Preface

This text was written for the three-credit trigonometry course at Grand Valley State University (MTH 123 – Trigonometry). This text begins with a circular function approach to trigonometry and transitions to the study of triangle trigonometry, vectors, trigonometric identities, and complex numbers.

The authors are very interested in constructive criticism of the textbook from the users of the book, especially students who are using or have used the book. Please send any comments you have to

trigtext@gmail.com

Important Features of the Textbook

This book is meant to be used and studied by students and the important features of the textbook were designed with that in mind. Please see the *Note to Students* on page (v) for a description of these features.

Content and Organization

The first two chapters of the textbook emphasize the development of the cosine and sine functions and how they can be used to model periodic phenomena. The other four trigonometric functions are studied in Section 1.6 and Section 2.4. Triangles and vectors are studied in Chapter 3, trigonometric identities and equations are studied in Chapter 4, and finally, using trigonometry to better understand complex numbers is in Chapter 5. Following is a more detailed description of the sections within each chapter.

Chapter 1 – The Trigonometric Functions

Section 1.1 introduces the unit circle and the wrapping function for the unit circle. This develops the important relationship between the real numbers and points on

the unit circle, which leads to the idea of associating intervals of real numbers with arcs on the unit circle. This is necessary for the development of the cosine and sine functions in Section 1.2. Understanding the ideas in this section is critical for proceeding further in the textbook.

The next two sections are intended to provide a rationale as to why we use radian measure in the development of the trigonometric functions. In addition, calculators and graphing devices are ubiquitious in the study of mathematics now, and when we use a calculator, we need to set the angle mode to radians. One of the purposes of Section 1.3 is to explain to students why we set our calculators to radian mode. It seems somewhat intellectually dishonest to simply tell students that they must use radian mode and provide no explanation as to why. Section 1.4 can be considered an optional section since it is not used later in the textbook. However, it does provide interesting applications of the use of radian measure when working with linear and anglular velocity.

The common arcs $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$ are introduced in Section 1.5. The exact values of the cosine and sine functions for these arcs are determined using information about the right triangle with two 45° angles and the right triangle with angles of 30° and 60°. An alternate development of these results using points on the unit circle and the distance formula is given in Exercises (9) and (10). Section 1.5 concludes with a discussion of the use of reference arcs, and Section 1.6 introduces the tangent, secant, cosecant, and cotangent functions.

Chapter 2 – Graphs of the Trigonometric Functions

The first three sections of this chapter deal with the graphs of sinusoidal functions and their use in modeling periodic phenomena. The graphs of the cosine and sine functions are developed in Section 2.1 using the unit circle. Geogebra applets are used in this development. Section 2.2 deals with the graphs of sinusoidal functions of the form $y = A\sin(B(x-C))+D$ or $y = A\cos(B(x-C))+D$. In this section, it is emphasized that the amplitude, period, and vertical shift for a sinusoidal function is independent of whether a sine or cosine is used. The difference in using a sine or cosine will be the phase shift. Sinusoidal models of periodic phenomena are discussed in Section 2.3. With the use of technology, it is now possible to do sine regressions. Although the textbook is relatively independent of the choice of technology, instructions for doing sine regressions using Geogebra are given in this section.

The graphs of the other four trigonometric functions are developed in Section 2.4. Most of this section can be considered as optional, but it is important to

at least discuss the material related to the graph of the tangent function since the inverse tangent function is part of Section 2.5. The inverse sine functionand inverse cosine function are, of course, also developed in this section. In order to show how inverse functions can be used in mathematics, solutions of trigonometric equations are studied in Section 2.6.

Chapter 3 – Triangles and Vectors

This chapter contains the usual material dealing with triangle trigonometry including right triangle trigonometry, the Law of Sines and the Law of Cosines, which are both handled in Section 3.3. The emphasis in this section is how to use these two laws to solve problems involving triangles. By having them both in the same section, students can get practice deciding which law to use for a particular problem. The proofs of these two laws are included as appendices for Section 3.3.

More work with the Law of Cosines and the Law of Sines is included in Section 3.4. In addition, this section contains problems dealing with the area of a triangle including Heron's formula for the area of a triangle. (The proof of Heron's formula is also in an appendix at the end of the section.)

The last two sections of this chapter deal with vectors. Section 3.5 deals with the geometry of vectors, and Section 3.6 deals with vectors from an algebraic point of view.

Chapter 4 – Identities and Equations

The first section of this chapter introduces the concept of a trigonometric identity. The emphasis is on how to verify or prove an identity and how to show that an equation is not an identity. The second section reviews and continues the work on trigonometric equations from Section 2.6.

The last three sections of the chapter cover the usual trigonometric identities in this type of course. In addition, the sections show how identities can be used to help solve equations.

Chapter 5 – Complex Numbers

It is assumed that students have worked with complex numbers before. However, Section 5.1 provides a good summary of previous work with complex numbers. In addition, this section introduces the geometric representation of complex numbers in the complex plane. Section 5.2 introduces the trigonometric or polar form of a complex number including the rules for multiplying and dividing complex numbers

in trigonometric form. Section 5.3 contains the material dealing with DeMoivre's Theorem about the powers of complex numbers and includes material on how to find roots of complex numbers.