# Appendix B

# Answers and Hints to Selected Exercises

Section 1

1. (a) Consider the equations

$$b_1x_1 + b_2x_2 + b_3x_3 = s,$$

and

$$k(b_1x_1 + b_2x_2 + b_3x_3) = ks.$$

   (b) Similar to part (a)

3. A cup of coffee costs \$1.95, a muffin costs \$2.05, and a scone costs \$2.15.

5. (a)

$$\begin{aligned} x_1 &= \frac{x_2 + 200 + 0 + 0}{4} \\ x_2 &= \frac{x_1 + x_3 + 200 + 0}{4} \\ x_3 &= \frac{x_2 + 0 + 400 + 0}{4} \end{aligned}$$

   (b) $x_1 = 75$, $x_2 = 100$ and $x_3 = 125$.

7. (a)

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1. \end{aligned}$$

   (b) Impossible.

   (c) Impossible.

   (d)

$$\begin{aligned} x_1 + \ \ x_2 + \ \ x_3 &= 0 \\ 2x_1 + 2x_2 + 2x_3 &= 0. \end{aligned}$$

9. (a) This system has the solution $x_1 = \frac{h-10}{3h-5}$ and $x_2 = \frac{5}{3h-5}$ as long as $h \neq \frac{5}{3}$.

   (b) When $h = \frac{5}{3}$, $x_1 = 2 - \frac{5}{3}x_2$ and $x_2$ is free.

11. (a) F
    (c) F
    (e) F

Section 2

1. (a) $h = 6$

   (b) $h = 6$ and $k \neq -2$.

   (c) $h = 6$ and $k = -2$.

3.

$$\begin{aligned} x_1 + x_2 \qquad &= \ \ 1 \\ -x_1 + x_2 \qquad &= -3 \\ x_1 + x_2 + x_3 &= \ \ 1. \end{aligned}$$

5. Student 2's hunch is correct.

6. (a) F
   (c) T
   (e) T

(g) F

(i) F

(k) F

Section 3

1. Augmented matrix:
$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 4 \\ 1 & 2 & 2 & 3 \\ 2 & 3 & -3 & 11 \end{array}\right],$$
solution $x_1 = 1$, $x_2 = 2$, and $x_3 = -1$

3. Row operations show this.

5. $x_1 = -4 - 6x_3$, $x_2 = -3 - 4x_3$, $x_3$ is free, $x_4 = 3$

7. (a) Yes

   (b) $\left[\begin{array}{cc|c} \blacksquare & * & * \\ 0 & \blacksquare & * \end{array}\right], \left[\begin{array}{cc|c} 0 & \blacksquare & * \\ 0 & 0 & * \end{array}\right]$

9. This is not possible.

11. $\left[\begin{array}{cc} \blacksquare & * \\ 0 & * \end{array}\right]$ and $\left[\begin{array}{cc} 0 & * \\ 0 & 0 \end{array}\right]$.

12 (a) T

(c) T

(e) F

(g) T

(i) F

(k) F

Section 4

1. $\mathbf{w} = \left(-\frac{9}{4}\right)\mathbf{u} + \left(\frac{7}{4}\right)\mathbf{v}$

3. Only when $w_1 = 2w_2 - 3w_3$. $0\mathbf{u} + 0\mathbf{v} = \mathbf{0}$

5. (a) We cannot make the desired solution.

   (b) We can make the desired chemical solution with $\frac{3}{4}$ of solution $\mathbf{v}_1$ for every $\frac{1}{4}$ of solution $\mathbf{v}_2$.

   (c) If $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ can be made from our original two chemical solutions, then $w_3 + \frac{3}{23}w_1 - \frac{7}{23}w_2 = 0$.

7. (a) Span$\{\mathbf{v}_1\}$ is the line in $\mathbb{R}^2$ through the origin and the point $(1,1)$.

   (b) Span$\{\mathbf{v}_1, \mathbf{v}_3\}$ is the plane in $\mathbb{R}^3$ through the origin and the points $(1,1,1)$ and $(2,0,1)$.

9. Yes.

11. (a) F

(c) T

(e) T

(g) F

(i) T

(k) T

(m) F

Section 5

1. The matrix-vector form of the system is
$$A\mathbf{x} = \mathbf{b}, \text{ where } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

3. The system corresponding to this matrix-vector equation is
$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 4 \\ x_1 - 2x_2 + 3x_3 &= -6. \end{aligned}$$
The solution to the system is
$$\begin{aligned} x_1 &= -\frac{17}{7}x_3 - \frac{10}{7} \\ x_2 &= \frac{2}{7}x_3 + \frac{16}{7} \end{aligned}$$
$x_3$ is free.

5. Both methods give
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1a + x_2b \\ x_1c + x_2d \end{bmatrix}.$$

7. (a) $b_3$

   (b) If $b_3 = 9$, then $a = 2$.

9. $A \begin{bmatrix} -1 \\ 6 \\ -5 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$.

11. The general solution to the system is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}.$$

The solution is the plane in $\mathbb{R}^3$ through the points $(3, 0, 0)$, $(5, 1, 0)$, and $(2, 0, 1)$.

13. $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

14. (a) F
    (c) T
    (e) T
    (g) T
    (i) T
    (k) T

Section 6

1. Linearly dependent. Change the last entry in $\mathbf{v}_3$ to a 0.

3. Only two of the solutions are necessary.

5. (a) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

   (b) $\mathbf{u} = \mathbf{v}_1 - 2\mathbf{v}_2 + 3\mathbf{v}_3$

7. This is not possible.

9. (a) F
   (c) T
   (e) T
   (g) T
   (i) T
   (k) T

Section 7

1. Ff $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then $T(\mathbf{x}) = \begin{bmatrix} x_1 + 2x_2 + x_3 \\ x_1 - 3x_3 \end{bmatrix}$.

3. $\begin{bmatrix} 12 \\ -11 \end{bmatrix}$.

5. $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

7. $T$ defined by $T(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}$

9. Not possible.

11. $L = \mathbf{v} + c\mathbf{w}$, $T(L) = T(\mathbf{v} + c\mathbf{w}) = T(\mathbf{v}) + cT(\mathbf{w})$

12. (a) F
    (c) F
    (e) T
    (g) F
    (i) F
    (k) F
    (m) T
    (o) T

Section 8

1. (a) $AB = \begin{bmatrix} a & b \\ c & d \\ 0 & 0 \end{bmatrix}$

   (b) $AB = [-2 \ -2]$.

3. (a) $X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

   (b) There is no matrix $X$ with this property.

(c) $X = \begin{bmatrix} 2x_{21} & 1+2x_{22} \\ x_{21} & x_{22} \end{bmatrix}$, where $x_{21}$ and $x_{22}$ can be any scalars

5. $A^m\mathbf{v} = 2^m\mathbf{v}$

7. $A^2 = 0$ and $B^3 = 0$

9. (a) $[a_{ij} + b_{ij}] = [b_{ij} + a_{ij}]$
   (c) $[a_{ij} + 0] = [a_{ij}]$
   (e) $[(a+b)a_{ij}] = [aa_{ij}] + [ba_{ij}]$
   (g) $[(ab)a_{ij}] = a[ba_{ij}]$

11. (a) Let $A^{\mathsf{T}} = [a'_{ij}]$. Then $a'_{ij} = a_{ji}$ by definition of the transpose. Let $\left(A^{\mathsf{T}}\right)^{\mathsf{T}} = [a''_{ij}]$. Then $a''_{ij} = a'_{ji} = a_{ij}$. So the $ij$th entry of $\left(A^{\mathsf{T}}\right)^{\mathsf{T}}$ is the same as the $ij$th entry of $A$, and we conclude that $\left(A^{\mathsf{T}}\right)^{\mathsf{T}} = A$.
    (c) The $ij$th entry of $aA$ is $aa_{ij}$, so the $ij$th entry of $(aA)^{\mathsf{T}}$ is $aa_{ji}$. But $aa_{ji}$ is also the $ij$th entry of $aA^{\mathsf{T}}$. We conclude that $(aA)^{\mathsf{T}} = aA^{\mathsf{T}}$.

13. If $A = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$, $B = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}$, then $AB$ equals $\begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

14. (a) F
    (c) F
    (e) F
    (g) T

## Section 9

1. (a) Eigenvector with eigenvalue $-1$.
   (b) Eigenvector with eigenvalue 3.
   (c) Eigenvector with eigenvalue 2.

3. (a) Eigenvalue
   (b) Eigenvalue
   (c) Not an eigenvalue
   (d) Not an eigenvalue

5. (a) $a_3 = 804,000$ $s_3 = 196,000$, $a_4 = 782,400$, $s_4 = 217,600$
   (b) $a_{k+1} = 0.9a_k + 0.3s_k$, $s_{k+1} = 0.1a_k + 0.7s_k$
   (c) $\begin{bmatrix} a_{k+1} \\ s_{k+1} \end{bmatrix}$ equals $\begin{bmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} a_k \\ s_k \end{bmatrix}$

6. (a) F
   (c) T
   (e) T
   (g) T
   (i) F
   (k) T

## Section 10

1. $C = (I)C = C = (BA)C = BAC = B(AC) = B(I) = B$

3. (a) $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -k \\ 0 & 1 \end{bmatrix}$
   (b) Row reduce $[A \mid I_2]$
   (c) Row reduce $[A \mid I_3]$ to see that $A^{-1} = \begin{bmatrix} 1 & -k & mk-\ell \\ 0 & 1 & -m \\ 0 & 0 & 1 \end{bmatrix}$.

5. $c \neq -4$

8. (a) T
   (c) T
   (e) T
   (g) T
   (i) F

## Section 11

1. $A$ is invertible as long as $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is not in the plane in $\mathbb{R}^3$ through the origin and the points $(1, -1, 1)$ and $(2, 1, 1)$

3. (a) T
   (c) T
   (e) T
   (g) T

Section 12

1. (a) Closed under addition, not closed under multiplication by scalars, contains the zero vector.
   (b) Not closed under addition, closed under multiplication by scalars, contains the zero vector.
   (c) Not closed under addition, not closed under multiplication by scalars, does not contain the zero vector.
   (d) Not closed under addition, closed under multiplication by scalars, contains the zero vector.

3. The only such set is the empty set.

5. $\mathbb{R}^2$.

7. (a) As an example, let $\mathbf{v} = [2\ 1]^{\mathsf{T}}$ in $\mathbb{R}^2$.
      i. $I_2$
      ii. $2I_2$
      iii. $[3\mathbf{e}_1\ \ -\mathbf{e}_2]$
      iv. $\left[\frac{a}{2}\mathbf{e}_1\ b\mathbf{e}_2\right][2\ \ 1]^{\mathsf{T}} = a\mathbf{e}_1 + b\mathbf{e}_2 = [a\ b]^{\mathsf{T}}$
   (b) $\mathbf{w}_1 + \mathbf{w}_2 = A_1\mathbf{v} + A_2\mathbf{v} = (A_1 + A_2)\mathbf{v}$, $c\mathbf{w}_1 = c(A_1\mathbf{v}) = A_1(c\mathbf{v})$, $0\mathbf{v} = \mathbf{0}$.
   (c) $\{\mathbf{0}\}$, $\mathbb{R}^m$

9. $W_1 \cap W_2$ is a subspace of $\mathbb{R}^n$, $W_1 \cup W_2$ is not in general a subspace of $\mathbb{R}^n$

11. Not in general a subspace of $\mathbb{R}^n$.

13. (a) F
    (c) T
    (e) T
    (g) F
    (i) T
    (k) F

Section13

1. A basis for Col $A$ is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \right\}$, a basis for Nul $A$ is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

3. $a = -3$ and $b = 0$

5. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

7. Not possible

8. (a) T
   (c) T
   (e) T
   (g) F
   (i) F

Section 14

1. (a) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$
   (b) $\left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$
   (c) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$
   (d) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$

(f) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. $a = 0$, $b = 4$

5.

(a) Eigenvalue 0, eigenspace $\mathbb{R}^2$

(b) Eigenvalue 0, eigenspace $\mathbb{R}^n$

6. (a) F

(c) F

(e) T

(g) T

Section 15

1. (a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$, $\dim(\text{Col } A) = 3$, $\dim(\text{Nul } A) = 2$

(b) $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

3. (a) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$, $\text{rank}(A) = 1$

(b) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, $\text{nullity}(A) = 3$

(c) $\text{rank}(A) + \text{nullity}(A) = 1 + 3 = 4$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix} \right\}$, dimension 1

5. (a) What are the solutions to $A\mathbf{x} = \mathbf{0}$?

(b) Use part (a) and the Rank-Nullity Theorem.

7. $\dim(W)$ can be 0, 1, 2, 3, or 4 corresponding to $\{\mathbf{0}\}$, a line through the origin in $\mathbb{R}^4$, a plane containing the origin in $\mathbb{R}^4$, a copy of $\mathbb{R}^3$ through the origin in $\mathbb{R}^4$, $\mathbb{R}^4$

9. $\dim(\text{Col } A) = 3$, $\dim(\text{Nul } A) = 2$

11.

(a) Not possible.

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

13. (a) F

(c) F

(e) T

(g) T

(i) F

(k) T

Section 16

1. $\det(2A) = (8a_{11})\left[(a_{22})(a_{33}) - (a_{23})(a_{32})\right] - (8a_{12})\left[(a_{21})(a_{33}) - (a_{23})(a_{31})\right] + (8a_{13})\left[(a_{21})(a_{32}) - (a_{22})(a_{31})\right] = 8\det(A)$.

3. (a) $\det(A^2) = \det(AA) = \det(A)\det(A) = [\det(A)]^2$.

(b) $\det(A^k) = \det(AA^{k-1}) = \det(A)[\det(A)]^{k-1} = [\det(A)]^k$.

(c) Yes.

5. $(\det(A))^2$

7. (a) F

(c) F

(e) T

(g) F

(i) T

(k) F

Section 17

1. (a) i. $\det(B - \lambda I_2) = (1 - \lambda)(-2-\lambda) - 2 = \lambda^2 + \lambda - 4$

ii. $B^2 + B - 4I_2 = 0$.

(b) $\det(A - AI_n) = \det(0) = 0$.

3. (a) Eigenvalues 0, 3; algebraic multiplicities 2, 1; geometric multiplicities 2,1

(b) Eigenvalues 1, 2; algebraic multiplicities 1, 2; geometric multiplicities 1,1

5. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

7. (a) F

(c) T

(e) F

(g) T

(i) T

Section 18

1. (a) Not diagonalizable.

(b) Diagonalizable by $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$, $P_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $P_2 = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, $D_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

5. Yes

7.

(a) Eigenvalues, 1; eigenvectors : Span$\{[1\ 0]^{\mathsf{T}}$

(b) Diagonalizable

9. (a) The $ii$ entry of $RS$ is $r_{i1}s_{1i} + r_{i2}s_{2i} + \cdots + r_{in}s_{ni}$. The $jj$ entry of $SR$ is $s_{j1}r_{1j} + s_{j2}r_{2j} + \cdots + s_{jn}r_{nj}$. Sum as $i$ and $j$ go from 1 to $n$.

(b) i. trace$(D)$
$= \text{trace}\left(P^{-1}(AP)\right)$
$= \text{trace}\left((AP)P^{-1}\right)$
$= \text{trace}(A)$

ii. trace$(A) = \text{trace}(D) = \sum_{i=1}^{n} \lambda_i$

11. (a) $e^A = \begin{bmatrix} e & e-1 \\ 0 & 1 \end{bmatrix}$

(b) $e^B = I_2 + B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

(c) $e^{A+B} = \begin{bmatrix} e & 0 \\ 0 & 1 \end{bmatrix}$

(d) No

13. (a)

$$\begin{aligned} e^A &= e^{PDP^{-1}} \\ &= \sum_{k\geq 0} \frac{1}{k!}\left(PDP^{-1}\right)^k \\ &= \sum_{k\geq 0} \frac{1}{k!} PD^kP^{-1} \\ &= P\left(\sum_{k\geq 0} \frac{1}{k!} D^k\right)P^{-1} \\ &= Pe^DP^{-1}. \end{aligned}$$

(b)

$$\begin{aligned} \det\left(e^A\right) &= \det\left(Pe^DP^{-1}\right) \\ &= \det\left(e^D\right) \\ &= e^{\lambda_1}e^{\lambda^2}\cdots e^{\lambda_n} \\ &= e^{\lambda_1+\lambda^2+\cdots+\lambda_n} \\ &= e^{\text{trace}(A)}. \end{aligned}$$

14. (a) T
    (c) F
    (e) T
    (g) F
    (i) T

Section 19

1. (a) Eigenvalues: 3 and $-1$; T$[1\ -1]^{\mathsf{T}}$ is an eigenvector for $A$ with eigenvalue $-1$ and $[1\ 1]^{\mathsf{T}}$ is an eigenvector for $A$ with eigenvalue 3

   (b) As $k$ increases, the vectors $A^k\mathbf{x}_0$ are approaching the vector $[1\ 1]^{\mathsf{T}}$, which is a dominant eigenvector of $A$.

   (c) The Rayleigh quotients $r_k = \frac{\mathbf{x}_{k+1}\cdot\mathbf{x}_k}{\mathbf{x}_k\cdot\mathbf{x}_k}$ approach the dominant eigenvalue 3.

   (d) Apply the power method to $B = (A - 0I_2)^{-1} = A^{-1}$. As $k$ increases, the vectors $B^k\mathbf{x}_0$ are approaching the vector $\frac{1}{2}[1\ -1]^{\mathsf{T}}$, which is an eigenvector of $A$.

   The Rayleigh quotients approach the other eigenvalue $-1$ of $A$.

3. $[1\ 1]^{\mathsf{T}}$ is an eigenvector for $A$ with eigenvalue 1, so the vectors $A^k\mathbf{x}_0$ are all equal to $\mathbf{x}_0$. We can adjust the seed to a non-eigenvector.

5. 8 is the dominant eigenvalue of $A$

7. (a) Since $\mathbb{R}^n$ has dimension $n$, it follows that any set of $n+1$ vectors is linearly dependent.

   (b) Proceed down the list $c_{n-1}$, $c_{n-2}$, etc., until you reach a weight that is non-zero.

   (c) $\mathbf{0} = q(A)\mathbf{v} = (A - \lambda I_n)Q(A)\mathbf{v}$

   (d) i. $q(t) = 24t - 10t^2 + t^3$

   ii. 0, 4, and 6

   iii. For $t = 0$, we have $Q(t) = (4-t)t-4)(t-6)$, and $[6\ -6\ 0]^{\mathsf{T}}$ an eigenvector for $A$ with eigenvalue 0.
   For $t = 4$ we have $Q(t) = t(t-6)$, and $[24\ 24\ 0]^{\mathsf{T}}$ is an eigenvector for $A$ with eigenvalue 4.
   For $t = 6$ we have $Q(t) = t(t-4)$, and $[0\ -6\ -12]^{\mathsf{T}}$ is an eigenvector for $A$ with eigenvalue 6.

9. If $\beta$ is an eigenvalue of $B$ with eigenvector $\mathbf{x}$, then $\frac{1}{\beta} + \alpha$ is an eigenvalue of $A$ with eigenvector $\mathbf{x}$.

10. (a) F
    (c) T

Section 20

1. (a) Eigenvalues:$\lambda_1 = 2 + 2\sqrt{2}i$ and $\lambda_2 = 2 - 2\sqrt{2}i$; Eigenvectors: $[-\sqrt{2}i\ 1]^{\mathsf{T}}$ and $[\sqrt{2}i\ 1]^{\mathsf{T}}$

   (b) Eigenvalues: $\lambda_1 = 2+i$ and $\lambda_2 = 2-i$; Eigenvectors: $[-(1+i)\ 1]^{\mathsf{T}}$, $[-(1-i)\ 1]^{\mathsf{T}}$

   (c) Eigenvalues: $\lambda_1 = -1+2i$ and $\lambda_2 = -1-2i$; Eigenvectors: $[(1+i)\ 2]^{\mathsf{T}}$ and $[(1-i)\ 2]^{\mathsf{T}}$

3. Just the rotation matrices

5. $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

7. (a) Characteristic polynomial $\lambda^2 + a_1\lambda + a_0$; $\begin{bmatrix} 0 & -2 \\ 1 & 2 \end{bmatrix}$

   (b) $\det(C - \lambda I_3) = -\lambda^3 - a_2\lambda^2 - a_1\lambda - a_0$

8. (a) T
   (c) F
   (e) T

Section 21

1. $\det(rA) = r^n \det(A)$

3. (a) $A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

   (c) $125 = \det(B) = \det(A_1) = \det(A)$

   (d) $\det(A) = (n+1)^{n-1}$

5. $A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$
$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

7. (a) 6

   (b) $-16$

9. $|c-7|$, volume is 0 when the parallelepiped is two-dimensional.

10. (a) T
    (c) T
    (e) F
    (g) T

Section 22

1. (a) Use the fact that $\mathbf{0}_1 + \mathbf{v} = \mathbf{v}$ for any vector $\mathbf{v}$ in our vector space.

   (b) Same reasoning as in part (a).

   (c) Use the transitive property of equality.

3. Use the fact that $-1 + 1 = 0$.

5. The intersection $W_1 \cap W_2$ is a subspace of $V$, but the union $W_1 \cup W_2$ is not in general a subspace of $V$.

7. The space $W$ is the span of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$.

9. Mimic the proof of Theorem 12.5.

11. (a) Closure is by definition, the sequence $\{0\}$ is the additive identity, the sequence $\{-x_n\}$ is the additive inverse of the sequence $\{x_n\}$. The other properties follow from the definitions of addition and multiplication by scalars.

    (b) The answer is no.

    (c) The answer is yes.

    (d) The answer is no.

    (e) The answer is yes.

    (f) To show that $\ell^2$ is closed under addition, expand the square.

13. (a) T
    (c) T
    (e) T
    (g) T
    (i) F

Section 23

1. (a) This set is a basis for $\mathbb{R}^3$.

   (b) The set is linearly independent in $\mathbb{P}_3$ but does not span $\mathbb{P}_3$.

   (c) The set is a basis for $\mathbb{P}_3$.

   (d) The set is linearly independent in $M_{3\times 2}$ but does not span $M_{3\times 2}$.

3. The set $\{M_{ij}\}$ where $M_{ij}$ is the matrix with a 1 in the $ij$th position and zeros everywhere else is a basis for $\mathcal{M}_{2\times 2}$, as is the set $\{M'_{ij}\}$, where $M'_{ij}$ is the matrix with a $-1$ in the $ij$th position and zeros everywhere else.

5. The set is a basis for $V$.

7. It is not possible.
9. Mimic the proof of Theorem 6.2.
11. Mimic the proof of Theorem 6.4.
13. (a) F
   (c) T
   (e) T
   (g) F
   (i) F
   (k) T

Section 24

1. The set $\{1+t^2, 2+t+2t^2+t^3, 1+t+t^3\}$ is a basis for $W$ and $\dim(W) = 3$.
3. (a) No.
   (b) The set $S$ is linearly dependent.
   (c) The set $\{A, B, E\}$ forms a basis for Span $S$ and $C = 2A - 3B$, $D = 3A - 2B$.
   (d) Let $F = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, and $H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. The set $\{A, B, E, F, G, H\}$ is a basis for $\mathcal{M}_{2\times 3}$.
5. The set $\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$ is a basis for $W$. It follows that $\dim(W) = 3$.
7. The set $\mathcal{B} = \{-1+t, -1+t^2\}$ is a basis for $W$ and $\dim(W) = 2$.
9. Consider dimensions.
11. (a) What happens if If $\mathbf{v}_i$ is in $W_1$ for each $i$?
   (b) Assume that $\mathbf{v}_1 \notin W_1$.
      i. Consider the set $\mathcal{S} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{v}_1\}$.
      ii. Write $\mathbf{v}_2$ and $\mathbf{v}_3$ in terms of the vectors in $\mathcal{S}$.

11.

(1) (a) T
   (c) T
   (e) F
   (g) T
   (i) F
   (k) T

Section 25

1. $[\mathbf{b}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.
3. Two such bases are $\{[1\ 0]^{\mathsf{T}}, [3\ 3]^{\mathsf{T}}\}$ and $\{[1\ 1]^{\mathsf{T}}, [3\ 1]^{\mathsf{T}}\}$.
5. $\mathcal{B} = \{[1\ 0\ 2]^{\mathsf{T}}, [2\ 1\ 1]^{\mathsf{T}}\}$.
7. (a) Show that $\mathcal{B}$ is linearly independent.
   (b) i. $[p_1(t)]_{\mathcal{B}} = [1\ \ 0\ \ 1]^{\mathsf{T}}$, $[p_2(t)]_{\mathcal{B}} = [1\ \ 1\ \ 1]^{\mathsf{T}}$, and $[p_3(t)]_{\mathcal{B}} = [2\ \ -1\ \ -1]^{\mathsf{T}}$.
      ii. Row reduce the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$.
9. (a) Show that $\mathcal{B}$ is linearly independent.
   (b) $[A]_{\mathcal{B}} = [0\ \ 0\ \ 0\ \ 1]^{\mathsf{T}}$, $[B]_{\mathcal{B}} = [0\ 1\ 1\ 0]^{\mathsf{T}}$, $[C]_{\mathcal{B}} = [1\ 1\ 1\ 0]^{\mathsf{T}}$, and $[D]_{\mathcal{B}} = [1\ \ -1\ \ -1\ 1]^{\mathsf{T}}$.
   (c) Row reduce $[[A]_{\mathcal{B}}\ [B]_{\mathcal{B}}\ [C]_{\mathcal{B}}\ [D]_{\mathcal{B}}]$.
11. Use the fact that $c\mathbf{u} = c(u_1\mathbf{v}_1 + u_2\mathbf{v}_2 + \cdots + u_n\mathbf{v}_n) = (cu_1)\mathbf{v}_1 + (cu_2)\mathbf{v}_2 + \cdots + (cu_n)\mathbf{v}_n$.
13. (a) Write $\mathbf{x}$ as a linear combination of basis vectors.

(b) Apply $T$ to an appropriate vector and use the fact that $T$ is one-to-one.

15. Find a vector $\mathbf{y}$ in $V$ such that $\mathbf{x} = [\mathbf{y}]_{\mathcal{B}}$.

17. (a) F
    (c) F
    (e) T
    (g) T
    (i) T

Section 26

1. (a) $\begin{bmatrix} 5 & -1 & -1 \\ 1 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$.

   (b) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & -1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

   (c) $\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 2 & 0 & 2 & 1 \end{bmatrix}$.

3. (a) $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} \frac{5}{2} & -2 \\ \frac{1}{2} & 1 \end{bmatrix}$ and $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = \begin{bmatrix} \frac{2}{7} & \frac{4}{7} \\ -\frac{1}{7} & \frac{5}{7} \end{bmatrix}$.

   (b) i. The columns are linearly independent.
       ii. Show that $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}^{-1}[\mathbf{x}]_{\mathcal{C}} = [\mathbf{x}]_{\mathcal{B}}$. Then show that $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}^{-1}$ and $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$ have the same corresponding columns.

5. (a) Use the trigonometric identities $\cos(\theta + \pi/2) = -\sin(\theta)$ and $\sin(\theta + \pi/2) = \cos(\theta)$.

   (b) $[\mathbf{x}]_{\mathcal{C}} \approx \begin{bmatrix} 3.2 \\ 1.6 \end{bmatrix}$.

   (c) $[\mathbf{y}]_{\mathcal{B}} \approx \begin{bmatrix} 0.2 \\ 3.6 \end{bmatrix}$.

7. (a) What is $[\mathbf{v}]_{\mathcal{S}}$ if $\mathcal{S}$ is a standard basis?

   (b) Use properties of change of basis matrices.

   (c) See the discussion after Activity 26.1.

8. (a) T
   (c) T

Section 27

1. (a) The angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{2}$. The distance between $\mathbf{u}$ and $\mathbf{v}$ is $\sqrt{10}$. The orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\mathbf{0}$.

   (b) The angle between $\mathbf{u}$ and $\mathbf{v}$ is $0$. The distance between $\mathbf{u}$ and $\mathbf{v}$ is $\sqrt{2}$. $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u}$.

   (c) The angle between $\mathbf{u}$ and $\mathbf{v}$ is approximately $104.96°$. The distance between $\mathbf{u}$ and $\mathbf{v}$ is $\sqrt{17}$. $\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{v}||^2}\mathbf{v} = -\frac{1}{10}[1\ 3]^{\mathsf{T}}$.

   (d) The angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{2}$. The orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\mathbf{0}$. The distance between $\mathbf{u}$ and $\mathbf{v}$ is $\sqrt{11}$.

   (e) The angle between $\mathbf{u}$ and $\mathbf{v}$ is approximately $54.74°$. The distance between $\mathbf{u}$ and $\mathbf{v}$ is $\sqrt{2}$. $\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{v}||^2}\mathbf{v} = \frac{1}{3}[1\ 1\ 1]^{\mathsf{T}}$.

3. $h = 6 \pm 4\sqrt{3}$

5. Write the dot product as a matrix-vector product.

7. (a) Write the norm as a dot product.

   (b) From part (a), what can we say about $2(\mathbf{u} \cdot \mathbf{v})$?

9. Consider cases of $\mathbf{u} \cdot \mathbf{v} \leq 0$ separately. Then write the norm as a dot product.

11. Use the definition of $W^\perp$.

13. If $\mathbf{w} \in W_2^\perp$ and $\mathbf{v} \in W_1$, in what other set is $\mathbf{v}$?

14. (a) F
    (c) F
    (e) F
    (g) F
    (i) T
    (k) T
    (m) F
    (o) T
    (q) T

Section 28

1. $\left\{[0\ 1\ 0]^\mathsf{T}, \left[\frac{3}{4}\ 0\ 1\right]^\mathsf{T}\right\}$

3. (a) Where are $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k$?
   (b) Take the dot product of $\mathbf{z}$ with $\mathbf{w}_i$.
   (c) Use parts (a) and (b).
   (d) Collect terms in $W$ and in $W^\perp$.

5. Consider $(P\mathbf{e}_i)\cdot(P\mathbf{e}_j)$ where $\mathbf{e}_t$ is the $t$th standard basis vector for $\mathbb{R}^n$.

7. (a) Simplify $(AB)^\mathsf{T}(AB)$.
   (b) Take the transpose of $A^\mathsf{T}$.
   (c) What is $\left(A^{-1}\right)^\mathsf{T} A^{-1}$?

8. (a) F
   (c) T
   (e) F
   (g) T
   (i) T

Section 29

1. Use properties of continuous functions.

3. (a) $2x^2 + 3y^2 = 1$
   (b) An ellipse centered at the origin with major axis the segment from $\left(0, -\frac{1}{\sqrt{3}}\right)$ to $\left(0, \frac{1}{\sqrt{3}}\right)$ and minor axis the segment from $\left(-\frac{1}{\sqrt{2}}, 0\right)$ to $\left(\frac{1}{\sqrt{2}}, 0\right)$.

5. (a) Verify the inner product properties. Why is the assumption that the $a_i$ are positive necessary?
   (b) $\begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ 0 & a_2 & 0 & \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & a_n \end{bmatrix}$.

7. (a) No.
   (b) If $A$ is a diagonal matrix with positive diagonal entries.

9. (a) Evaluate each side of the inequality.
   (b) Write $||\mathbf{w}||^2$ as an inner product and expand.

11. (a) Expand the inner product.
    (b) Expand the inner product.
    (c) Convert $A$ and $B$ to vectors in $\mathbb{R}^{n^2}$ whose entries are the entries in the first row followed by the entries in the second row and so on.

13. (a) Compute the inner product of the vectors.
    (b) Try to write $\mathbf{v}$ in terms of the basis vectors for $W$.
    (c) $[1\ 2\ 1]^\mathsf{T}$. Approximately 1.41.

15. (a) Use the fact that $\mathbf{0} = \mathbf{0} + \mathbf{0}$.
    (b) Use the fact that $\langle \mathbf{u}, \mathbf{v}\rangle = \langle \mathbf{v}, \mathbf{u}\rangle$.
    (c) Same hint as part (b).
    (d) Use the fact that $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$.

17. Mimic Theorem 28.3.

19. (a) F

(c) T

(e) F

(g) F

(i) T

19. (a) F

(c) T

(e) T

(g) T

Section 30

1. (a) $\text{proj}_W \mathbf{v} = \frac{1}{2}[1\ 0\ 1]^{\mathsf{T}}$

(b) $\frac{1}{2}[1\ 0\ 1]^{\mathsf{T}}$, $\sqrt{\frac{1}{2}}$.

3. (a) $\text{proj}_W h(t) \approx -0.386t^2 - 0.721t + 2.06$

(b) $2 - 2t^2$

(c) $\text{proj}_W h(t)$

5. (a) $\{\frac{1}{\sqrt{3}}[1\ 1\ 1]^{\mathsf{T}}, \frac{1}{\sqrt{2}}[1\ -1\ 0]^{\mathsf{T}}\}$

(b) $\left\{\frac{\sqrt{6}}{4}(1+t), \quad \frac{\sqrt{2}}{4}(1 - 3t), \quad \frac{\sqrt{10}}{4}\left(\frac{1}{3} - t^2\right)\right\}$

(c) $\left\{\frac{1}{\sqrt{14}}[1\ 0\ 2]^{\mathsf{T}}, \quad \frac{\sqrt{35}}{70}[-9\ \ 1\ \ 3]^{\mathsf{T}}, \quad \frac{1}{\sqrt{10}}[1\ 2\ -2]^{\mathsf{T}}\right\}$

(d) $$\left\{ \frac{1}{\sqrt{5}}\begin{bmatrix} 1\\0\\1\\0\\1\\0\\1 \end{bmatrix}, \quad \frac{\sqrt{3}}{6}\begin{bmatrix} -2\\2\\2\\0\\0\\0\\0 \end{bmatrix}, \quad \frac{1}{\sqrt{5}}\begin{bmatrix} 0\\-1\\1\\-1\\0\\0\\-1 \end{bmatrix}, \frac{\sqrt{105}}{210}\begin{bmatrix} 5\\1\\4\\6\\0\\15\\-9 \end{bmatrix} \right\}$$

7. (a) Evaluate an appropriate equation at well-chosen points.

(b) $\left\{1, \cos(t), \sin(t) - \frac{2}{\pi}\right\}$

9. We cannot find a QR factorization for this matrix.

Section 31

1. (a) $\frac{1}{\sqrt{5}}\begin{bmatrix} -2 & 2\\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

(c) $A$ is not orthogonally diagonalizable.

3. $\frac{1}{2}\begin{bmatrix} 9 & 3 & 0 & 0\\ 3 & 9 & 0 & 0\\ 0 & 0 & 4 & 0\\ 0 & 0 & 0 & 4 \end{bmatrix}$

5. (a) Determine $P_i^{\mathsf{T}}$

(b) Show that every column of $P_i$ is a scalar multiple of $\mathbf{u}_i$.

(c) Use the orthonormal basis to simplify $P_i^2$.

(d) Use the orthonormal basis to simplify $P_iP_j$.

(e) Use the orthonormal basis to simplify $P_i\mathbf{u}_i$.

(f) Use the orthonormal basis to simplify $P_i\mathbf{u}_j$.

(g) Recall that $\text{proj}_{W_i}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{u}_i}{\mathbf{u}_i\cdot\mathbf{u}_i}\mathbf{u}_i$.

7. (a) A basis for the eigenspace of $A$ corresponding to the eigenvalue $-1$ is $\{[0\ 0\ -2\ 1]^{\mathsf{T}}, [-2\ 1\ 0\ 0]^{\mathsf{T}}\}$ and a basis for the eigenspace of $A$ corresponding to the eigenvalue 4 is $\{[0\ 0\ 1\ 2]^{\mathsf{T}}, [1\ 2\ 0\ 0]^{\mathsf{T}}\}$. $P_1$ is $\frac{1}{5}\begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 4 & -2\\ 0 & 0 & -2 & 1 \end{bmatrix}$,

$P_2$ is $\frac{1}{5}\begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

$P_3$ is $\frac{1}{5}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 41 \end{bmatrix}$,

$P_4$ is $\frac{1}{5}\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

$\mu_1 = -1$, $\mu_2 = 4$, $Q_1$ is $\frac{1}{5}\begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix}$, and

$Q_2$ is $\frac{1}{5}\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$.

(b) i. Collect matrices with the same eigenvalues.

ii. Use the fact that each $P_i$ is a symmetric matrix.

iii. Use Theorem 31.8.

iv. Use Theorem 31.8.

v. Explain why $\{\mathbf{u}_{1_j}, \mathbf{u}_{2_j}, \ldots, \mathbf{u}_{m_j}\}$ is a orthonormal basis for $E_{\mu_j}$.

(c) The rank of $Q_j$ is $m_j$.

8. (a) T
   (c) F
   (e) T
   (g) T
   (i) T
   (k) F

Section 32

1. (a) $\begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & 0 & 2 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 1 & -\frac{1}{2} \\ 1 & 5 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ -\frac{1}{2} & 0 & 2 & 8 \end{bmatrix}$

3. (a) i. Let $P$ be a matrix that orthogonally diagonalizes $A$, with $P^{\mathsf{T}}AP = D$. Use this to calculate $Q(\mathbf{x})$.

ii. Substitute in part i.

iii. Make an argument similar to part ii.

(b) The maximum value of $Q(\mathbf{x})$ on the unit circle is 1 and it occurs at the input $\frac{1}{\sqrt{2}}[1\ -1]$.

5. (a) $1 = \lambda_1 y_1^2 + \lambda_2 y_2^2$.
   (b) An ellipse.
   (c) A hyperbola.
   (d) Two lines.

7. Use Exercise 6 (a) to compare $Q_A(\mathbf{e}_i)$ and $Q_B(\mathbf{e}_i)$, then compare $Q_A(\mathbf{e}_i + \mathbf{e}_j)$ to $Q_B(\mathbf{e}_i + \mathbf{e}_j)$ for $i \neq j$.

9. (a) F
   (c) T
   (e) T
   (g) F
   (i) T
   (k) T
   (m) T
   (o) T

Section 33

1. (a) $U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Sigma = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}$, $V = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

(b) $U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\Sigma = [\sqrt{2}]$, $V = [1]$.

(c) $U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $\Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $V = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$.

(d) $U = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & \frac{2}{\sqrt{5}} & 0 \\ \frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & 0 & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{10}} & -\frac{2}{\sqrt{10}} & 0 & \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{10}} & \frac{2}{\sqrt{10}} & -\frac{2}{\sqrt{5}} & 0 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 5 & 0 \\ 0 & \sqrt{5} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $V = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

(e) $U = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$, $\Sigma = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $V = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

3. (a) $\sqrt{28}$

(b) $U = \begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{5}} & \frac{3}{\sqrt{70}} \\ \frac{2}{\sqrt{14}} & -\frac{1}{\sqrt{5}} & \frac{6}{\sqrt{70}} \\ \frac{3}{\sqrt{14}} & 0 & -\frac{5}{\sqrt{70}} \end{bmatrix}$, $\Sigma = \begin{bmatrix} \sqrt{28} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $V = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

(c) i. $\left\{\frac{1}{\sqrt{2}}[-1\ 1]^{\mathsf{T}}\right\}$.

ii. $\left\{\frac{1}{\sqrt{14}}[1\ 2\ 3]^{\mathsf{T}}\right\}$.

iii. $\left\{\frac{1}{\sqrt{2}}[1\ 1]^{\mathsf{T}}\right\}$.

5. Find the transpose of an SVD for $A$.

7. $||A|| = \lambda_1$

9. Mimic Exercise 5 in Section 31.

9. (a) F
(c) T
(e) F
(g) F

Section 34

1. (a) 60, 15, and 6

(b) $A = 60\left(\frac{1}{5}[3\ 0\ 4]^{\mathsf{T}}\right)\left(\frac{1}{3}[2\ 1\ 1]\right) + 15\left(\frac{1}{5}[4\ 0\ -3]^{\mathsf{T}}\right)\left(\frac{1}{3}[-1\ -2\ 2]\right) + 6\left(\frac{1}{5}[0\ 5\ 0]^{\mathsf{T}}\right)\left(\frac{1}{3}[-2\ 2\ 1]\right)$.

(c) $\begin{bmatrix} 24 & 12 & 24 \\ 0 & 0 & 0 \\ 32 & 16 & 32 \end{bmatrix}$, $\frac{261}{3861} \approx 0.068$.

(d) $\begin{bmatrix} 20 & 4 & 32 \\ 0 & 0 & 0 \\ 35 & 22 & 26 \end{bmatrix}$, $\frac{36}{3861} \approx 0.009$.

3. (a) $f(x) \approx 35.5273 - 0.0527x$.

(b) Approximately 32.10.

5. (a) The plot demonstrates that the data is not linear.

(b) $g(x) = \frac{266}{97} + \frac{12213}{970}x - \frac{1423}{9700}x^2$.

(c) Approximately 23.9 and 61.9 degrees.

7. (a) $\det(A) = 1$, eigenvalue 1

(b) i. $(2, 1, 1)$

ii. $(4, 2, 1, 1)$

iii. $\mathbf{x} = [x_1\ x_2\ \ldots\ x_n]^{\mathsf{T}}$ with $x_1 = 2^{n-2}$, $x_2 = 2^{n-3}$, ..., $x_{n-2} = 2$, $x_{n-1} = 1$, and $x_n = 1$

iv. $||A - B||_F = \frac{1}{2^{n-2}}$

9. (a) Use the fact that $\Sigma\Sigma^+$ is a symmetric matrix.

   (b) Take the transpose of $A^+A$.

11. $A^+ = 0$

13. If $A^{\mathsf{T}}A\mathbf{x} = \mathbf{0}$, what is $\mathbf{x}^{\mathsf{T}}A^{\mathsf{T}}A\mathbf{x}$?

14. (a) F

    (c) F

    (e) F

Section 35

1. Use properties of the definite integral from calculus.

3. The property that $T^{-1}(\mathbf{w}) = \mathbf{v}$ whenever $T(\mathbf{v}) = \mathbf{w}$ is the key to this problem.

5. (a) linear transformation, one-to-one, onto

   (b) linear transformation, one-to-one, onto

7. Use the fact that $T$ is one-to-one to show that is linearly independent, and that $T$ is onto to show that $\mathcal{C}$ spans $W$.

9. Yes, but the vector space needs to be infinite dimensional.

11. Show that $T(\mathbf{v}) = \mathbf{w}$ has at most one solution for each $\mathbf{w}$ in $W$.

13. (a) Think of $T(V')$ as the range of a suitable restriction of $T$.

    (b) Use Exercise 12. It is possible.

    (c) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be a basis for Ker($T$). Extend this basis to a basis $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k, \mathbf{v}_{k+1}, \ldots, \mathbf{v}_n\}$ of $V$. Use this basis to find a basis for Range($T$).

    (d) This follows from fact that coordinate transformations are linear.

18. (a) F

    (c) T

    (e) F

    (g) T

    (i) T

Section 36

1. (a) $r(t) = r_0(1) + r_1(t) + r_2(t_2)$ and $[r(t)]_{\mathcal{B}} = \begin{bmatrix} r_0 \\ r_1 \\ r_2 \end{bmatrix}$.

   (b) $T$ is a linear transformation

   (c) The coordinate mapping is a linear transformation.

   (d) $[T(p_0(t))]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $[T(p_1(t))]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $[T(p_2(t))]_{\mathcal{C}} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (f) $[T(1 + t - t^2)]_{\mathcal{C}} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}$, $T(1 + t - t^2) = 2(1) + 0(t) + 1(t^2) + (-1)t^3 = 2 + t^2 - t^3$.

   (g) $T(1 + t - t^2) = 2 + t^2 - t^3$.

3. (a) Use the linearity of both $S$ and $T$.

(b) True

5. $\mathbf{x} \in \text{Ker}(T)$ if and only if $[\mathbf{x}]_{\mathcal{B}} \in \text{Nul } [T]_{\mathcal{B}}^{\mathcal{C}}$
T

7. (a) Since $\mathbf{w}$ is in the range of $T$, there is a vector $\mathbf{v}$ so that $T(\mathbf{v}) = \mathbf{w}$.

   (b) Since $\mathbf{y}$ is in the range of $T'$, there exists a vector $\mathbf{x}$ in $\mathbb{R}^n$ so that $T'(\mathbf{x}) = \mathbf{y}$. What is $[\mathbf{w}]_{\mathcal{C}}$?

   (c) How do we tell if $T'$ is onto?

9. (a) F

   (c) T

   (e) T

   (g) T

   (i) T

Section 37

1. (a) Use properties of the derivative.

   (b) $[T]_{\mathcal{S}} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$.

   (c) $-1$, $-2$, and $-3$ with bases $\{1\}$, $\{-1+t\}$, $\{1-2t+t^2\}$

   (d) $\mathcal{B} = \{-1, -1+t, 1-2t+t^2\}$

3. (a) Use properties of differentiable functions.

   (b) Use properties of the derivative.

   (c) What is $D\left(e^{\lambda x}\right)$?

5. (a) Use properties of the matrix transpose.

   (b) For which matrices is $T(A) = A$?

   (c) When is it possible to have $A^{\mathsf{T}} = \lambda A$?

6. (a) F

   (c) T

   (e) T

   (g) T