + \text{LRF}_2) \dot{\theta}_1 + \text{LRF}_2 \dot{\theta}_2 \\
(\text{LMCP} \cos(\theta_3 + \theta_2) + \text{LPIP} \cos \theta_3 + \text{LRF}_3) \dot{\theta}_1 + (\text{LPIP} \cos \theta_3 + \text{LRF}_3) \dot{\theta}_2 + \text{LRF}_3 \dot{\theta}_3
\end{bmatrix}
\] (3.56)

\[
\mathbf{f} = \begin{bmatrix}
f_1 \\
f_2 \\
f_3
\end{bmatrix}
\] (3.57)

Expression 3.55 can now be written as Equation 3.58.
\[ Vf = W\dot{\theta} \] (3.58)

The matrix \( V \) itself can be represented by the multiplication of two matrices by factoring out the rotational velocity terms, \( \dot{\theta}_i \), and leaving behind a Jacobian matrix [61] shown by Equation 3.59.

\[
\begin{bmatrix}
LRF_1 & 0 & 0 \\
LRF_2 + LMCP \cos(\theta_2) & LRF_2 & 0 \\
LRF_3 + LPIP \cos(\theta_3) + LMCP \cos(\theta_3 + \theta_2) & LRF_3 + LPIP \cos(\theta_3) & LRF_3
\end{bmatrix}
\] (3.59)

Therefore, Equation 3.58 can be rewritten as Equation 3.60.

\[
\dot{\theta}^T Jf = W\dot{\theta}
\] (3.60)

Since Equation 3.60 has a \( \dot{\theta} \) on either side of the equation it can be dropped leaving Equation 3.61.

\[
Jf = W
\] (3.61)

Now the three reaction forces can be solved for by taking the inverse of the Jacobian, \( J \), matrix and multiplying it by the \( W \) matrix as shown in Equation 3.62.

\[
f = J^{-1}W
\] (3.62)

The full solution for the magnitudes of each of the reaction forces may be found in Appendix A.2
3.1.13 Mathematica Code

For the interested reader, a Mathematica Notebook can be found in Appendix A.1 that goes through the process detailed in Sections 3.1.2 through 3.1.12 above. The code also contains the function used to compare the experimental values with those that the model predicts.

3.2 Experimental Validation

The Mathematica model was tested for its accuracy by using a model finger and test rig to assess the total force output of the system.

3.2.1 Equipment

The equipment used in the experimentation was a mix of off the shelf components, what was available, along with custom designed and manufactured parts.

3.2.1.1 Test Fingers

Test fingers were designed in SolidWorks. Each finger was made up of three segments just like the human finger. Figure 3.15 shows the three segments color coded with the proximal segment in Purple, the middle segment in light blue and, the distal segment in light green. A detailed design can be found in Appendix B.
Fig. 3.15: Colored CAD rendering of the test rig

From the CAD model, STL files were generated, and 3D printed using a Dimension 1200es printer. Figure 3.16 is a partially assembled print of one of the rigs, the green coloring was a marker used in experiments.

Fig. 3.16: 3D Printed Finger Rig

In the final assembly, the models were pinned together using .125 inch diameter by .75 inch long stainless steel dowel pins. The segments were sized so that the pin was pressed into one segment and had a clearance fit into the following one. Figure 3.17 shows an assembled test rig with the tendon threaded through it.
For the testing, a total of two identical rigs were designed and built with the segment lengths shown Table 3.2. These are the lengths of the segments measured between the centers of the hinges in the case of the MCP and PIP segments, and the length from the joint center to the end of the segment for the DIP segments.

### Table 3.2: Segment Lengths

<table>
<thead>
<tr>
<th></th>
<th>L$_{\text{prox}}$</th>
<th>L$_{\text{mid}}$</th>
<th>L$_{\text{dis}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.058 m</td>
<td>.028 m</td>
<td>.026 m</td>
</tr>
</tbody>
</table>

The rigs differed by the positions of the attachment points, LTA$_j$. Table 3.3 shows the design length of the attachment points on each of the rigs.

### Table 3.3: Rig Attachment Point Summary

<table>
<thead>
<tr>
<th>Rig #</th>
<th>LTA$_1$</th>
<th>LTA$_2$</th>
<th>LTA$_3$</th>
<th>LTA$_4$</th>
<th>LTA$_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.014 m</td>
<td>.041 m</td>
<td>.015 m</td>
<td>.018 m</td>
<td>.022 m</td>
</tr>
<tr>
<td>2</td>
<td>.014 m</td>
<td>.041 m</td>
<td>.012 m</td>
<td>.018 m</td>
<td>.022 m</td>
</tr>
</tbody>
</table>

#### 3.2.1.2 Force Sensor

The force sensor that was used was an Elastisens ES-90-150/60-10 flexible pressure pad sensor by Novel. Its dimensions are 150 $\times$ 60 mm, and it is made up of 90
sensors arranged in a $15 \times 6$ grid. The sensor received power from the DAQ and was calibrated before being used.

The sensor was unable to accurately report the individual contact forces of the test rigs because it is primarily designed to report a matrix of pressures. The DAQ software did, however, report the total force on the pad allowing it to be manually recorded. This inability to resolve individual contact forces required a slight modification of Equation 3.62 to the expression shown in Equation 3.63 for the experimentation.

$$f_{tot} = \Sigma f \quad (3.63)$$

### 3.2.1.3 Test Stand

The test stand was custom fabricated for the experiment out of materials available in the machine shop. The stand consists of three major components, an interchangeable test diameter holder to test different gripping orientations, a frame to hold a webcam, and a component to hold the test rigs. The test diameter holder consists of two steel rods pressed into an aluminum base, one of which is test diameter for gripping and the other provides support for the sensor. For the experimental tests, there were three different diameters shown in Table 3.4 that were selected after reviewing an EVA tools catalog [78]. See Table D.1 in for a full list of the tools and diameters. Appendix C contains a detailed description of the test diameter holder design. The webcam is part of a custom vision measurement system to measure the angular orientation of the test rigs.
Table 3.4: Test Diameter Sizes

<table>
<thead>
<tr>
<th>Diameter #</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19 mm (.75 in)</td>
</tr>
<tr>
<td>2</td>
<td>35 mm (1.38 in)</td>
</tr>
<tr>
<td>3</td>
<td>38 mm (1.5 in)</td>
</tr>
</tbody>
</table>

Diameter 1 represents the smallest tool that would need to be gripped, Diameter 2 is the median of the sampled tools which were nearly identical to the mean, and Diameter 3 is the mode of the set.

To aid the vision system in determining the point of contact, the test diameter is brightly colored. The vision system measures the angular orientation of the system and the point of contact between the test rig and the grip diameter. For the webcam stand, aluminum extrusions were used to build its frame. Figure 3.18 shows the experimental setup with call-outs for specific parts.

![Fig. 3.18: Test Rig Setup Where 1 is the Test Rig, 2 is the sensor sad, 3 is the webcam, 4 is the Test Diameter, and 5 is the stand for the webcam.](image)

73
3.2.2 Testing

The tests were conducted by wrapping the test finger around the pressure pad, which itself was wrapped around the test diameter. The tendon was then routed through the test apparatus such that the free end was hanging over a pulley and the fixed end was mounted in the test rig. A known mass was then hung on the free end of the tendon to generate a known tendon force. Figure 3.19 is an example of one of the tests, showing the known masses creating tension in the tendon.

Each of the test rigs was tested using four different tendon tensions gripping different test diameters for a total of eight experiments. The eight experiments were each repeated ten times for a total of 80 tests. During each of the tests, the orientation of the system was measured using the vision system, and the total contact load was measured with the pressure pad.
CHAPTER 4

Results

4.1 Measured Forces

4.1.1 Rig 1

Table 4.1 is a summary of the measured forces from Rig 1, and it represents the averages of the raw values after outliers have been removed using Peirce’s criterion from the original set of twenty measurements [79].

Table 4.1: Rig 1 Measured contact force summary

<table>
<thead>
<tr>
<th>Test</th>
<th>Tendon Tension (N)</th>
<th>Measured Contact Force (N)</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.90</td>
<td>4.73</td>
<td>0.53</td>
</tr>
<tr>
<td>2</td>
<td>9.80</td>
<td>9.77</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>14.70</td>
<td>13.76</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td>19.60</td>
<td>18.22</td>
<td>1.90</td>
</tr>
</tbody>
</table>

4.1.2 Rig 2

Table 4.2 is a summary of the measured forces from Rig 2, and it represents the averages of the raw values after outliers have been removed using Pierce’s criterion from the original set of twenty measurements.

Table 4.2: Rig 2 Measured contact force summary

<table>
<thead>
<tr>
<th>Test</th>
<th>Tendon Tension (N)</th>
<th>Measured Contact Force (N)</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.90</td>
<td>4.08</td>
<td>0.55</td>
</tr>
<tr>
<td>6</td>
<td>9.80</td>
<td>10.53</td>
<td>2.78</td>
</tr>
<tr>
<td>7</td>
<td>14.70</td>
<td>11.86</td>
<td>1.39</td>
</tr>
<tr>
<td>8</td>
<td>19.60</td>
<td>13.75</td>
<td>0.85</td>
</tr>
</tbody>
</table>
4.2 Predicted forces

The orientation of system was measured during each of the experiments in Section 4.1, conditioned with Pierce’s criterion then averaged. Standard deviations and known uncertainties were incorporated into the Mathematica program found in Appendix A.1 using the Experimental Data Analyst (EDA) package. The Mathematica model emulates Equation 3.63 in its output of the total contact force in order to predict what the pressure pad would report.

Table 4.3 contains the calculated uncertainties for the tendon tensions based on the resolution of the scale used to measure the weights and the error in the gravity model used to calculate weight.

Table 4.3: Calculated uncertainties for tendon tensions

<table>
<thead>
<tr>
<th>Tendon Tension (N)</th>
<th>Uncertainty (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.9018</td>
<td>0.001</td>
</tr>
<tr>
<td>9.8017</td>
<td>0.001</td>
</tr>
<tr>
<td>14.7035</td>
<td>0.0014</td>
</tr>
<tr>
<td>19.5877</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Measurements of the test Rigs have an accuracy of $\pm 1 \times 10^{-5}$ m.

4.2.1 Rig 1

The relevant dimensions that were input into the model for the Rig 1 predictions are shown in Table 4.4 has the segment lengths and Table 4.5 has the lengths of the attachment points.

Table 4.4: Rig Measured Segment Lengths

<table>
<thead>
<tr>
<th>$L_{\text{prox}}$</th>
<th>$L_{\text{mid}}$</th>
<th>$L_{\text{dis}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05823 m</td>
<td>.02792 m</td>
<td>.02643 m</td>
</tr>
</tbody>
</table>
Table 4.5: Rig 1 Attachment Point Summary

<table>
<thead>
<tr>
<th>LTA1</th>
<th>LTA2</th>
<th>LTA3</th>
<th>LTA4</th>
<th>LTA5</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01501 m</td>
<td>.04059 m</td>
<td>.01524 m</td>
<td>.01855 m</td>
<td>.02158 m</td>
</tr>
</tbody>
</table>

During each experiment, the angular orientation of the test rig was recorded. After the data was conditioned the means and standard deviations of the joint angles and contact points were calculated for each test. Table 4.6 contains the joint angles and contact points for the Rig 1 test series as a percentage of length along the segment.

Table 4.6: Rig 1 Measured Force result summary

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>σ</td>
<td>Mean</td>
<td>σ</td>
<td>Mean</td>
</tr>
<tr>
<td>MCP</td>
<td>10.65 N</td>
<td>0.545 N</td>
<td>10.998 N</td>
<td>0.412 N</td>
<td>12.181 N</td>
</tr>
<tr>
<td>Mid Contact %</td>
<td>0.361</td>
<td>0.057</td>
<td>0.346</td>
<td>0.036</td>
<td>0.357</td>
</tr>
<tr>
<td>Dis Contact %</td>
<td>0.768</td>
<td>0.064</td>
<td>0.782</td>
<td>0.046</td>
<td>0.838</td>
</tr>
</tbody>
</table>

Table 4.7 are the results of the predictions made by the math model for the Rig 1 experiment series, again incorporating known uncertainties into the model.

Table 4.7: Rig 1 predicted contact force summary

<table>
<thead>
<tr>
<th>Test</th>
<th>Tendon Tension (N)</th>
<th>Predicted Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.9</td>
<td>4.9 ± 0.82</td>
</tr>
<tr>
<td>2</td>
<td>9.8</td>
<td>10.54 ± 0.91</td>
</tr>
<tr>
<td>3</td>
<td>14.7</td>
<td>13.9 ± 3.5</td>
</tr>
<tr>
<td>4</td>
<td>19.6</td>
<td>19.3 ± 3.2</td>
</tr>
</tbody>
</table>
Figure 4.1 is an error bar plot comparing the measured and predicted values for Rig 1.

Fig. 4.1: Rig 1 comparison of experimental and predicted values with data from Table 4.1 and Table 4.7 respectively.
4.2.2 Rig 2

The relevant dimensions that were input into the model for the Rig 2 predictions are shown in Table 4.8 has the segment lengths and Table 4.9 has the lengths of the attachment points.

Table 4.8: Rig 2 Measured Segment Lengths

<table>
<thead>
<tr>
<th></th>
<th>L_{prox}</th>
<th>L_{mid}</th>
<th>L_{dis}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.05763 m</td>
<td>.02738 m</td>
<td>.02694 m</td>
</tr>
</tbody>
</table>

Table 4.9: Rig 2 Attachment Point Summary

<table>
<thead>
<tr>
<th></th>
<th>LTA_1</th>
<th>LTA_2</th>
<th>LTA_3</th>
<th>LTA_4</th>
<th>LTA_5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.01538 m</td>
<td>.04168 m</td>
<td>.01207 m</td>
<td>.01849 m</td>
<td>.02187 m</td>
</tr>
</tbody>
</table>

During each experiment, the angular orientation of the test rig was recorded. After the data was conditioned the means and standard deviations of the joint angles and contact points were calculated for each test. Table 4.10 contains the joint angles and contact points for the Rig 2 test series as a percentage of length along the segment.

Table 4.10: Rig 2 Measured Force result summary

<table>
<thead>
<tr>
<th>Value</th>
<th>Test 5</th>
<th>Test 6</th>
<th>Test 7</th>
<th>Test 8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>σ</td>
<td>Mean</td>
<td>σ</td>
</tr>
<tr>
<td>MCP</td>
<td>13.04 N</td>
<td>0.078 N</td>
<td>12.806 N</td>
<td>0.246 N</td>
</tr>
<tr>
<td>PIP</td>
<td>12.264 N</td>
<td>1.031 N</td>
<td>12.667 N</td>
<td>0.586 N</td>
</tr>
<tr>
<td>DIP</td>
<td>66.822 N</td>
<td>0.573 N</td>
<td>67.058 N</td>
<td>0.288 N</td>
</tr>
<tr>
<td>Mid Contact %</td>
<td>0.635</td>
<td>0.023</td>
<td>0.61</td>
<td>0.004</td>
</tr>
<tr>
<td>Dis Contact %</td>
<td>0.474</td>
<td>0.01</td>
<td>0.467</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4.11 are the results of the predictions for the Rig 2 experiment series, again incorporating known uncertainties into the model.
Table 4.11: Rig 2 predicted contact force summary

<table>
<thead>
<tr>
<th>Test</th>
<th>Tendon Tension (N)</th>
<th>Predicted Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.9</td>
<td>4.78 ± 0.88</td>
</tr>
<tr>
<td>6</td>
<td>9.8</td>
<td>9.8 ± 2.1</td>
</tr>
<tr>
<td>7</td>
<td>14.7</td>
<td>10.28 ± 0.55</td>
</tr>
<tr>
<td>8</td>
<td>19.6</td>
<td>14.08 ± 0.44</td>
</tr>
</tbody>
</table>

Figure 4.2 is an error bar plot comparing the measured and predicted values for the Rig 2 tests.

Fig. 4.2: Rig 2 comparison of experimental and predicted values from Table 4.11 and Table 4.2
5.1 Discussion of results

Figure 4.1 shows that the model was able to predict the measured values accurately. The figure shows that the means of each of the sets are within the error bars of the other, and are nearly on top of one another. The Pearson Correlation between the measured and predicted values is .998, indicating a high degree of correlation [80]. As Figure 4.2 shows, like the Rig 1 series, the Rig 2 values all lie within the error bars of each other. For the Rig 2 case, the Pearson Correlation is slightly lower at .971, but it is still indicative of a high degree of correlation.

From the plots, it can be seen that the error bars on the predictions are much larger than the error bars of the measurements. This is due to the sensitivity of the model brought on by the singularities that bound it. The slope near the singularities increases rapidly to infinity causing error analysis to report impossibly large deviations if the error calculation gets too close. Figure 5.1 is an example of what the singularities look like if the model were to be plotted in 3 dimensions. This plot was generated for a given rig in a specific angular orientation while varying the possible points of contact along the middle and proximal segments with a 1 Newton tendon tension. The contour lines in the plot show how extreme the slope becomes as the plot approaches the singularities. This rapid change in slope is responsible for the large deviations that the error analysis reports.

Additionally, these singularities are also why not all of the known uncertainties were incorporated into the analysis. Their additions would push the errors to unreasonably large values, if not causing the analysis to fail altogether. The uncer-
tainties that were omitted are the angular deviations because the joint angles shift
the positions of the singularities.

Fig. 5.1: 3D surface plot of the results of the mathematical model

5.1.1 Comparison to a grip dynamometer

The goal of this work is to determine if the conceptual model is a viable candidate for improving the grip strength of the astronaut. Unfortunately, a suitable grip dynamometer was not available at the time of testing to properly assess the accuracy of the math model. Fortunately, it is possible to correlate the total contact force to the results that a grip dynamometer would give. Below are two examples of methods to compare the results of the model with those of a standard like the Jamar grip dynamometer [81].
5.1.1.1 Mühldorfer-Fodor et al.

In a study conducted by Mühldorfer-Fodor et al., the authors develop a relationship between a Manugraphy 200mm circumference cylindrical pressure pad and a Jamar grip dynamometer at position 4 [4]. Equation 5.1 is the relationship that Mühldorfer-Fodor et al. developed.

\[
\text{Manugraphy}(N) = -12.0 + 15.1 \times \text{Jamar(kg)} \tag{5.1}
\]

Equation 5.2 is Equation 5.1 rearranged to solve for Jamar grip force that a given pressure pad reading would be equivalent to.

\[
\frac{\text{Manugraphy}(N) + 12.0}{15.1} = \text{Jamar(kg)} \tag{5.2}
\]

The authors note that this is not a direct comparison because there are different geometries between the two systems. Furthermore, the paper only provides the relationship between the Manugraphy 200mm circumference cylindrical pressure pad and not the 150mm pressure pad, which would be a more accurate representation of this work’s experimental setup. Nevertheless, the relationship derived by Equation 5.2 was used to gauge the magnitude of grip strength that the conceptual system may recover.

This model currently only accounts for the grip produced by one finger, whereas Equation 5.2 accounts for all the fingers involved in a grasp. Therefore, a contribution from each finger in a grasp needs to be used to add the additional force that the model is currently missing. Kjoshita et al. report that the contributions of each finger during a static grasp for the index, middle, ring, and little fingers were 42.0%, 27.4%, 17.6% and 12.9%, respectively [82]. It should be noted that these values depend on a whole host of factors, the friction of the object gripped, the force of the grip, and the person; however, for this work, they will be used as reported.
For the estimation of the Jamar grip dynamometer reading, the geometric parameters from Test 6 are used. Those parameters can be found in Tables 4.8, 4.9, & 4.10. Feeding those values into the model yields $F_t(0.72 \pm 0.022)$ for the total contact force, where $F_t$ is the tendon tension. If one imagines a case where the tendon for each finger is capable of operating at a tension equivalent to those in [59], 23 kg [sic] or 225 N, then the force that the Rig system generates is $161.5 \pm 5.1$ N. Table 5.1 are the results of calculating the contribution of each finger, assuming that Rig 2 is the Index finger.

<table>
<thead>
<tr>
<th>Finger</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index Finger</td>
<td>$161.5 \pm 5.1$</td>
</tr>
<tr>
<td>Middle Finger</td>
<td>$105.4 \pm 3.3$</td>
</tr>
<tr>
<td>Ring Finger</td>
<td>$67.8 \pm 2.1$</td>
</tr>
<tr>
<td>4th Finger</td>
<td>$49.8 \pm 1.5$</td>
</tr>
</tbody>
</table>

This yields a total load of $384.5 \pm 6.6$ newtons, which when used in Equation 5.2 gives a grip strength of $26.16 \pm 0.44$ kg ($57.8 \pm 0.97$ lbf). Comparing this recovery to the maximum grip strength lost in the study by Thompson et al. [1] from Table 1.1, 69 lbf, means the system can recover %84 of the lost strength. Compared to the mean case, this is a %114 recovery of the lost strength.

5.1.1.2 Single Component Magnitude

The Jamar grip dynamometer is only capable of measuring loads in a single direction, the line of action between the fingers and the area between the proximal palm and thumb. That is to say the measurement of a unidirectional force. Figure 5.2 is an illustration of the forces that a Jamar grip Dynamometer measures.
This would be equivalent to the model only predicting the forces in the y-direction, as shown in Figure 5.3.

Equation 3.62 yields the magnitudes of the contact forces and when those values are fed back into Equations 3.43 and 3.44 the x and y components of each contact force can be calculated. Table 5.2 contains the calculated y component contributions of the fingers in a grasp.
Table 5.2: Calculated Test 6 parameters for y-components of a grasp

<table>
<thead>
<tr>
<th>Finger</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index Finger</td>
<td>120.3 ± 2.6</td>
</tr>
<tr>
<td>Middle Finger</td>
<td>70.8 ± 1.5</td>
</tr>
<tr>
<td>Ring Finger</td>
<td>50.4 ± 1.1</td>
</tr>
<tr>
<td>4th Finger</td>
<td>36.96 ± 0.81</td>
</tr>
</tbody>
</table>

This yields a total of 278.5 ± 3.3 Newtons or a grip strength of 28.3 ± 0.33 kg (62 ± 0.74 lbf). Comparing this recovery to the maximum grip strength lost in the study by Thompson et al. [1] from Table 1.1, 69 lbf, means the system can recover %90 of the lost strength. Compared to the average case, it is a %124 recovery.
5.2 Future work

In light of the difficulties encountered with the testing the model, one potential path for future work, is to repeat the experiments with a sensor designed specifically to report forces magnitudes rather than pressures. This would make it easier to record the loading generated by the system and prevent the inaccuracies due to unknown contact points. Lessons learned about the movement of the system as a response to successive loadings and unloading can also be applied to make the test stand more resistant to drift. Lastly, a full hand can be constructed and tested.

5.3 Conclusions

Work in space will always remain a challenge, and as humanity continues to expand outward, its hazards will become no less infrequent. Favetto et al. predicts that the 2020’s will see at least ten times as many EVA hours as all the previous spaceflight combined. With this increase in the work in space the hazards produced by the spacesuit itself must be addressed. A spacesuit, despite being a marvel of engineering, has shortcomings that make the work of the user more difficult and frequently leaves them injured. Technological advancement will be the key to solving this problem, and there are already possible solutions in development. To date, no solution has been found which adequately addresses the issues caused by the astronaut gloves. This work shows that the proposed conceptual model is a viable candidate for making an astronaut’s work in space less strenuous.
REFERENCES


APPENDICES
Appendix A

Model

A.1 Full Program

The Mathematica program below represents the model developed in Section 3.1.
Startup Parameters

Sysconfig, clear

Clear["Global`*"] (* Clears all loaded variables for a clean slate *)
Needs["EDA`"]
Needs["ErrorBarPlots`"]

SetOptions[EvaluationNotebook[], ShowGroupOpener -> True]

$Assumptions = True;
$Assumptions = {LTA1 > 0, LTA2 > 0, LTA3 > 0, LTA4 > 0, LTA5 > 0,
    LRF1 > 0, LRF2 > 0, LRF3 > 0, LMCP > 0, LPIP > 0, LDIP > 0, ΘMCP[t] < 0, ΘPIP[t] < 0,
    ΘDIP[t] < 0, LMCP > 0, LPIP > 0, LDIP > 0, Ft > 0, _ ∈ Reals, Indeterminate -> 0};

Off[Power::infy];

vecang[v1_ ? VectorQ, v2_ ? VectorQ] := Module[{n1 = Norm[v1], n2 = Norm[v2]}, 2 ArcTan[Norm[v1 n2 + n1 v2], Norm[v1 n2 - n1 v2]]]

RemoveAbs[x_] := ComplexExpand[Abs[x]] (* Usage: func /. Abs→ RemoveAbs *)

Math Model

Analysis Setup

Angle Definitions

Here the angular positions of each of the joints are defined with respect to time so that the derivatives can be taken for the virtual power calculation.
Where ΘMCP is the angle of the Metacarpophalangeal or MCP joint, ΘPIP is the angle of the Proximal Interphalangeal or PIP joint and ΘDIP is the angle of the Distal Interphalangeal or DIP joint.

System Geometry

Segment Geometry

The linkages are defined using vectors containing the x and y components of each of the linkages.

\[
\begin{align*}
\text{ProximalSegment} &= \text{LMCP} \times (-\sin[A1] / \cos[A1]); \\
\text{MedialSegment} &= \text{LMCP} \times (-\sin[A1] / \cos[A1]) + \text{LPIP} \times (-\sin[A1 + A2] / \cos[A1 + A2]); \\
\text{DistalSegment} &= \text{LMCP} \times (-\sin[A1] / \cos[A1]) + \text{LPIP} \times (-\sin[A1 + A2] / \cos[A1 + A2]) + \text{LDIP} \times (-\sin[A1 + A2 + A3] / \cos[A1 + A2 + A3]);
\end{align*}
\]

Where LMCP is the length of the Metacarpophalangeal segment of the finger or the length between the first knuckle and the second knuckle, LPIP is the length of the Proximal segment or the length between the second knuckle and the third knuckle, and LDIP is the length of the Distal Segment or the distance between the third knuckle and the tip of the finger.

Reaction Force Positions

The locations of the contact points or the reaction force locations, are defined using vectors.

\[
\begin{align*}
\text{RFpos1} &= \text{LRF1} \times (-\sin[A1] / \cos[A1]); \\
\text{RFpos2} &= \text{LMCP} \times (-\sin[A1] / \cos[A1]) + \text{LRF2} \times (-\sin[A1 + A2] / \cos[A1 + A2]); \\
\text{RFpos3} &= \text{LMCP} \times (-\sin[A1] / \cos[A1]) + \text{LPIP} \times (-\sin[A1 + A2] / \cos[A1 + A2]) + \text{LRF3} \times (-\sin[A1 + A2 + A3] / \cos[A1 + A2 + A3]);
\end{align*}
\]

Where LRF1 is the length of the contact point along the Metacarpophalangeal segment or put another way the distance between the point of contact and the MCP joint. LRF2 is the length of the contact point along the proximal segment or put another way the distance between the point of contact and the PIP joint.

Tendon Attachment Positions

Likewise for the tendon attachment points
(*) Tendon Attachment Positions *)

\[
\begin{align*}
T\text{Apos}_1 &= L\text{T}A_1 \cdot \begin{pmatrix}
-\sin[A_1] \\
\cos[A_1]
\end{pmatrix}; \\
T\text{Apos}_2 &= L\text{T}A_2 \cdot \begin{pmatrix}
-\sin[A_1] \\
\cos[A_1]
\end{pmatrix}; \\
T\text{Apos}_3 &= L\text{MCP} \cdot \begin{pmatrix}
-\sin[A_1] \\
\cos[A_1]
\end{pmatrix} + L\text{T}A_3 \cdot \begin{pmatrix}
-\sin[A_1 + A_2] \\
\cos[A_1 + A_2]
\end{pmatrix}; \\
T\text{Apos}_4 &= L\text{MCP} \cdot \begin{pmatrix}
-\sin[A_1] \\
\cos[A_1]
\end{pmatrix} + L\text{T}A_4 \cdot \begin{pmatrix}
-\sin[A_1 + A_2] \\
\cos[A_1 + A_2]
\end{pmatrix}; \\
T\text{Apos}_5 &= L\text{MCP} \cdot \begin{pmatrix}
-\sin[A_1] \\
\cos[A_1]
\end{pmatrix} + L\text{PIP} \cdot \begin{pmatrix}
-\sin[A_1 + A_2] \\
\cos[A_1 + A_2]
\end{pmatrix} + L\text{T}A_5 \cdot \begin{pmatrix}
-\sin[A_1 + A_2 + A_3] \\
\cos[A_1 + A_2 + A_3]
\end{pmatrix};
\end{align*}
\]

Where LTA1 and LTA2 are the distances between the first 2 attachment points and the MCP joint, LTA3 and LTA4 are the distances between the next two attachment points and the PIP joint, and LTA5 is the distance between the last attachment point and the DIP joint.

### Reaction Force Definitions

The components of reaction forces are defined as acting perpendicular to their respective segment.

\[
\begin{align*}
\text{RF1} &= \text{Flatten}\left[f_1 \cdot \begin{pmatrix}
-\cos[A_1] \\
-\sin[A_1]
\end{pmatrix}\right]; \\
\text{RF2} &= \text{Flatten}\left[f_2 \cdot \begin{pmatrix}
-\cos[A_1 + A_2] \\
-\sin[A_1 + A_2]
\end{pmatrix}\right]; \\
\text{RF3} &= \text{Flatten}\left[f_3 \cdot \begin{pmatrix}
-\cos[A_1 + A_2 + A_3] \\
-\sin[A_1 + A_2 + A_3]
\end{pmatrix}\right];
\end{align*}
\]

Where f1 is the magnitude of the first reaction force, f2 is the magnitude of the second reaction force, and f3 is the magnitude of the third reaction force. This is what is being solved for during the virtual power calculation.

### System Jacobian

#### Velocity Derivatives

For the virtual power, the velocities of the reaction forces are calculated by taking the time derivative.

\[
\begin{align*}
\text{RF1vel} &= \text{Flatten}\left[D[\text{RFpos}_1, t]\right]; \\
\text{RF2vel} &= \text{Flatten}\left[D[\text{RFpos}_2, t]\right]; \\
\text{RF3vel} &= \text{Flatten}\left[D[\text{RFpos}_3, t]\right];
\end{align*}
\]
Power Calculation

All the power produced by the reaction forces is equal to all of the power produced by the tendon.

\[
\text{Sysconfig ReactionPower} = \text{RF1.RF1vel} + \text{RF2.RF2vel} + \text{RF3.RF3vel};
\]

\[
\text{Sysconfig ReactionPower} \ //\ \text{Simplify}
\]

\[
\text{Out}[40] = \text{Sysconfig f3 LRF3} \theta'\{t\} + \text{f1 LRF1} + \text{f2 LRF2} + \text{f3 LRF3} + \text{f3 LPIP Cos}\{\theta\{t\}\} + \text{f2 LMCP Cos}\{\theta\{t\}\} + \text{f3 LMCP Cos}\{\theta\{t\}\} \theta'\{t\}
\]

Jacobian Matrix Creation

Force Factoring & Matrix Construction

From the reaction power equation, the 3 reaction forces are factored out leaving a 1x3 matrix of the coefficients.

\[
\text{Sysconfig ReactionVelocityMatrix} = \text{Normal[CoefficientArrays[ReactionPower, \{f1, f2, f3\}]][[2]]};
\]

\[
\text{Print[ReactionVelocityMatrix} //\ \text{MatrixForm} //\ \text{Simplify}
\]

\[
\text{Sysconfig}
\]

Velocity Factoring

The velocities are factored out then factored out leaving behind the 3x3 Jacobian matrix.

\[
\text{Sysconfig SystemJacobian} = \text{CoefficientArrays[ReactionVelocityMatrix[[\#]], \{\theta MCP'[t], \theta PIP'[t], \theta DIP'[t]\}][[2]]} /\@\text{Range}[3];
\]

\[
\text{Inverse[SystemJacobian} //\ \text{MatrixForm} //\ \text{Simplify}
\]
Wrenches

MCP

\[
\text{In[45]}:= \text{Sysconfig}
\]

\[
\text{Wrench1} = \text{LTA1} + \text{LTA2} \times \sin[\text{vecang}[	ext{Flatten[ProximalSegment]}, \text{Flatten[TApos3 - TApos2]}]]
\]

PIP

\[
\text{In[46]}:= \text{Sysconfig}
\]

\[
\text{Wrench2} = \text{LTA3} \times \sin[\text{vecang}[	ext{Flatten[MedialSegment - ProximalSegment]}, \text{Flatten[TApos3 - TApos2]}]] + \\
\text{LTA4} \times \sin[\text{vecang}[	ext{Flatten[MedialSegment - ProximalSegment]}, \text{Flatten[TApos5 - TApos4]}]]
\]

DIP

\[
\text{In[47]}:= \text{Sysconfig}
\]

\[
\text{Wrench3} = \text{LTA5} \times \sin[\text{vecang}[	ext{Flatten[DistalSegment - MedialSegment]}, \text{Flatten[TApos5 - TApos4]}]]
\]

Reaction Power Matrix

\[
\text{In[48]}:= \text{Sysconfig}
\]

\[
\text{WrenchPower} = \{\text{Wrench1}, \text{Wrench2}, \text{Wrench3}\} \times \text{Ft}
\]

Force Outputs

3 Reaction Forces

\[
\text{In[49]}:= \text{Sysconfig}
\]

\[
\text{Forces} = \text{Inverse}[\text{Transpose}[\text{SystemJacobian}]].\text{WrenchPower};
\]

2 Reaction Forces

\[
\text{In[50]}:= \text{Sysconfig}
\]

\[
\text{Forces} = \text{Inverse}[\text{Transpose}[\text{SystemJacobian}[\{2 ; 3, 2 ; 3\}]].\text{WrenchPower}[\{2 ; 3\}];
\]

Model Function

The results of the two reaction force case are then used to generate a function that the data from the testing procedure
can be fed into in order to generate the possible output loading.

```
In[51]=
Sysconfig
model[xA1_, xA2_, xA3_, xLMCP_, xLPIP_, xLDIP_,
    xLRF1_, xLRF2_, xLRF3_, xLTA1_, xLTA2_, xLTA3_, xLTA4_, xLTA5_] :=
Forces //. {LMCP \[Rule] xLMCP, LPIP \[Rule] xLPIP, LDIP \[Rule] xLDIP, LRF1 \[Rule] xLRF1, LRF2 \[Rule] xLRF2, LRF3 \[Rule] xLRF3,
    LTA1 \[Rule] xLTA1, LTA2 \[Rule] xLTA2, LTA3 \[Rule] xLTA3, LTA4 \[Rule] xLTA4, LTA5 \[Rule] xLTA5,
```
A.2 Full Equations

There is no real practical way to display the equations from the model, other than to display their simplified from the Mathematica output.
Full Equation for Contact Force on the Proximal Segment

\[
\text{ln}[109] := \text{Forces}[[1]] /. \{\text{ΘMCP}[t] \to \text{ΘMCP}, \text{ΘPIP}[t] \to \text{ΘPIP}, \text{ΘDIP}[t] \to \text{ΘDIP}\} // \text{Simplify}
\]

\[
\text{Out}[109] := \frac{1}{\text{LRF1}} \left( \text{LTA1} + \text{LTA2} \sin \left(2 \text{ArcTan}\left[\sqrt{\left(\text{LMCP} \cos[\ThetaMCP] (\text{LMCP} - \text{LTA2} + \sqrt{\text{LMCP}^2 - 2 \text{LMCP} \text{LTA2} + \text{LTA2}^2 + \text{LTA3}^2 + 2 (\text{LMCP} - \text{LTA2}) \text{LTA3} \cos[\ThetaPIP]) + \text{LMCP} \text{LTA3} \cos[\ThetaMCP + \ThetaPIP]]^2 + \text{LMCP}^2 (\text{LMCP} - \text{LTA2} + \sqrt{\text{LMCP}^2 - 2 \text{LMCP} \text{LTA2} + \text{LTA2}^2 + \text{LTA3}^2 + 2 (\text{LMCP} - \text{LTA2}) \text{LTA3} \cos[\ThetaPIP])}
\right) \right] \right) \sin[\text{ΘMCP} + \text{LTA3} \sin[\ThetaMCP + \ThetaPIP]]^2} + \sqrt{\left(\text{LPIP} (\text{LMCP} - \text{LTA2}) \text{cos}[\ThetaMCP] + \text{LP}[\text{LTA3} + \sqrt{\text{LMCP}^2 - 2 \text{LMCP} \text{LTA2} + \text{LTA2}^2 + \text{LTA3}^2 + 2 (\text{LMCP} - \text{LTA2}) \text{LTA3} \cos[\ThetaPIP])^2 + \text{LP}^2 \text{cos}[\ThetaMCP + \ThetaPIP] + \sqrt{\text{LPIP} (\text{LMCP} - \text{LTA2}) \text{sin}[\ThetaMCP] + \text{LP} \text{LTA3} + \sqrt{\text{LMCP}^2 - 2 \text{LMCP} \text{LTA2} + \text{LTA2}^2 + \text{LTA3}^2 + 2 (\text{LMCP} - \text{LTA2}) \text{LTA3} \cos[\ThetaPIP])^2 + \text{LP}^2 \text{sin}[\ThetaMCP + \ThetaPIP] + \text{LP} \text{LTA4} \sin[2 \text{ArcTan}\left[\sqrt{\text{LPIP} \text{LTA4} + \sqrt{\text{LP}^2 - 2 \text{LP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LP} - \text{LTA4}) \text{LTA5} \cos[\ThetaDIP])^2 + \text{LP}^2 (\text{LPIP} - \text{LTA4} + \sqrt{\text{LP}^2 - 2 \text{LP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LP} - \text{LTA4}) \text{LTA5} \cos[\ThetaDIP])^2 + \text{LP}^2 \text{sin}[\ThetaMCP + \ThetaPIP] + \text{LP} \text{LTA5} \sin[\ThetaDIP + \ThetaMCP + \ThetaPIP]]^2}\right]}
\right]}
\]

\]

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Full Equation for Contact Force on the Middle Segment

\[ \text{In[110]} = \text{Forces[[2]]} /. \{\theta\text{MCP} \to \theta\text{MCP}, \theta\text{PIP} \to \theta\text{PIP}, \theta\text{DIP} \to \theta\text{DIP}\} \] // Simplify

\[ \text{Out[110]} = \frac{1}{\text{LRF}_2 \text{LRF}_3} \text{LTA}_5 \left( \left( \text{LRF}_3 + \left( \text{LPI}P - \text{LRF}_2 \right) \cos[\theta\text{DIP}] \right) \sin[\theta\text{DIP}] + \cos[\theta\text{DIP}] \right) \]

\[ \times \left( \sin[2 \arctan]\left( \left( \text{LDIP} \left( \text{LPI}P - \text{LTA}_4 \right) \cos[\theta\text{MCP} + \theta\text{PIP}] + \text{LDIP} \left( \text{LTA}_5 + \left( \sqrt{\text{LPI}P^2 - 2 \text{LPI}P \text{LTA}_4 + \text{LTA}_4^2 + \text{LTA}_5^2 + 2 (\text{LPI}P - \text{LTA}_4) \text{LTA}_5 \cos[\theta\text{DIP}] \right) \cos[\theta\text{DIP} + \theta\text{MCP} + \theta\text{PIP}] \right)^2 + \left( \text{LDIP} \left( \text{LPI}P - \text{LTA}_4 \right) \sin[\theta\text{MCP} + \theta\text{PIP}] + \text{LDIP} \left( \text{LTA}_5 + \sqrt{\text{LPI}P^2 - 2 \text{LPI}P \text{LTA}_4 + \text{LTA}_4^2 + \text{LTA}_5^2 + 2 (\text{LPI}P - \text{LTA}_4) \text{LTA}_5 \cos[\theta\text{DIP}] \right) \sin[\theta\text{DIP} + \theta\text{MCP} + \theta\text{PIP}] \right)^2 \right), \right) \]

\[ \sqrt{\left( \left( \text{LDIP} \left( \text{LPI}P - \text{LTA}_4 \right) \cos[\theta\text{MCP} + \theta\text{PIP}] + \text{LDIP} \left( \text{LTA}_5 + \sqrt{\text{LPI}P^2 - 2 \text{LPI}P \text{LTA}_4 + \text{LTA}_4^2 + \text{LTA}_5^2 + 2 (\text{LPI}P - \text{LTA}_4) \text{LTA}_5 \cos[\theta\text{DIP}] \right) \cos[\theta\text{DIP} + \theta\text{MCP} + \theta\text{PIP}] \right)^2 + \left( \text{LDIP} \left( \text{LPI}P - \text{LTA}_4 \right) \sin[\theta\text{MCP} + \theta\text{PIP}] + \text{LDIP} \left( \text{LTA}_5 + \sqrt{\text{LPI}P^2 - 2 \text{LPI}P \text{LTA}_4 + \text{LTA}_4^2 + \text{LTA}_5^2 + 2 (\text{LPI}P - \text{LTA}_4) \text{LTA}_5 \cos[\theta\text{DIP}] \right) \sin[\theta\text{DIP} + \theta\text{MCP} + \theta\text{PIP}] \right)^2 \right) \right) \]
\[
\begin{align*}
&\left( LTA3 + \sqrt{LMCP^2 - 2 LMCP LTA2 + LTA2^2 + LTA3^2 + 2 (LMCP - LTA2) LTA3 \cos[\Theta PIP]} \right) \\
&\sin[\Theta MCP + \Theta PIP] \right)^2, \sqrt{\left( \left( LPIP \ (LMCP - LTA2) \cos[\Theta MCP] + LPIP \right) \left( LTA3 - \sqrt{LMCP^2 - 2 LMCP LTA2 + LTA2^2 + LTA3^2 + 2 (LMCP - LTA2) LTA3 \cos[\Theta PIP]} \right) \cos[\Theta MCP + \Theta PIP] \right)^2 + \left( LPIP \ (LMCP - LTA2) \sin[\Theta MCP] + LPIP \right) \left( LTA3 - \sqrt{LMCP^2 - 2 LMCP LTA2 + LTA2^2 + LTA3^2 + 2 (LMCP - LTA2) LTA3 \cos[\Theta PIP]} \right) \sin[\Theta MCP + \Theta PIP] \right)^2] \right), \sqrt{\left( \left( LPIP \ (LPIP - LTA4 + \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \cos[\Theta MCP + \Theta PIP] + LPIP \ LTA5 \cos[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2 + \left( LPIP \ (LPIP - LTA4 + \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \sin[\Theta MCP + \Theta PIP] + LPIP \ LTA5 \sin[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2] \right), \sqrt{\left( \left( LPIP \ (LPIP - LTA4 + \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \cos[\Theta MCP + \Theta PIP] + LPIP \ LTA5 \cos[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2 + \left( LPIP \ (LPIP - LTA4 + \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \sin[\Theta MCP + \Theta PIP] + LPIP \ LTA5 \sin[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2] \right), \\
&\frac{1}{LRF3} \left( LRF3 + LPIP \cos[\Theta DIP] \right) \sin[\Theta MCP + \Theta PIP] + \frac{1}{2} \arctan[\sqrt{\left( \left( LPIP \ (LPIP - LTA4) \cos[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \cos[\Theta DIP + \Theta MCP + \Theta PIP] + LPIP \right)^2 + \left( LPIP \ (LPIP - LTA4) \sin[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \sin[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2}], \\
&\sqrt{\left( \left( LPIP \ (LPIP - LTA4) \cos[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \cos[\Theta DIP + \Theta MCP + \Theta PIP] + LPIP \right)^2 + \left( LPIP \ (LPIP - LTA4) \sin[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \sin[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2], \\
&\frac{1}{LRF3} \left( LRF3 + LPIP \cos[\Theta DIP] \right) \sin[\Theta MCP + \Theta PIP] + \frac{1}{2} \arctan[\sqrt{\left( \left( LPIP \ (LPIP - LTA4) \cos[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \cos[\Theta DIP + \Theta MCP + \Theta PIP] + LPIP \right)^2 + \left( LPIP \ (LPIP - LTA4) \sin[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \sin[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2}], \\
&\frac{1}{LRF3} \left( LRF3 + LPIP \cos[\Theta DIP] \right) \sin[\Theta MCP + \Theta PIP] + \frac{1}{2} \arctan[\sqrt{\left( \left( LPIP \ (LPIP - LTA4) \cos[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \cos[\Theta DIP + \Theta MCP + \Theta PIP] + LPIP \right)^2 + \left( LPIP \ (LPIP - LTA4) \sin[\Theta MCP + \Theta PIP] + LPIP \right) \left( LTA5 - \sqrt{LPIP^2 - 2 LPIP LTA4 + LTA4^2 + LTA5^2 + 2 (LPIP - LTA4) LTA5 \cos[\Theta DIP]} \right) \sin[\Theta DIP + \Theta MCP + \Theta PIP] \right)^2)].
\end{align*}
\]
Full Equation for Contact Force on the Distal Segment

\[
\text{LTA5} - \sqrt{\text{LPIP}^2 - 2 \text{LPIP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LPIP} - \text{LTA4}) \text{LTA5} \cos[\Theta_{DIP}]}
\]

\[
\sin[\Theta_{DIP} + \Theta_{MCP} + \Theta_{PIP}]^2\]

\[
\text{LDIP} \left(\text{LTA5} - \sqrt{\text{LPIP}^2 - 2 \text{LPIP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LPIP} - \text{LTA4}) \text{LTA5} \cos[\Theta_{DIP}]}
\]

\[
\cos[\Theta_{DIP} + \Theta_{MCP} + \Theta_{PIP}]^2 + \text{LDIP} (\text{LPIP} - \text{LTA4}) \sin[\Theta_{MCP} + \Theta_{PIP}] +
\]

\[
\text{LDIP} \left(\text{LTA5} + \sqrt{\text{LPIP}^2 - 2 \text{LPIP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LPIP} - \text{LTA4}) \text{LTA5} \cos[\Theta_{DIP}]}
\]

\[
\sin[\Theta_{DIP} + \Theta_{MCP} + \Theta_{PIP}]^2\right)\right]^{\frac{1}{2}} + \text{LDIP} (\text{LPIP} - \text{LTA4}) \sin[\Theta_{MCP} + \Theta_{PIP}] +
\]

\[
\text{LDIP} \left(\text{LTA5} - \sqrt{\text{LPIP}^2 - 2 \text{LPIP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LPIP} - \text{LTA4}) \text{LTA5} \cos[\Theta_{DIP}]}
\]

\[
\cos[\Theta_{DIP} + \Theta_{MCP} + \Theta_{PIP}]^2 + \text{LDIP} (\text{LPIP} - \text{LTA4}) \sin[\Theta_{MCP} + \Theta_{PIP}] +
\]

\[
\text{LDIP} \left(\text{LTA5} + \sqrt{\text{LPIP}^2 - 2 \text{LPIP} \text{LTA4} + \text{LTA4}^2 + \text{LTA5}^2 + 2 (\text{LPIP} - \text{LTA4}) \text{LTA5} \cos[\Theta_{DIP}]}
\]

\[
\sin[\Theta_{DIP} + \Theta_{MCP} + \Theta_{PIP}]^2\right)\right)\right)]^{\frac{1}{2}}\]
Appendix B

Finger Test Rig

The rigs are intended to be 3D printed so only their most relevant dimensions have been called out. They are designed to be as rigid as possible in order to keep up with the assumptions in the mathematical model. The design of the test rigs was driven by numerous iterative tests and the resources that were available for manufacturing.

Figure B.1 shows the relevant dimensions in the proximal Segment. Unless otherwise specified the dimensions are in meters. Each of the holes for the joints is undersized so that they can be finished to the final size to avoid the limitations of the 3D printer. The MCP segment is the same for all the test rigs.

The middle segment design shown in Figure B.2 shows the relevant dimensions...
of the segments. There are two different segments used in the testing with the only differences between them being the tendon attachment points. The width of the contact face was determined through preliminary testing and selected so that it is large enough to interact with the pressure pad.

![Middle Segment Rig 1 and 2](image1)

**Fig. B.2: Middle Segment relevant dimensions**

Figure B.3 shows the distal segment’s relevant dimensions. There are two different segments used in the testing with the only differences between them being the tendon attachment points.

![Distal Segment Rig 1 & 2](image2)

**Fig. B.3: Distal Segment relevant dimensions**
Appendix C

Test Stand Design

Figure C.1 is an example of one of the three test diameter holders that were constructed for the project. They are fabricated out of an aluminum plate with the test diameter and the retaining pin pressed into it. All are identical except for the test diameter size; a different one was manufactured for each test diameter. The test diameters are .015 inches undersized to account for the thickness of the sensor. Units are in inches.

![Test Diameter Dimensions](image)

Fig. C.1: Test Diameter Dimensions
Figure C.2 is the test rig holder. Its function is to hold the test rigs beneath the camera in the proper position to grip the test diameter and to allow the tendon to be hung from the table. It is made up of four parts; the main holder body machined from a piece of steel. The test rig holding pin is machined from stock and pressed into the main holder body, and the tendon routing pin is a half inch dowel pin held in place with a setscrew.

![Test Rig Holder Dimensions](image)

Figure C.3 is the camera frame that holds the webcam in place to take measurements of the experiment. It is fabricated out of the available 80/20 T-Slotted aluminum parts.
Figure C.4 is a CAD model of the full test stand with all of its components and a represents the actual test setup.
Fig. C.4: Test Stand CAD
Appendix D

NASA EVA Tool Summary

The EVA Tool survey is shown in Table D.1. It was generated by going through a NASA EVA handbook and identifying the tools or parts of the space station structure that an Astronaut would grab onto [78]. Then, if listed, recording the dimensions of the “handle” or “handhold” as either the diameter or the maximum and minimum values listed. All units are inches as reported by the handbook.

Diameters selected for the testing procedure are shown in green. The modes of each part of the data set were selected because it represented the most common sizes of tools that the astronaut may need to grasp. The decision was made to select the median of the data sets over the mean for ease of manufacturability and the fact that the differences between the mean and the median are minimal.
### Table D.1: EVA Tool Survey

<table>
<thead>
<tr>
<th>Tool</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt Puller</td>
<td>1.38</td>
<td>1.27</td>
</tr>
<tr>
<td>pin straightener tool</td>
<td>1.38</td>
<td>1.27</td>
</tr>
<tr>
<td>coax connector tool</td>
<td>2.06</td>
<td>1.38</td>
</tr>
<tr>
<td>antenna cutter</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>cutter, tube</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>door latch tool, External tank</td>
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<td></td>
</tr>
<tr>
<td>door support bracket</td>
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<td></td>
</tr>
<tr>
<td>Drill, 1/4 in</td>
<td>0.62</td>
<td>0.7</td>
</tr>
<tr>
<td>Drive unit preload tool</td>
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<td>0.9</td>
</tr>
<tr>
<td>Force Measurement tool</td>
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<td></td>
</tr>
<tr>
<td>Hammer</td>
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<td>1.25</td>
</tr>
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<td>Handle, GFE</td>
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<tr>
<td>Handle, Oval</td>
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<td>0.75</td>
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<td>Hydrazine Brush</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Hydrazine Detector</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Mirror, Inspection</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Pip Pine, Lock-Lock</td>
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<td></td>
</tr>
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<td>Power Drive Unit Disconnect</td>
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</tr>
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<td>Power Ratchet Tool</td>
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<td>Power Tool, Mini</td>
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<tr>
<td>Probe</td>
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<td>1.75</td>
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<tr>
<td>Pry Bar</td>
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<td></td>
</tr>
<tr>
<td>Ratchet 3/8 inch drive</td>
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<td>0.75</td>
</tr>
<tr>
<td>Ratchet, 3/8 inch drive EVA</td>
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<td>0.75</td>
</tr>
<tr>
<td>Ratchet, 3/8-Inch Drive Tether</td>
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<td>0.75</td>
</tr>
<tr>
<td>Ratchet, 3/8-Inch Drive, with 7/16-inch socket</td>
<td>1.4</td>
<td>0.75</td>
</tr>
<tr>
<td>Ratchet, 3/8-INCH Drive With 7/16-Inch SOCKET</td>
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<td>0.75</td>
</tr>
<tr>
<td>Screwdriver, Extension</td>
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<td></td>
</tr>
<tr>
<td>Steering Wheel</td>
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<td></td>
</tr>
<tr>
<td>Torque Limiter</td>
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<td>Wrench</td>
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<td>Wrench, RMS MPM</td>
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<td>Handrail/Handhold</td>
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<tr>
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<td>Wrench, Spanner</td>
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<td>Wrench, Shuttle Umbilical Retraction System</td>
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<td>MMU Range Finder</td>
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<td>Wrench, 1/2-inch open end</td>
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<tr>
<td>Hand Hold</td>
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<td>Handrail, on-orbit installed</td>
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<td><strong>Median</strong></td>
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Appendix E
Experiment Information

E.0.1 Tendon

The tendon is a 1mm thick, 3mm wide Vectran woven cord. Vectran was selected for the tendon material because of its strength and resistance to stretch; NASA also uses this material in their Robonaut tendon [59].

E.0.2 Pressure Sensor DAQ

For collecting the sensor data, the Pliance-xf-16 wireless sensor is used. It is capable of reading from 256 sensors at a rate of approximately 75 Hz. Its wireless radio is Bluetooth which it connects to a computer using a USB receiver that comes with it. It receives power from a battery pack.

E.0.3 Pressure Sensor Software

The software for the system is Pliance/E; it handles the acquisition and exportation of the pressure data. This version of the software requires a computer capable of running Windows 7 and has at least one available USB port.

E.0.4 Tendon Tension

Known masses were used to load the tendon in the test rig. These masses were selected so that the load could be stepped up in half kilogram increments starting with one-half kilogram and going all the way to two kilograms. Additional weights are also needed to overload the system to overcome friction. Before experimentation, their mass was measured on a calibrated scale and the local gravity in Grand Rapids.
Michigan is $9.8056 \text{ m/s}^2$ [83]. One value for the uncertainty in the Gravitation model reported by Bomfirm et al. is $\pm 6.4 \times 10^{-5} \text{ m/s}^2$ [84]. Table E.1 shows each of the four loads applied to the test rigs.

Table E.1: Load Summary

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<th>Load #</th>
<th>Tendon tension (N)</th>
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<tr>
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<tr>
<td>4</td>
<td>$19.5877 \pm 0.0014$</td>
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