To Study the Current Distributions of an Antenna in Magnetized Collisional Plasma via PFFDTD Simulations

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To Study the Current Distributions of an Antenna in Magnetized Collisional Plasma via PF-FDTD Simulations

Sindhuja Moravineni

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Abstract

To Study the Current Distributions of Electrically Short Dipoles in Magnetized Collisional Plasma via FDTD Simulations

This thesis paper talks about the study of current distributions on dipole antenna immersed in collisional plasma. Analytical models like Balmain’s[6] one dimensional triangular current distribution, Staras[8] proposed and developed by Nikitin’s[10] three-dimensional exponential current distribution and the PF-FDTD (Plasma Fluid- Finite Difference Time Domain) model[11] are compared against the Auroral Space Structure Probe (ASSP) flight data in order to show how data analysis can benefit. PF-FDTD model is used to study the antenna in weakly collisional plasma and the comparisons of the analytical and numerical solutions show that numerical model match better for data analysis of flight data sweeps than the analytical models. Advantages and disadvantages of these different models are discussed.
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Chapter 1

Introduction

The world of communication changed when Guglielmo Macaroni succeeded in receiving the first wireless radio transmission across the Atlantic ocean on December 12, 1901. This message travelled about 2200 miles (3500 km) from England to Canada and showed that the radio waves do not follow the curvature of the earth for transmission but are reflected off the ionosphere bouncing back to the receiving station[1]. Since then, researchers have been trying to explore this phenomenon which occurs in region from about 50 to 600 miles above the Earth’s atmosphere. High energy particles released from the Sun affect the neutral atoms of the upper atmosphere giving rise to the region called ionosphere. This effect on atoms ionizes them and creates a quasi-neutral vapor or fluid called plasma in which free electrons and ions react with each other. The increased density of the particles allows electrons to recombine with ions, causing the atoms to recombine into neutral atoms at lower altitudes.

The early experiments on ionosphere was done by Sir Robert-Wattson-Watt and his colleagues by sending electromagnetic waves at ionosphere and observing the return time of the various frequencies that were reflected[2]. It was noted that the higher concentration of free electrons the higher the cut-off frequency of the plasma, quantified as the plasma frequency($\omega_p$).

$$\omega_p = \sqrt{\frac{nq^2}{\epsilon m}} \quad (1.1)$$

Where $n$ the density of is free electrons, $q$ is the fundamental charge, $\epsilon$ is the permittivity of free space and $m$ is the mass of an electron. The charged nature of the plasma additionally produces absorption, reflection, and propagation of waves that can
be related to the ambient magnetic field, thermal temperatures of the various atomic structures, and the collisional rates of the various constituents [3]. In order to manage data analysis, simplifying assumptions are required. The frequency of charged particle moving perpendicular to direction of uniform magnetic field is called as cyclotron frequency ($\omega_c$).

$$\omega_c = \Omega \frac{|q|B_0}{m}$$ (1.2)

With the advent of rockets, it became feasible to perform additional in-situ experiments on the ionosphere. For example, the Langmuir probe, one of the oldest and most often used probe for measuring low temperature plasmas[4], works by inserting one or more electrodes into plasma with a constant or time-varying electric potential between the various electrodes. The attracted charges produce a current that can be correlated to the voltage in order to measure the plasma temperature, which relates to the kinetic energy associated with the plasma.

For the determination of in-situ free electron density, the most suitable probe is based on radio frequency called as Capacitive probe or C-probe. It is commonly used to back out the triboelectric charging that effects Langmuir type probes[5]. C-probes effectively treat the plasma as a dielectric between to electrodes, with the relative permittivity ($\varepsilon_r$) being

$$\varepsilon_r = 1 - \left(\frac{\omega_p}{\omega}\right)^2$$ (1.3)

Where $\omega = 2\pi f$, where $f$ is the radian drive frequency and $f$ is the frequency, is typically chosen well above the plasma frequency. Analytical theories like those proposed by Balmain [6], Adachi [7], and Staras [8], also proposed that the impedance of an RF probe could also be related to other plasma properties, if only the resulting antenna current distributions could be accounted for.
1.1 Fields and Currents

Classical antenna theory links the radiation pattern of a conductive surface to the current distribution on its structure[9]. The same can be said for an RF plasma probe in the ionosphere. However, the standard triangular and sinusoidal freespace current distributions become questionable in the free charged environment. Classical methods apply the Lorentz condition. In a freely charged environment, the combined force of electric and magnetic fields exerted on a point charge due to electromagnetic fields is called Lorentz condition. It is given by the equation

\[ F = q(E + v \times B) \]  
\( (1.4) \)

Where, F is the Lorentz force, q is the charged particle, E is the electric field, v is the velocity and B is the magnetic field. In the analysis of such problems related to radiated fields, the common practice is to specify the sources first and then obtain the radiated fields by the sources. One of the procedures in the analysis of the radiated fields make use of the auxiliary functions known as vector potentials, which help in the determination of electric fields (E) and magnetic fields (B)[9]. The vector potentials A (magnetic vector potential) and F (electric vector potential) relate to the electric current source (J) and magnetic current source (M) by their integrals respectively. The vector potential (A) to the prescribed current distribution (J) can be further linked to the radiative electric field (E) by

\[ E_A = -\nabla \Phi_e - j\omega A = -j\omega A - j \frac{1}{\omega \varepsilon \mu} \nabla (\nabla, A) \]  
\( (1.5) \)

Where \( E_A \) is the electric field due vector potential A, \( \Phi_e \) is the electric scalar potential, \( \varepsilon \) is the dielectric tensor, \( \mu \) is the permeability.
Similarly, the vector potential (F) to the prescribed magnetic current source (M) can be linked to radiative magnetic field (B) by

\[ H_F = -j \omega F - \frac{j}{\omega \varepsilon \mu} \nabla (\nabla \cdot F) \] (1.6)

Where \( H_F \) is the magnetic field due to vector potential F. Equations (1.5) and (1.6) are used for determining electric and magnetic fields once the vector potentials are derived. With the plasma modifying the dielectric constant (\( \varepsilon \)), Nikitin inverted this relationship to link the input impedance of a dipole antenna to the current distribution along the antenna and the plasma environment.

\[ Z(\omega) = \frac{j}{2\pi \omega |I_0|^2} \int \frac{[k \cdot J_{\text{ext}}(k)][k \cdot J_{\text{ext}}^*(k)]}{k \cdot \varepsilon \cdot k} dk \] (1.7)

Where \( Z(\omega) \) is the input impedance, \( I_0 \) is the magnitude of current at antenna terminals, \( k \cdot \varepsilon \cdot k \) is the dispersion relation for plasma waves, \( J_{\text{ext}} \) is the prescribed current distribution.

1.2 Antenna Currents and Antenna Impedances

Current distribution on the surface of antenna can only be inferred but not measured. K.G. Bakmain proposed one dimensional (1-D) triangular current distribution on a cylindrical dipole in cold plasma under quasi static conditions by applying standard electrically short antenna free-space. This model was expected to be accurate for operating frequencies above the upper hybrid (\( \omega_{uh} \)) of the plasma.

\[ \omega_{uh}^2 = \omega_p^2 + \Omega^2 \] (1.8)

Where \( \Omega \) is the gyro frequency. The energy absorption of the plasma was supposed to significantly attenuate the current distribution along the antenna at frequencies closer to
cut-off frequencies. As a result, Staras proposed a three-dimensional (3-D) exponential current distribution that could more accurately model the antenna plasma interactions near cut off frequencies. This proposal of Staras was later developed into closed analytical solution by Nikitin and Swenson[10]. The approach of electromagnetic numerical methods has paved way to self-consistent solvers, who are independent of assumed current distributions. One such self-consistent model is the Plasma Fluid- Finite Difference Time Domain(PF-FDTD) model[11]. Based on the Maxwell’s equations, plasma fluid equations and prescribed boundary conditions the electric fields, magnetic fields, plasma densities and plasma velocities are directly calculated by the PF-FDTD algorithm. With the help of these calculated electric fields and magnetic fields expected current distributions on an antenna can be inferred by using the Ampere’s integral law which is given by

\[ \oint B \, dl = \mu_0 I \] (1.9)

Where B is the magnetic field, \( dl \) is the infinitesimal length, \( \mu_0 \) is the permeability of the medium and I is the current flowing through the loop.

\[ I(t) = \oint H(t) \, dl \]

\[ = \frac{dx}{2\mu_0} \left[ B_{x}^{t+\frac{1}{2}}(i, j, k) - B_{x}^{t-\frac{1}{2}}(i, j, k) + B_{x}^{t+\frac{1}{2}}(i, j, k) - B_{x}^{t-\frac{1}{2}}(i, j + 1, k) + B_{x}^{t+\frac{1}{2}}(i, j + 1, k) \right] \]

\[ + \frac{dy}{2\mu_0} \left[ B_{y}^{t+\frac{1}{2}}(i + 1, j, k) - B_{y}^{t-\frac{1}{2}}(i, j, k) + B_{y}^{t+\frac{1}{2}}(i, j, k) - B_{y}^{t+\frac{1}{2}}(i, j + 1, k) \right] \] (1.10)
Equation (1.10) is used to find the current in the time domain, which is later converted to frequency domain for impedance calculation. Where $B_x$, $B_y$ are magnetic fields on X-axis and Y-axis of the antenna respectively. Based on the principle of equation (1.9), equation (1.10) was derived to monitor the currents based on the calculated electric and magnetic fields of PF-FDTD model.

1.3 Thesis Overview

This thesis is a comparison of the Balmain’s 1-D triangular current distribution model, Staras proposed 3-D exponential current distribution and the PF-FDTD numerical model. These models are compared against ASSP (Auroral Spatial Structures Probe) flight data recorded by scientists via Utah State University. The major objective of this thesis is to study the current distributions of short dipole antenna immersed in weakly magnetized collisional plasma and to study the current distributions experienced by the ASSP flight. Chapter 2 outlines about each of the analytical and numerical models in detail with limitations and assumptions. Chapter 3 compares the three models against each other and ASSP flight data sweeps as well. Chapter 4 discusses the variation of currents seen in simulations by PF-FDTD and links this to measurable variations in the datasets. The summary of all the work done is presented in the Chapter 5 with some possible ideas for future work.
Chapter 2

Models

2.1 Balmain’s Model

Balmain derived a closed form expression for the impedance of a short cylindrical dipole antenna using a quasi-static approximation. Assuming that free space current distributions dominate any other modes present on the antenna Balmain’s applied a 1-D current distribution and it is given by the equation (2.1). This assumption is also referred to as triangular current distribution on an antenna in this thesis.

\[
J(r) = I_o \left(1 - \frac{|z|}{l}\right) \frac{\delta(x)\delta(y)}{2\pi a} n_z
\]  \hspace{1cm} (2.1)

Where \(I_o\) is the magnitude of current at antenna terminals, \(l\) is the length of the antenna, \(a\) is the radius of the antenna, \(x\), \(y\) and \(z\) axes of the plane of the antenna.

When equation (2.1) is transformed to Fourier domain, it yields to

\[
J(k) = \frac{lI_o}{(2\pi)^3} sinc^2 \left(\frac{k_z l}{2}\right) n_z
\]  \hspace{1cm} (2.2)

Where \(l\) is the length of the antenna is, \(I_o\) is the magnitude of current at antenna terminals, \(k_z\) is the K-space transform in axis \(z\).
Balmain used the anisotropic form of equation (1.3) to account for the effect of earth’s magnetic field.

\[
\varepsilon = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \tag{2.3}
\]

Where

\[
\varepsilon_1 = 1 - \frac{\omega_p^2(1 - j\frac{\nu}{\omega})}{\omega^2(1 - j\frac{\nu}{\omega})^2 - \omega_c^2}
\]

\[
\varepsilon_2 = \frac{\omega_p^2\omega_c}{\omega(\omega^2 - \omega_c^2)}
\]

\[
\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2(1 - j\frac{\nu}{\omega})}
\]

Where \(\omega_p\) is the plasma frequency, \(\omega_c\) is the cyclotron or gyro frequency, \(\nu\) is the collision frequency. When these equations (2.2) and (2.3) are applied to equation (1.4) gives the impedance of the dipole antenna equation (2.4).

\[
Z = -\frac{j}{2\pi\omega\varepsilon_0\varepsilon_1 l} \left[ \ln \frac{l}{a} - 1 + \ln \left( \frac{\varepsilon_1}{\varepsilon_3} \right)^{1/2} \right] \tag{2.4}
\]

The input impedance of dipole antenna with length of 0.5m and radius 0.01m for plasma frequency (Fp) of 4.10MHZ, gyro frequency (Fg) of 2.25MHz and collisional frequency (v) of 0.1% Fp are shown in figure (2.2). Figures (2.3), (2.4) and (2.5) are plotted for change in plasma frequency from 4.10 MHz to 6.10 MHz with keeping other parameters constant as in figure (2.2), change in gyro frequency from 2.25MHz to 3.25 MHz while keeping other parameters same as figure (2.2) and collisional frequency from 0.1% Fp to 0.4% Fp while keeping other parameters constant with respect to figure (2.2) respectively. Taking figure (2.2) as reference, it can be observed that the amplitude of the
plots for varying conditions has decreased from $4.5 \times 10^4$ to $4 \times 10^4$, $3 \times 10^4$ and $1.5 \times 10^4$ respectively.

Figure 2.2: Impedance of antenna in plasma, using Balmain’s model for $F_p$ of $4.10\text{MHz}$, $F_g$ of $2.25\text{MHz}$ and $v$ of 0.1% $F_p$.

Figure 2.3: Impedance of antenna in plasma, using Balmain’s model for $F_p$ of $6.10\text{MHz}$, $F_g$ of $2.25\text{MHz}$ and $v$ of 0.1% $F_p$. (Change in plasma frequency with respect to figure(2.2))
Figure 2.4: Impedance of antenna in plasma, using Balmain’s model for F_p of 4.10MHz, F_g of 3.25MHz and v of 0.1% F_p. (Change in gyro frequency with respect to figure (2.2))

Figure 2.5: Impedance of antenna in plasma, using Balmain’s model for F_p of 4.10MHz, F_g of 2.25MHz and v of 0.4% F_p. (Change in collisional frequency with respect to figure (2.2))
The upper hybrid frequency is obtained at \((F_{uh})^2 = (F_p)^2 + (F_g)^2\), where \(F_{uh}\) is the upper hybrid frequency (parallel resonance), \(F_p\) is the plasma frequency (plasma resonance) and \(F_g\) is the gyro or the cyclotron frequency (series resonance). The effective coupling of energy from the plasma region into the antenna is called plasma resonance. There are two aspects of discussion called the series resonance and the parallel resonance. As the series resonance occurs at the gyro frequency, figure (2.7) shows that the Balmain’s model matches the flight data at series resonance. Discrepancy from the flight data sweep to Balmain’s model can be observed at parallel resonance due to reactive coupling of energy. At the region near to plasma frequency and upper hybrid frequency, there are discrepancies in the Balmain’s impedance curve, giving room for errors in Balmain’s model. When we look at the upper hybrid frequency \((F_{uh})\), in Figure (2.6), it lies at a frequency range of 4.80MHz. As the frequency range reaches the upper hybrid frequency the input impedance seems more triangular. Researchers tried to develop additional theories by varying current distributions or the analysis techniques to overcome these limitations of the Balmain’s model. Due to the complex nature of plasma, the additional theories could not be validated. Hence, Balmain’s model is suitable for the analysis above the upper hybrid frequency and not near the plasma resonance. Due to the simplicity of the model, it can be used in analyzing simple current distributions but when the current distributions are complex in nature, Balmain’s model is not accurate.

While a more through discussion will occur in chapters 3 and 4, a preliminary comparison of Balmain’s model to ASSP flight data, seen in Figure 2.7, yields:

1. The free space propagation modes dominate for frequencies significantly above the cut-off frequency \((W_{uh})\) and Balmain’s model closely matches experimental data sets.
2. Significant differences between the model and data sets occur for frequencies near known plasma wave propagation modes and is speculated that this error occurs due to the assumed triangular current distribution of Balmain’s model.

3. There are also known regions between the plasma resonance modes where energy propagation can return to standard free space models as Ordinary, Extraordinary and Whistler Waves can be launched [12].

Figure 2.6: Impedance of a dipole antenna using Balmain’s model for \( F_p \) of 4.10MHz, \( F_g \) of 2.25MHz and \( v \) of 0.1\% \( F_p \) against ASSP flight data sweep.
2.2 Nikitin’s Model

The discrepancies between the Balmain’s model and the experimental data sets near the upper hybrid resonances showed that reactively coupled energy near the antenna feed point could significantly modify the antenna current distribution. Staras proposed a cylindrically, symmetrically decaying current which could more accurately model the current loss via evanescent modes. This was later developed into closed form solution by Pavel Nikitin and Charles Swenson. This model assumes exponential form of current distribution. The biggest limiting factor of this method is that the 3-D exponential current distribution is no longer just confined to the antenna but instead it decays exponentially with radial distance from the surface.
Figure 2.8: Theoretical representation of exponential current distribution

\[ J(r) = \frac{I_o e^{-\frac{r^2}{a^2} + \frac{z^2}{l^2}}}{\sqrt{\frac{r^2}{a^2} + \frac{z^2}{l^2}}} n_z \]  

(2.5)

Where \( I_o \) is the magnitude of current at antenna terminals, \( a \) is the radius of the antenna, \( l \) is the length of the antenna, \( z \) is the axis of the antenna.

As discussed in the section 2.1, the impedance integral is computed by the Fourier transform of the current distribution.

\[ J(k) = \frac{2\pi l_o}{(2\pi)^{3/2} + (1 + k_r^2 a^2 + k_z^2 l^2)^{1/2}} n_z \]  

(2.6)

Where \( k_r \) and \( k_z \) are wave numbers based on different permittivity tensor (\( \varepsilon \)). When equation (2.6) and equation (2.3) are applied to equation (1.4) the result turns into

\[ Z = -\frac{j}{\pi^2 \omega \varepsilon_0 \varepsilon_3 l} \int_0^{+\infty} \int_0^{+\infty} \frac{s^2 t ds dt}{(s^2 + \frac{\varepsilon_1}{\varepsilon_3} t^2)(s^2 + 1 + \frac{a^2}{l^2} t^2)^2} \]  

(2.7)
Where $s = k l$, and $t = k l$ are unitless variables. For the usage of data analysis, this equation (2.7) impedance integral can be further simplified using the Cauchy's theorem

\[
Z = -\frac{j}{2\pi \omega \varepsilon_0 \varepsilon_3} l \int_0^\infty \frac{tdt}{p(p - jq)^2} = R_e(Z) + j \text{Im}(Z) \tag{2.8}
\]

where

\[
R_e(Z) = \frac{1}{2\pi \omega \varepsilon_0 l} \left[ \frac{\nu \omega_p^2}{\omega^3} I_1 + (1 - \frac{\omega_p^2}{\omega^2})I_2 \right] \left[ \left(1 - \frac{\omega_p^2}{\omega^2}\right) + \frac{\nu^2}{\omega^2} \right] \tag{2.9}
\]

and

\[
\text{Im}(Z) = \frac{1}{2\pi \omega \varepsilon_0 l} \left[ \frac{\nu \omega_p^2}{\omega^3} I_2 - (1 - \frac{\omega_p^2}{\omega^2})I_1 \right] \left[ \left(1 - \frac{\omega_p^2}{\omega^2}\right) + \frac{\nu^2}{\omega^2} \right] \tag{2.10}
\]

Where $I_1$ and $I_2$ are defined by numerically solved integrals:

\[
I_1 = \int_0^1 \frac{\left(1 + y^2 + \beta(1 - y^2)\right)^2 - \alpha^2(1 - y^2)^2}{\left((1 + y^2 + \beta(1 - y^2))^2 + \alpha^2(1 - y^2)^2\right)^2} (1 - y^2)dy \tag{2.11}
\]

\[
I_2 = \int_0^1 \frac{2\alpha(1 - y^2)^2(1 + y^2 + \beta(1 - y^2))}{\left((1 + y^2 + \beta(1 - y^2))^2 + \alpha^2(1 - y^2)^2\right)^2} dy \tag{2.12}
\]
This integrals can be derived in closed form for any given parameters of $\alpha$ and $\beta$:

$$
\alpha = \frac{l}{a} R_e \left\{ \sqrt{\frac{\varepsilon_1}{\varepsilon_3}} \right\}
$$

$$
\beta = \frac{l}{a} \text{Im} \left\{ \sqrt{\frac{\varepsilon_1}{\varepsilon_3}} \right\}
$$

The input impedance of dipole antenna with length of 0.5m and radius 0.01m for plasma frequency ($F_p$) of 4.10MHz, gyro frequency ($F_g$) of 2.25MHz and collisional frequency ($v$) of 0.1% $F_p$ are shown in figure (2.9) similar to the figures shown in section 2.1. The amplitude changes from $2 \times 10^6$ to $5.5 \times 10^6$ when the gyro frequency has changed and further decreased to $1.25 \times 10^3$ when the collisional frequency changed. The trend of change in the amplitudes for various plasma conditions vary significantly between the Balmain’s model and Nikitin’s model.

![Figure 2.9: Impedance of antenna in plasma, using Nikitin’s model for $F_p$ of 4.10MHz, $F_g$ of 2.25MHz and $v$ of 0.1% $F_p$.](image)

Figure 2.9: Impedance of antenna in plasma, using Nikitin’s model for $F_p$ of 4.10MHz, $F_g$ of 2.25MHz and $v$ of 0.1% $F_p$. 
Figure 2.10: Impedance of antenna in plasma, using Nikitin’s model for Fp of 6.10MHz, Fg of 2.25MHz and v of 0.1% Fp. (Change in plasma frequency with respect to figure (2.9))

Figure 2.11: Impedance of antenna in plasma, using Nikitin’s model for Fp of 4.10MHz, Fg of 3.25MHz and v of 0.1% Fp. (Change in gyro frequency with respect to figure (2.9))
Figure 2.12: Impedance of antenna in plasma, using Nikitin’s model for Fp of 4.10MHz, Fg of 3.25MHz and v of 0.1% Fp. (Change in collisional frequency with respect to figure (2.9))

The series resonance occurring at the gyro frequency near 2.25MHz for the figure (2.13), shows that the Nikitin’s impedance curve matches the experimental dataset at this region. At the plasma resonance region the Nikitin’s model has a larger amplitude than the flight data causing a mismatch. At the region of parallel resonance it can be seen that Nikitin’s curve match the flight data imperfectly. Though the Nikitin’s impedance curve do not match closely the flight data after the upper hybrid frequency but it still follows the trend like the flight data.

The comparison of Nikitin’s model to ASSP flight data set in figure (2.13) and (2.14) yields to

1. For frequencies above the cut-off region (W_{th}), the Nikitin’s model matches the flight datasets.
2. Differences in the curves occur in the known areas of the plasma wave propagation modes.

3. The divergence of the curve occurs in the regions where the current distribution is more triangular as expected only in electromagnetic wave modes.

Figure 2.13: Impedance of a dipole antenna using Nikitin’s model for Fp of 4.10 MHz, Fg of 2.25 MHz and v of 0.1% Fp against ASSP flight data sweep.

Figure 2.14: Figure 2.13 with focus on ASSP flight data sweep.
2.3 The Numerical Model

The models in this paper so far are based on analytical assumptions and calculations. When the analytical models have limitations for analyzing the antennas, the numerical model steps in. The numerical model discussed in this thesis paper is the Plasma Fluid-Finite Difference Time Domain model (PF-FDTD)[11]. This numerical approach can be done using the Finite Difference Time Domain (FDTD) model and plasma fluid equations. PF-FDTD is a full wave self-consistent model. It is constructed with the help of Maxwell’s equations, Ohms law and plasma fluid equations. This numerical model not only helps in studying how an antenna behaves with warm, collisional, magnetized plasma but also helps in inferring the electron density, ion density, electron velocity, ion velocity in the plasma. The numerical model helps in analyzing the region around the critical electron resonances for different ionospheric conditions. These parameters are difficult to model and analyze analytically as they have limiting assumptions detailed in the discussion in section 2.1 and section 2.2, numerical models are approached.

Maxwell’s equations

The PF-FDTD model uses 5-moment Maxwellian equations and the ideal gas law equation for temperature effect. These equations include Faraday’s equation (2.13), Ampere’s equation (2.14), Ohm’s equation (2.15).

\[
\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.13}
\]

\[
\nabla \times B = \varepsilon \mu \frac{\partial E}{\partial t} + \mu J \tag{2.14}
\]

\[
J = \sum_{s} q_{s} n_{s} U_{s} \tag{2.15}
\]
There are certain approximations made to these equations like a.) the loss term and the production terms are set to zero, b.) Plasma will be assumed to be isothermal medium.

Equation (2.16) is the continuity of mass equation and the equation (2.17) is the continuity of momentum equation. The third approximation made to the Maxwellian equation is that c.) the plasma will be subsonic and the compression term in the momentum equation is ignored because it's a smaller value [12].

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s U_s) = P - L
\]  \hspace{1cm} (2.16)

\[
m_s n_s \left( \frac{\partial U_s}{\partial t} + (U_s, \nabla) U_s \right) = q_s n_s (E + U_s \times B) - \nabla P_s - m_s n_s \sum_{\alpha \neq s} v_{s\alpha} (U_s - U_\alpha)
\]  \hspace{1cm} (2.17)

Equation (2.18) is the ideal gas law and the ideal gas law mixed with momentum equations and the fluid current put into ampere’s equation.

\[
P_s = n_s k_b T
\]  \hspace{1cm} (2.18)

Equation (2.14) is substituted into equation (2.13) and is re-written as equation (2.19):

\[
\nabla \times B = \epsilon \mu \frac{dE}{dt} + \mu \sum_s q_s n_s U_s
\]  \hspace{1cm} (2.19)

Equation (2.18) is substituted into equation (2.17) and is re-written as equation (2.21).

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s U_s) = 0
\]  \hspace{1cm} (2.20)

\[
m_s n_s \frac{\partial U_s}{\partial t} = q_s n_s (E + U_s \times B) - k_b T \nabla n_s - m_s n_s \sum_{\alpha \neq s} v_{s\alpha} (U_s - U_\alpha)
\]  \hspace{1cm} (2.21)
Then these equations are discretized for a physical location and respective time at which the field values will be noted. These discretized equations help in solving the Maxwell’s equations numerically.

**Yee Cell:**

The various field components of E(Electric) and B(Magnetic) fields can be decomposed into their orthogonal components. These electric and magnetic components can then be allocated according to the Yee cell model[13].

![Extended Yee Cell](image)

**Figure 2.15: Extended Yee Cell (PF-FDTD cell)**

An extended Yee cell is used in the PF-FDTD model that centralizes all particles and prescribes the associated vector velocities directly to all the common particles. Then the spatial values are used to calculate comparable values where needed.

For minimizing the calculations for the time derivatives, the electric and magnetic fields are alternated every half time step. This technique is also called the Leap frog Scheme. A central difference technique helps the components to be spatially (Yee cell) and temporally (Leap frog) separated.
Figure 2.16: Timeline for one iteration (Leap frog scheme)

According to Ward in his paper “Numerical Model of Antenna in plasma”, When the PF-FDTD cell and the leap frog scheme technique are applied to vector components of the earlier discussed Maxwell’s equations, they can be re-written in the discrete form. A sample of these resulting Ampere’s and Faraday’s equations is (2.22) and equation (2.23).

\[(i,j,k)\]
\[= E_x^{t-1}(i,j,k)\]
\[+ \frac{dt}{\varepsilon_0 \mu_0} \left[ \frac{B_z^{t-\frac{1}{2}}(i,j+\frac{1}{2},k) - B_z^{t-\frac{1}{2}}(i,j-\frac{1}{2},k)}{dy} - \frac{B_y^{t-\frac{1}{2}}(i,j,k+\frac{1}{2}) - B_y^{t-\frac{1}{2}}(i,j,k-\frac{1}{2})}{dz} \right.\]
\[- \mu_0 \sum_s n_s(i,j,k) U_{xz}^s(i,j,k) + n_s(i-1,j,k) U_{xz}^s(i-1,j,k) \left. \right] \quad (2.22)\]

\[B_{x}^{t+\frac{1}{2}}(i,j,k) = B_{x}^{t-\frac{1}{2}}(i,j,k)\]
\[+ dt \left[ \frac{E_y^t(i,j,k+\frac{1}{2}) - E_y^t(i,j,k-\frac{1}{2})}{dz} \right.\]
\[- \frac{E_z^t(i,j+\frac{1}{2},k) - E_z^t(i,j-\frac{1}{2},k)}{dy} \left. \right] \quad (2.23)\]
With the help of these equations, electric fields and magnetic fields are calculated for the simulations done using this PF-FDTD model. Once all the equations for the parameters like the future velocity of the charged particles, future densities of the cells are all calculated. These equations are used for generating the PF-FDTD model. After the model is generated, different Plasma frequency ($f_p$), gyro frequency ($f_g$), temperature ($T$), collisional frequency ($f_c$) and other parameters and values are plugged into the model and simulated, and the input impedance curve is obtained by using MATLAB. Simulations generated from PF-FDTD model are used throughout this paper. The figure (2.17) was generated using the PF-FDTD model for analyzing the current distributions of an antenna immersed in cold collisional plasma. It can be seen from the figure that the PF-FDTD curve matches the flight data accurately at series resonance. At the parallel resonance the impedance curve matches the flight data but there is discrepancy in the amplitude and also the PF-FDTD has low data points. With close observation it can be noticed that the PF-FDTD matches the flight data at series, plasma, and parallel resonances. Figures (2.18), (2.19), (2.20) and (2.21) show the impedance plots for varying conditions of plasma similar to section 2.1 and section 2.2. Though the plots has changed based on the parameter changed with figure (2.18) as reference, there is no major change in the amplitude of the impedance plots unlike the analytical models.
Figure 2.17: Impedance of an antenna in cold plasma using PF-FDTD model for $F_p$ of 4.10 MHz, $F_g$ of 2.35 MHz, $F_{uh}$ of 4.85 Mhz against flight data sweep from ASSP flight data sweep.

Figure 2.18: Impedance of an antenna in plasma using PF-FDTD model for $F_p$ of 4.10 MHz, $F_g$ of 2.35 MHz, $\nu$ of 0.1% $F_p$. 
Figure 2.20: Impedance of an antenna in plasma using PF-FDTD model for $F_p$ of 4.10 MHz, $F_g$ of 3.35 MHz, $\nu$ of 0.1% $F_p$. (Change in the gyro frequency with respect to figure (2.18))

Figure 2.21: Impedance of an antenna in plasma using PF-FDTD model for $F_p$ of 4.10 MHz, $F_g$ of 2.35 MHz, $\nu$ of 0.4% $F_p$. (Change in the collisional frequency with respect to figure (2.18))
Chapter 3

Comparison

3.1 Flight Data

On 26th January 2015 NASA Auroral Space Structures Probe (ASSP) was flown into the Aurora to measure the temporal and spatial variation of energy occurring in the upper atmosphere and around the aurora[14]. The ASSP was flown into the space on a NASA ORION IV sounding rocket from poker flat research range in Alaska. Figure (3.1) shows the image of the sounding rocket with payloads at the poker flat research center. This ASSP mission was the first of its kind, started by NASA sounding rockets mission which made use of constellation of mini-pay loads for the research study. The electric and magnetic observations were made by seven payloads in rapid successions through same volumes. The data retrieved from these payloads will be combined with ground-based observations and studied.

Each of the payloads on the rocket carried a crossed pair of double pair sensors to measure the in-situ electric fields, and a Langmuir probe, GPS receiver, 3- axis magnetometer. The data is recorded from sounding rockets through ASSP. The payloads generated up-leg and down-leg RF probe impedance sweeps from 164 km up to 519 km. Complex impedance measurements at 128 frequencies between 1.6 MHz and 21.5 MHz were recorded and extracted from the data sets. This extracted dataset will be used and compared with the different analytical models and numerical model discussed in this paper and observations will be made.
3.2 Comparison of the models

In the chapter 2, the two analytical models (i.e) Balmain’s model of current distribution, Nikitin’s model of current distribution and PF-FDTD the numerical model were discussed. In this section of the chapter, these models will be compared against each other keeping the ASSP flight data as baseline reference for comparison. The analysis of the comparison will be done in two steps, a.) Observation and b.) Error calculation.

Analysis of Sweep number 2356:

In the figure (3.2), the flight data sweep is compared against the models for the plasma frequency($F_p$) of 4.7MHz, gyro frequency($F_g$) of 2.8Mhz, with upper hybrid($F_{uh}$) lying at 5.47MHz. In the observation stage, it can be seen that Nikitin’s assumption of current distribution’s impedance curve has a shift in the amplitude towards gyro frequency. The PF-FDTD’s impedance curve and the Balmain’s assumption of current distribution’s impedance curve match the flight data. In the next stage of observation of
the error calculation, we will compare PF-FDTD’s impedance curve, and Balmain’s impedance curve to the flight data as Nikitin’s impedance curve was eliminated in the observation stage of analysis.

Figure 3.2: Comparison of Impedance of an antenna in cold collisional plasma using different models for $F_p$ of 4.7 MHz, $F_g$ of 2.8 MHz, length(l) of 0.5m, radius(a) of 0.01m and flight data sweep(2356).

Figure 3.3: Error calculation for PF-FDTD’s curve, Balmain’s Curve against flight data.
<table>
<thead>
<tr>
<th>Error</th>
<th>Reference (Flight Data)</th>
<th>Balmain</th>
<th>PF-FDTD</th>
<th>Least Error</th>
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</thead>
<tbody>
<tr>
<td>Error 1</td>
<td>0.025</td>
<td>0.05</td>
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<td>Error 2</td>
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<td>0.025</td>
<td>Balmain/PF-FDTD</td>
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<td>0.025</td>
<td>Balmain/PF-FDTD</td>
</tr>
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</tr>
<tr>
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<td>PF-FDTD</td>
</tr>
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<td>Error 9</td>
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<td>Error 13</td>
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<td>PF-FDTD</td>
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<td>Error 15</td>
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<td>0.08</td>
<td>0.07</td>
<td>PF-FDTD</td>
</tr>
</tbody>
</table>

Table 3.1: Error points for analysis of sweep number 2356 shown in figure 3.3.

The error calculation step includes identification of error points in the figure and then least error impedance curve with respect to the flight data curve is identified. In the figure (3.3), there are fifteen errors at different data points for the impedance curve. The alternate PF-FDTD data points are considered as error data points and around cut-off frequency all the data points are considered. These errors are calculated with flight data.
curve as reference and the least error returning curve will be chosen. Also, these error data points will be observed around the same frequency range for all sweep observations for better results. Besides these fifteen error points, further observation is also used to analyze the better match to the flight data from the analytical and numerical models. For the error calculation, the x-axis and y-axis for the impedance curve has been divided into scale of 0 to 1.

**Sample Error data point calculation:**

**Error 4:** Error 4 data point lies around 3.5MHz. It lies in between the gyro frequency and plasma frequency. For this data point, the flight data lies at the 0.02 point of the y-axis. Hence, the Curve which is farther from 0.02 of the y-axis will have the greater error value.

- Balmain’s curve data point lies at 0.04 of the reference axis for error calculation.
  
  \[0.04 - 0.02 = 0.02\]. The error for Balmain is 0.02.

- PF-FDTD’s curve data point lies at 0.1 of the reference axis.
  
  \[0.1 - 0.02 = 0.08\]. The error for PF-FDTD is 0.08.

From the above error calculation and observation of the curves at data point of error 4 in the impedance section of the figure 3.3, Balmain’s curve for input impedance matches the flight data better at this point of the curve. In the similar manner, all the error data points are calculated and the curve with least error is picked. From table 3.1, it is evident that PF-FDTD matches the experimental flight data at maximum number of points than Balmain’s curve and hence for this frequency range PF-FDTD is a better match for flight data analysis than Balmain’s model.
Analyses of Sweep number 3854 and 3567:

Similar to the analysis of sweep number 2356, two more sweep analyses (Sweep 3854 and Sweep 3567) are presented in this paper. Figures (3.4) and (3.5) show the impedance plots of various models for sweep number 3854 and corresponding error details are shown in table (3.2). Similar to the sweep number 3854, figures (3.6) and (3.7) show the impedance plots for flight data sweep 3567 and the errors are tabulated in table (3.3).

**Figure 3.4:** Comparison of Impedance of an antenna in cold collisional plasma using different models with \( F_p \) of 4.25 MHz, \( F_g \) of 2.35 MHz, length \( l \) of 0.5m, radius \( a \) of 0.01m and flight data sweep(3854).

**Figure 3.5:** Error calculation for PF-FDTD’s curve, Balmain’s Curve against flight data.
<table>
<thead>
<tr>
<th>Error</th>
<th>Reference (Flight Data)</th>
<th>Balmain</th>
<th>PF-FDTD</th>
<th>Least Error</th>
</tr>
</thead>
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<td>0</td>
<td>0</td>
<td>Balmain/PF-FDT D</td>
</tr>
<tr>
<td>Error 2</td>
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<td>0</td>
<td>0</td>
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<td>Balmain</td>
</tr>
<tr>
<td>Error 4</td>
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<td>Balmain/PF-FDT D</td>
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</table>

Table 3.2: Error points for analysis of sweep number 3854 shown in figure 3.5.
Figure 3.6: Comparison of Impedance of an antenna in cold collisional plasma using different models with $F_p$ of 4.10 MHz, $F_g$ of 2.25 MHz, length (l) of 0.5m, radius (a) of 0.01m and flight data sweep(3567).

Figure 3.7: Error calculation for PF-FDTD’s curve, Balmain’s Curve against flight data.
<table>
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<tr>
<th>Error</th>
<th>Reference (Flight Data)</th>
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<th>Least Error</th>
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<td>0.04</td>
<td>0.05</td>
<td>Balmain/PF-FDTD</td>
</tr>
</tbody>
</table>

Table 3.3: Error points for analysis of sweep number 3567 shown in figure 3.7.

The observations were made for different sets of data and error points were calculated with flight data sweeps as reference points. The last column of the error table shows the curve with least error or the curve that is closer to flight data curve. It can be seen from the error tables of the three sweeps presented, that PF-FDTD’s impedance curve looks like a closer match to the flight data than Balmain’s impedance curve and Nikitin’s impedance curve. Though the Balmain’s curve matched the flight data sweeps at series
resonance (Fg) and after the cut-off frequencies (Fuh), it had discrepancies near the known plasma wave propagation regions, as discussed in section 2.1 of this paper. The Nikitin’s impedance curve looks like it follows the flight data near upper hybrid but has problems near the plasma frequencies. The PF-FDTD’s curve matches the Flight data more accurately then the Balmain’s curve and Nikitin’s curve by observing the plots and from the error calculation tables, but it can see noticed that the PF-FDTD’s amplitude has discrepancy from the flight data. This difference in the amplitude opens the discussion of the accuracy of PF-FDTD in matching the flight data sweeps. It can be also seen that there are no enough PF-FDTD data points around the region to justify the differences due to other influences. Discrepancies of the amplitudes show that though PF-FDTD has been accurate, there is a problem in matching the parameters with the flight data. This shows that there still remains a challenge to properly gauge the PF-FDTD parameters to match with flight data, which is not studied in this thesis.
Chapter 4

Current Distributions

The antenna current distributions are dominated by how energy couples into the plasma based upon the natural plasma resonances. As discussed earlier in this thesis paper, PF-FDTD is used for all the simulations required for impedance and current distribution analysis. The reason why PF-FDTD model is preferred over other models is that, PF-FDTD computes its current distribution self consistently while the current distributions in the Balmain’s model is assumed based upon the logical arguments. Through the discussions in chapter 3, it was identified that the self-consistency of the PF-FDTD enabled it to match the flight data better at all frequencies. Based on this argument, what follows is a study of current distributions of weakly magnetized plasma on 1m dipole antenna around and below the upper hybrid.

4.1 Current Distributions of Collisional Plasma using PF-FDTD

The interactions and reaction between plasma particles and the existing fields is one of the major properties of plasma. As the particles react with each other the words collision and interaction are used interchangeably.

There are two types of collisions known as elastic and inelastic collision. In inelastic collision the particles may be excited or re-combined, and the internal states of particles involved will be changed. Also, in inelastic collision electrons can be removed from the atom by increasing the energy state of electrons which will result in ionization. Whereas in elastic collision the internal states of particles involved are not changed. The collision assumed in this PF-FDTD model is elastic collision with the help of the momentum equation. The plasma frequency and the gyro frequency are kept constant while the collision frequency is varied keeping the temperature constant.
Figure 4.1: Impedance curve for varied collision frequencies with a gyro frequency (F\textsubscript{g}) of 2.8MHz, plasma Frequency (F\textsubscript{p}) of 4.7MHz and upper hybrid frequency lying at 5.47MHz.

In figure (4.1), three different collision frequencies (i.e) 2% F\textsubscript{p}, 10% F\textsubscript{p} and 20% F\textsubscript{p} have been simulated for plasma frequency of 4.7MHz, gyro frequency of 2.8MHz. It can be noted that the simulation data points are low near the upper hybrid frequency and hence lines had to be used along with data points to indicate the flow of impedance curve. In the observation of figure (4.1), we can see that the amplitude of lower collision frequency impedance is higher than the higher collision frequency impedance curve. The impedance curves for 10% F\textsubscript{p} and 20% F\textsubscript{p} almost overlap each other with slight variation in the beginning of the simulation. These variations of collision frequencies are further investigated with the help of current distribution curves for each collision variation.
Figure 4.2: Current distributions for 2% Fp collision frequency.

Analysis:

<table>
<thead>
<tr>
<th>Drive Frequency</th>
<th>0.86MHz</th>
<th>2.97MHz</th>
<th>4.8MHz</th>
<th>5.48MHz</th>
<th>5.94MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature</td>
<td>Triangular/exponential</td>
<td>Triangular</td>
<td>Exponential</td>
<td>Triangular/exponential</td>
<td>Triangular/exponential</td>
</tr>
<tr>
<td>Sym/asym</td>
<td>asymmetric</td>
<td>symmetric</td>
<td>symmetric</td>
<td>symmetric</td>
<td>Symmetric</td>
</tr>
</tbody>
</table>

Table 4.1: Current distribution analysis table for 2% Fp collision frequency.

For the analysis of these current distributions five different drive frequencies have been chosen. These drive frequencies are chosen based on the discrepancies in the impedance curve.

1. The first drive frequency is chosen before the gyro frequency at 0.86MHz. The current distribution at this frequency looks triangular in nature to some extent and also shows signs of exponential nature. The current distribution looks asymmetrical with 0.25 on the x-axis as the feed gap or center of the antenna. But this asymmetrical nature is assumed to be not caused by plasma effects.
2. The second drive frequency was chosen between the gyro frequency and the plasma frequency around 2.97MHz on the impedance curve. The 2.97MHz drive frequency current distributions in figure 4.2 show that it is triangular in nature. It looks symmetrical on either side from the center.

3. The third drive frequency is chosen between plasma frequency and the upper hybrid frequency and looks symmetrical on either side of the antenna. The current distribution exhibits exponential nature.

4. The fourth drive frequency is chosen just at the upper hybrid frequency at 5.48MHz. It exhibits both exponential nature and triangular nature. The triangular current distribution pattern is observed closed to the center of the antenna. It looks symmetrical on both sides of the antenna.

5. The last drive frequency is identified above the upper hybrid frequency at 5.94MHz. It also exhibits exponential and a little triangular close to the center of the antenna. It looks symmetrical on both sides of the antenna.

From the above analysis we can observe that current distributions for 2% Fp collision frequency exhibit both triangular and exponential nature telling us about the presence of both standing wave and evanescent wave.

The similar analysis technique has been applied for 10% Fp collisional frequency and 20% Fp collisional frequency. The current distributions are plotted in figures (4.2) and (4.3). The analysis for these two plots are tabulated in tables (4.2) and (4.3) with the observation notes in the comments row of the table.
Figure 4.3: Current distributions for 10% Fp collision frequency.

<table>
<thead>
<tr>
<th>Drive Frequency</th>
<th>0.86MHz</th>
<th>2.97MHz</th>
<th>4.8MHz</th>
<th>5.48MHz</th>
<th>5.94MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature</td>
<td>Triangular</td>
<td>Triangular</td>
<td>Exponential</td>
<td>Triangular/exponential</td>
<td>Triangular/exponential</td>
</tr>
<tr>
<td>Sym/asym</td>
<td>asymmetric</td>
<td>symmetric</td>
<td>symmetric</td>
<td>symmetric</td>
<td>symmetric</td>
</tr>
<tr>
<td>Comments</td>
<td>Abnormal distribution</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
</tr>
</tbody>
</table>

Table: 4.2: Current distribution analysis table for 10% Fp collision frequency.

The abnormal distribution near the series resonance (0.86MHz) is assumed not to be the effects of plasma.

<table>
<thead>
<tr>
<th>Drive Frequency</th>
<th>0.86MHz</th>
<th>2.97MHz</th>
<th>4.8MHz</th>
<th>5.48MHz</th>
<th>5.94MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature</td>
<td>Triangular</td>
<td>Triangular</td>
<td>Exponential</td>
<td>Triangular/exponential</td>
<td>Triangular/exponential</td>
</tr>
<tr>
<td>Sym/asym</td>
<td>asymmetric</td>
<td>symmetric</td>
<td>symmetric</td>
<td>symmetric</td>
<td>symmetric</td>
</tr>
<tr>
<td>Comments</td>
<td>Abnormal distribution</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
</tr>
</tbody>
</table>

Table: 4.3: Current distribution analysis table for 20% Fp collision frequency.

The abnormal distribution near the series resonance (0.86MHz) is assumed not to be the effects of the plasma.
Figure 4.4: Current distributions for 20% Fp collision frequency.

The observations made from the above analyses are:

1. The varied collision frequencies have slight differences in the amplitude near the parallel resonance of the impedance curve based on the percentage of collision frequencies but they tend to have similar trend in the impedance curves.

2. The current distribution observations show both triangular nature and exponential nature of current distributions through PF-FDTD model which was already shown by J.Ward[11].

3. The drive frequencies close to (series resonance) gyro frequencies show triangular nature for all three collision frequencies.

4. The drive frequencies close to plasma frequencies for all the analyses show exponential nature.

5. The drive frequencies close to (cut-off frequencies) upper hybrid and above upper hybrid exhibit both exponential and triangular nature.

6. Much of collision effects could not be seen for the above range of frequencies.
4.2 Possible current distributions experienced on ASSP flight

In the previous section, we have seen the effects of collision frequency on the plasma frequency in weakly magnetized plasma. In this section, we will study the current distributions possibly experienced by ASSP flight. As we have seen in section 2 of chapter 3 that the PF-FDTD model better matches the flight data for analysis than the other analytical models, we will use PF-FDTD model to analyze the currents on the ASSP flight. As 3 different sweeps were used in the comparison of the models, the same sweeps will be studied.

Figure 4.5: Current distributions for sweep number 2356 of ASSP flight, based on the PF-FDTD simulation for F_P of 4.7 MHz, F_G of 2.8 MHz, F_uh of 5.47MHz.
Table 4.4: Current distributions analysis table for flight data sweep 2356.

<table>
<thead>
<tr>
<th>Drive Frequency</th>
<th>2.2MHz</th>
<th>2.85MHz</th>
<th>4.91MHz</th>
<th>5.37MHz</th>
<th>5.71MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature</td>
<td>Triangular</td>
<td>Triangular</td>
<td>Exponential</td>
<td>Triangular/exponential</td>
<td>Triangular/exponential</td>
</tr>
<tr>
<td>Sym/asym</td>
<td>symmetric</td>
<td>symmetric</td>
<td>asymmetric</td>
<td>asymmetric</td>
<td>symmetric</td>
</tr>
<tr>
<td>Comments</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Slight difference in the curve on either sides</td>
<td>Difference in the curve on either sides</td>
<td>Normal distribution</td>
</tr>
</tbody>
</table>

The PF-FDTD simulation used for analysis of this sweep has plasma frequency (Fp) of 4.7MHz and gyro frequency (Fg) of 2.8MHz. These values are chosen from figure 3.3, as these were the PF-FDTD values which matched the ASSP flight data sweep number 2356. Four drive frequencies were chosen for this analysis and these four drive frequencies lie closer to the error points used in the figure 3.4. 0.25 on the x-axis of figure 4.5 is the feed gap or center of the antenna that is used. The upper hybrid frequency lies at 5.47MHz.

**Analysis:**

1. The first drive frequency lies around 2.2MHz, which is the region below gyro frequency. The current distribution at this region appears to be triangular in nature. The distribution tends to be symmetrical on either sides of the antenna with feed gap in the center.

2. The second drive frequency is in the region close to gyro frequency at 2.85MHz.
The current distribution at this region appears to be triangular and symmetric on both sides of the antenna.

3. The third drive frequency appears to be in between plasma frequency and the upper hybrid frequency or just near the plasma frequency of 4.91MHz. Exponential nature of current distribution is observed here and the curve has differences on the either sides of the antenna slightly.

4. The fourth drive frequency lies just at the upper hybrid frequency of 5.37MHz. The current distribution at this region also has both exponential and triangular nature. The current distribution appears asymmetrical for this drive frequency. The curve on the right side of the antenna shows a greater decay, showing the possible effects of plasma.

5. The last drive frequency lies above upper hybrid frequency and it exhibits both triangular and exponential nature of distribution. It is symmetric on both sides of the antenna.

This observation tells that the current distributions possibly experienced in this sweep have both triangular and exponential nature. The drive frequency closer to upper hybrid frequency appears to show possible plasma effect.

The similar analysis for the current distributions experienced by the flight data sweeps 3567 and 3854 has been plotted in figures 4.6 and 4.7 and has been tabulated in their respective table number 4.5 and 4.6. The observations from all the three flight data sweeps has been presented at the end of this section.
Figure 4.6: Current distributions for sweep number 3567 of ASSP flight, based on the PF-FDTD simulation for with $F_p$ of 4.10 MHz, $F_g$ of 2.25 MHz, $F_{th}$ of 4.6MHz, length (l) of 0.5m, radius (a) of 0.01m.

<table>
<thead>
<tr>
<th>Drive Frequency</th>
<th>2.05MHz</th>
<th>3.43MHz</th>
<th>4.11MHz</th>
<th>4.68MHz</th>
<th>4.91MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature</td>
<td>Exponential</td>
<td>Exponential</td>
<td>Exponential</td>
<td>Triangular/ exponential</td>
<td>Triangular/ exponential</td>
</tr>
<tr>
<td>Sym/asym</td>
<td>symmetric</td>
<td>symmetric</td>
<td>asymmetric</td>
<td>symmetric</td>
<td>symmetric</td>
</tr>
<tr>
<td>Comments</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Difference in the curve on either sides</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
</tr>
</tbody>
</table>

Table: 4.5: Current distribution analysis table for flight data sweep 3567.
Figure 4.7: Current distributions for sweep number 3854 of ASSP flight, based on the PF-FDTD simulation for $F_p$ of 4.25 MHz, $F_g$ of 2.35 MHz, $F_{uh}$ of 4.85 MHz, length ($l$) of 0.5m, radius ($a$) of 0.01m.

<table>
<thead>
<tr>
<th>Drive Frequency</th>
<th>2.05MHz</th>
<th>3.43MHz</th>
<th>4.34MHz</th>
<th>4.80MHz</th>
<th>5.26MHz</th>
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</thead>
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<tr>
<td>Nature</td>
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<td>Exponential</td>
<td>Triangular/exponential</td>
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<td>Sym/asym</td>
<td>symmetric</td>
<td>symmetric</td>
<td>asymmetric</td>
<td>symmetric</td>
<td>symmetric</td>
</tr>
<tr>
<td>Comments</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
<td>Difference in the curve on either sides</td>
<td>Normal distribution</td>
<td>Normal distribution</td>
</tr>
</tbody>
</table>

Table: 4.6: Current distribution analysis table for flight data sweep 3854.
Observation: From the analysis of this section, it can be observed that the ASSP flight data has experienced possibly both the triangular nature of current distribution and the exponential nature, which was also proposed by PF-FDTD model that current distributions on antenna in plasma exhibit both triangular and exponential nature in section 4.1.

1. The drive frequencies near the gyro frequency (series resonance) of figure 4.5, shows that current distribution experiences triangular nature of distribution. But in figures 4.6 and 4.7 it can be noticed that they slightly experience exponential current distribution.

2. Near the known plasma frequencies, the figures 4.5 through 4.7, show that flight data have experienced exponential nature of current distributions. Also the variations in the current distributions on either sides of the antenna are assumed to be the effects of plasma oscillations.

3. Near the cut-off frequencies, the current distributions experienced by the flight data sweeps can be noticed as both exponential and triangular distribution in nature.

4. After the cut-off frequencies, the curves show that they still experience both triangular and exponential form of current distributions.

5. It is also observed that as the flight data sweep numbers increase (i.e) from 2356 to 3567 to 3854, the range of plasma frequency parameters decrease as seen in the above curves.
Chapter 5
Summary

5.1 Conclusion of the thesis

In this thesis, Balmain’s analytical model and Nikitin’s analytical model for current distributions on an antenna in plasma were compared with PF-FDTD model of current distribution which is the numerical model. It was found that PF-FDTD model is a better suitable model for analyzing current distributions than the other analytical models. This statement is backed up with the analysis and observations made in section 2 of chapter 3, in which ASSP flight data sweeps were used as references for the comparison of the models. After the best suitable model was chosen, the possible current distributions experienced by ASSP flight data sweeps were studied using the self-consistent PF-FDTD model in section 2 of chapter 4. It was observed that the current distributions experienced by the ASSP flight data sweeps were of both triangular and exponential in nature. This dual nature of current distributions was observed to be more prevalent near the cut-off frequencies. This dual nature of current distribution is a contradiction to Balmain’s theory or Staras theory which assumes triangular distribution of current or exponential distribution of current respectively. The PF-FDTD model was further used to study the current distributions of weakly magnetized collisional frequency. This was accomplished by using varied collision frequencies keeping the other plasma parameters constant. The simulations with varied collision frequencies also showed that the current distributions on an antenna in plasma have both triangular and exponential nature of distributions. Much of collisional effects could not be observed or studied with the current version of PF-FDTD model.
5.2 Future work

The arguments made in this thesis paper and the work done has opened the gates for more research in different directions. It was observed in chapter 3 that though PF-FDTD model matches the flight data better than the other models of current distributions, there still remains a challenge to match them accurately. One of the possible suggested future works would be to properly gauging the PF-FDTD parameters to match the flight data sweeps. The other possible future work would be to improve the stability of the model for collisional frequency effects and higher frequency ranges. These are some of the future work insights observed during this thesis paper.
References:


