Appendix A

Answers for the Progress Checks

Section 1.1

Progress Check 1.1

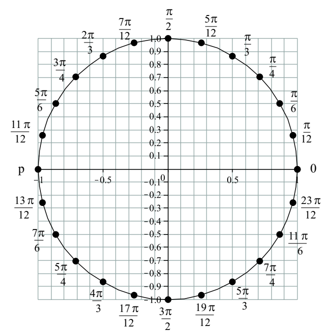
1. Some positive numbers that are wrapped to the point $(-1, 0)$ are $\pi, 3\pi, 5\pi$.
   Some negative numbers that are wrapped to the point $(-1, 0)$ are $-\pi, -3\pi, -5\pi$.

2. The numbers that get wrapped to $(-1, 0)$ are the odd integer multiples of π.

3. Some positive numbers that are wrapped to the point $(0, 1)$ are $\dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}$.
   Some negative numbers that are wrapped to the point $(0, 1)$ are $-\dfrac{3\pi}{2}, -\dfrac{7\pi}{2}, -\dfrac{11\pi}{2}$.

4. Some positive numbers that are wrapped to the point $(0, -1)$ are $\dfrac{3\pi}{2}, \dfrac{7\pi}{2}, \dfrac{11\pi}{2}$.
   Some negative numbers that are wrapped to the point $(0, -1)$ are $-\dfrac{\pi}{2}, -\dfrac{5\pi}{2}, -\dfrac{9\pi}{2}$.

Progress Check 1.2



1. For $t = \dfrac{\pi}{3}$, the point is approximately $(0.5, 0.87)$.

2. For $t = \dfrac{2\pi}{3}$, the point is approximately $(-0.5, 0.87)$.

3. For $t = \dfrac{4\pi}{3}$, the point is approximately $(-0.5, -0.87)$.

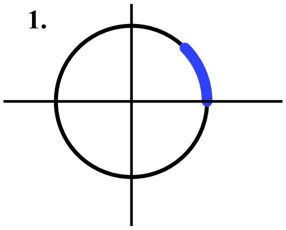
4. For $t = \dfrac{5\pi}{3}$, the point is approximately $(0.5, -0.87)$.

5. For $t = \dfrac{\pi}{4}$, the point is approximately $(0.71, 0.71)$.

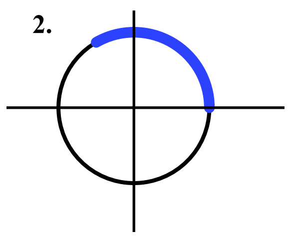
6. For $t = \dfrac{7\pi}{4}$, the point is approximately $(0.71, -0.71)$.

Progress Check 1.3

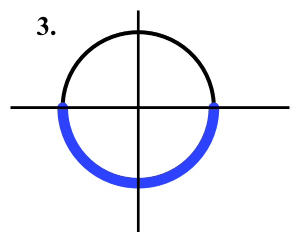
1.



2.



3.



Progress Check 1.4

1. We substitute $y = \dfrac{1}{2}$ into $x^2 + y^2 = 1$.

$$\begin{aligned} x^2 + \left(\frac{1}{2}\right)^2 &= 1 \\ x^2 &= \frac{3}{4} \\ x &= \pm\frac{\sqrt{3}}{2} \end{aligned}$$

The two points are $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$ and $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$.

2. We substitute $x = \dfrac{\sqrt{5}}{4}$ into $x^2 + y^2 = 1$.

$$\begin{aligned}\left(\frac{\sqrt{5}}{4}\right)^2 + y^2 &= 1\\ y^2 &= \frac{11}{16}\\ y &= \pm\frac{\sqrt{11}}{4}\end{aligned}$$

The two points are $\left(\dfrac{\sqrt{5}}{4}, \dfrac{\sqrt{11}}{4}\right)$ and $\left(\dfrac{\sqrt{5}}{4}, -\dfrac{\sqrt{11}}{4}\right)$.

Section 1.2

Progress Check 1.5

1. $\cos\left(\dfrac{\pi}{2}\right) = 0$
$\sin\left(\dfrac{\pi}{2}\right) = 1.$

2. $\cos\left(\dfrac{3\pi}{2}\right) = 0$
$\sin\left(\dfrac{3\pi}{2}\right) = -1.$

3. $\cos(0) = 1$
$\sin(0) = 0.$

4. $\cos\left(-\dfrac{\pi}{2}\right) = 0$
$\sin\left(-\dfrac{\pi}{2}\right) = -1.$

5. $\cos(2\pi) = 1$
$\sin(2\pi) = 1.$

6. $\cos(-\pi) = -1$
$\sin(-\pi) = 0.$

Progress Check 1.6

1. $\cos(1) \approx 0.5403,$
$\sin(1) \approx 0.8415.$

2. $\cos(2) \approx -0.4161$
$\sin(2) \approx 0.9093.$

3. $\cos(-4) \approx -0.6536$
$\sin(-4) \approx 0.7568.$

4. $\cos(5.5) \approx 0.7807$
   $\sin(5.5) \approx -0.7055$.

5. $\cos(15) \approx -0.7597$
   $\sin(15) \approx 0.6503$.

6. $\cos(-15) \approx -0.7597$
   $\sin(-15) \approx -0.6503$.

Progress Check 1.7

1. Since we can wrap any number onto the unit circle, we can always find the terminal point of an arc that corresponds to any number. So the cosine of any real number is defined and the domain of the cosine function is the set of all of the real numbers.

2. For the same reason as for the cosine function, the domain of the sine function is the set of all real numbers.

3. On the unit circle, the largest x-coordinate a point can have is 1 and the smallest x-coordinate a point can have is -1. Since the output of the cosine function is the x-coordinate of a point on the unit circle, the range of the cosine function is the closed interval $[-1, 1]$. That means $-1 \leq \cos(t) \leq 1$ for any real number t.

4. On the unit circle, the largest y-coordinate a point can have is 1 and the smallest y-coordinate a point can have is -1. Since the output of the sine function is the y-coordinate of a point on the unit circle, the range of the sine function is the closed interval $[-1, 1]$. That means $-1 \leq \sin(t) \leq 1$ for any real number t.

Progress Check 1.8

1. If $\dfrac{\pi}{2} < t < \pi$, then the terminal point of the arc t is in the second quadrant and so $\cos(t) < 0$ and $\sin(t) > 0$.

2. If $\pi < t < \dfrac{3\pi}{2}$, then the terminal point of the arc t is in the third quadrant and so $\cos(t) < 0$ and $\sin(t) < 0$.

3. If $\dfrac{3\pi}{2} < t < 2\pi$, then the terminal point of the arc t is in the fourth quadrant and so $\cos(t) > 0$ and $\sin(t) < 0$.

4. If $\dfrac{5\pi}{2} < t < 3\pi$, then the terminal point of the arc t is in the second quadrant and so $\cos(t) < 0$ and $\sin(t) > 0$.

5. Note that $\cos(t) = 0$ at $t = \dfrac{\pi}{2}$ and $t = \dfrac{3\pi}{2}$. Since $\cos(t)$ is the x-coordinate of the terminal point of the arc t, the previous response shows that $\cos(t)$ is positive when t is in one of the intervals $\left[0, \dfrac{\pi}{2}\right)$ or $\left(\dfrac{3\pi}{2}, 2\pi\right]$.

6. Note that $\sin(t) = 0$ at $t = 0$ and $t = \pi$. Since $\sin(t)$ is the y-coordinate of the terminal point of the arc t, the previous response shows that $\sin(t)$ is positive when t is in the interval $(0, \pi)$.

7. Note that $\cos(t) = 0$ at $t = \dfrac{\pi}{2}$ and $t = \dfrac{3\pi}{2}$. Since $\cos(t)$ is the x-coordinate of the terminal point of the arc t, the previous response shows that $\cos(t)$ is negative when t is in the interval $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$.

8. Note that $\sin(t) = 0$ at $t = \pi$ and $t = 2\pi$. Since $\sin(t)$ is the y-coordinate of the terminal point of the arc t, the previous response shows that $\sin(t)$ is positive when t is in the interval $(\pi, 2\pi)$.

Progress Check 1.9

1. Since $0 < \dfrac{\pi}{5} < \dfrac{\pi}{2}$, the terminal point of the arc $\dfrac{\pi}{5}$ is in the first quadrant. Therefore, $\cos\left(\dfrac{\pi}{5}\right)$ is positive.

2. Using the information about t in (1), $\sin\left(\dfrac{\pi}{5}\right)$ is positive.

3. We can write $\dfrac{\pi}{2}$ as $\dfrac{4\pi}{8}$ and π as $\dfrac{8\pi}{8}$, so $\dfrac{\pi}{2} < \dfrac{5\pi}{8} < \pi$. This puts the terminal point of the arc $\dfrac{5\pi}{8}$ in the second quadrant. Therefore, $\cos\left(\dfrac{5\pi}{8}\right)$ is negative.

4. Using the information about t in (3), $\sin\left(\dfrac{5\pi}{8}\right)$ is positive.

5. We can write $-\dfrac{\pi}{2}$ as $\dfrac{-8\pi}{16}$ and $-\pi$ as $\dfrac{-16\pi}{16}$, so $-\pi < \dfrac{-9\pi}{16} < -\dfrac{\pi}{2}$. This puts the terminal point of the arc $\dfrac{-9\pi}{16}$ in the third quadrant. Therefore, $\cos\left(\dfrac{-9\pi}{16}\right)$ is negative.

6. Using the information about t in (5), $\sin\left(\dfrac{-9\pi}{16}\right)$ is negative.

7. We can write -2π as $\dfrac{-24\pi}{12}$ and $-\dfrac{5\pi}{2}$ as $\dfrac{-30\pi}{12}$, so $-\dfrac{5\pi}{2} < \dfrac{-25\pi}{12} < -2\pi$. This puts the terminal point of the arc $\dfrac{-25\pi}{12}$ in the fourth quadrant. Therefore, $\cos\left(\dfrac{-25\pi}{12}\right)$ is positive.

8. Using the information about the arc t in (7), $\sin\left(\dfrac{-25\pi}{12}\right)$ is negative.

Progress Check 1.10

Any point on the unit circle satisfies the equation $x^2+y^2 = 1$. Since $(\cos(t), \sin(t))$ is a point on the unit circle, it follows that $(\cos(t))^2 + (\sin(t))^2 = 1$ or

$$\cos^2(t) + \sin^2(t) = 1.$$

Progress Check 1.12

1. Since $\cos(t) = \dfrac{1}{2}$, we can use the Pythagorean Identity to obtain

$$\begin{aligned}\left(\frac{1}{2}\right)^2 + \sin^2(t) &= 1\\ \frac{1}{4} + \sin^2(t) &= 1\\ \sin^2(t) &= \frac{3}{4}\\ \sin(t) &= \pm\sqrt{\frac{3}{4}}\end{aligned}$$

Notice that we cannot determine the sign of $\sin(t)$ using only the Pythagorean Identity. We need further information about the arc t. In this case, we are given that the terminal point of the arc t is in the fourth quadrant, and hence, $\sin(t) < 0$. Consequently,

$$\sin(t) = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}.$$

2. Since $\sin(t) = -\dfrac{2}{3}$, we can use the Pythagorean Identity to obtain

$$\begin{aligned}
\cos^2(t) + \left(-\frac{2}{3}\right)^2 &= 1 \\
\cos^2(t) + \frac{4}{9} &= 1 \\
\cos^2(t) &= \frac{5}{9} \\
\cos(t) &= \pm\sqrt{\frac{5}{9}}
\end{aligned}$$

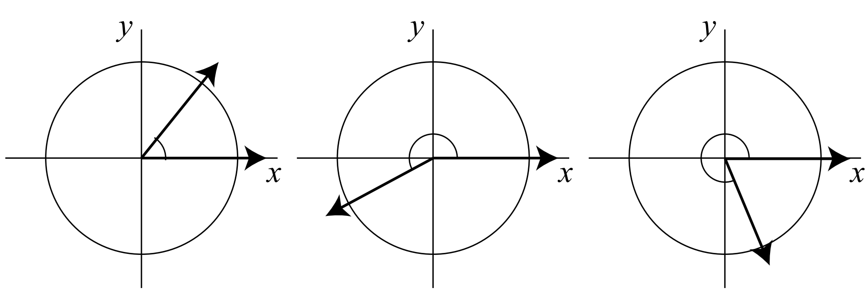
Once again, we need information about the arc t to determine the sign of $\cos(t)$. In this case, we are given that $\pi < t < \dfrac{3\pi}{2}$. Hence, the terminal point of the arc t is in the third quadrant and so, $\cos(t) < 0$. Therefore,

$$\cos(t) = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}.$$

Section 1.3

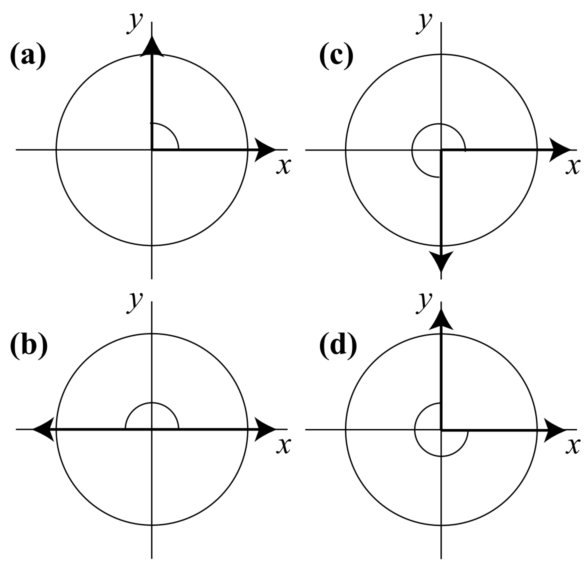
Progress Check 1.13

These graphs show positive angles in standard position. The one on the left has its terminal point in the first quadrant, the one in the middle has its terminal point in the third quadrant, and the one on the right has its terminal point in the fourth quadrant.



Progress Check 1.14

1.



2. (**a**) 90° (**c**) 270°
(**b**) 180° (**d**) −270°

Progress Check 1.15

Angle in radians	Angle in degrees	Angle in radians	Angle in degrees
0	$0°$	$\frac{7\pi}{6}$	$210°$
$\frac{\pi}{6}$	$30°$	$\frac{5\pi}{4}$	$225°$
$\frac{\pi}{4}$	$45°$	$\frac{4\pi}{3}$	$240°$
$\frac{\pi}{3}$	$60°$	$\frac{3\pi}{2}$	$270°$
$\frac{\pi}{2}$	$90°$	$\frac{5\pi}{3}$	$300°$

Angle in radians	Angle in degrees
$\dfrac{2\pi}{3}$	$120°$
$\dfrac{3\pi}{4}$	$135°$
$\dfrac{5\pi}{6}$	$150°$
π	$180°$

Angle in radians	Angle in degrees
$\dfrac{7\pi}{4}$	$315°$
$\dfrac{11\pi}{6}$	$330°$
2π	$360°$

Progress Check 1.16

Using a calculator, we obtain the following results correct to ten decimal places.

- $\cos(1) \approx 0.5403023059$,
  $\sin(1) \approx 0.8414709848$.
- $\cos(2) \approx -0.4161468365$
  $\sin(2) \approx 0.9092974268$.
- $\cos(-4) \approx -0.6536436209$
  $\sin(-4) \approx 0.7568024953$.
- $\cos(-15) \approx -0.7596879129$
  $\sin(-15) \approx -0.6502878402$.

The difference between these values and those obtained in Progress Check 1.6 is that these values are correct to 10 decimal places (and the others are correct to 4 decimal places). If we round off each of the values above to 4 decimal places, we get the same results we obtained in Progress Check 1.6.

Section 1.4

Progress Check 1.17

1. Use the formula $s = r\theta$.

$$s = r\theta = (10\text{ft})\frac{\pi}{2}$$
$$s = 5\pi$$

   The arc length is 5π feet.

2. Use the formula $s = r\theta$.

$$s = r\theta = (20\text{ft})\frac{\pi}{2}$$
$$s = 10\pi$$

The arc length is 10π feet.

3. First convert 22° to radians. So $\theta = 22° \times \left(\frac{\pi \text{ rad}}{180°}\right) = \frac{11\pi}{90}$, and

$$s = r\theta = (3\text{ft})\frac{11\pi}{90}$$
$$s = \frac{11\pi}{30}$$

The arc length is $\frac{11\pi}{30}$ feet or about 1.1519 feet.

Progress Check 1.18

1. We see that

$$\omega = 40\frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}}$$
$$\omega = 80\pi\frac{\text{rad}}{\text{min}}$$

2. The result from part (a) gives

$$v = r\left(\frac{\theta}{t}\right) = r\omega$$
$$v = (3 \text{ ft}) \times 80\pi\frac{\text{rad}}{\text{min}}$$
$$v = 240\pi\frac{\text{ft}}{\text{min}}$$

3. We now convert feet per minute to feet per second.

$$v = 240\pi\frac{\text{ft}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}}$$
$$v = 4\pi\frac{\text{ft}}{\text{sec}} \approx 12.566\frac{\text{ft}}{\text{sec}}$$

Progress Check 1.20

1. One revolution corresponds to 2π radians. So
$$\omega = \frac{2\pi \text{ rad}}{24 \text{ hr}} = \frac{\pi}{12} \frac{\text{rad}}{\text{hr}}.$$

2. To determine the linear velocity, we use the formula $v = r\omega$.
$$\begin{aligned} v = r\omega &= (3959 \text{ mi}) \left(\frac{\pi}{12} \frac{\text{rad}}{\text{hr}} \right) \\ &= \frac{3959\pi}{12} \frac{\text{mi}}{\text{hr}} \end{aligned}$$
The linear velocity is approximately 1036.5 miles per hour.

3. To determine the linear velocity, we use the formula $v = r\omega$.
$$\begin{aligned} v = r\omega &= (2800 \text{ mi}) \left(\frac{\pi}{12} \frac{\text{rad}}{\text{hr}} \right) \\ &= \frac{2800\pi}{12} \frac{\text{mi}}{\text{hr}} \end{aligned}$$
The linear velocity is approximately 733.04 miles per hour. To convert this to feet per second, we use the facts that there are 5280 feet in one mile, 60 minutes in an hour, and 60 seconds in a minute. So
$$\begin{aligned} v &= \left(\frac{2800\pi}{12} \frac{\text{mi}}{\text{hr}} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ hr}}{60 \text{ min}} \right) \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) \\ &= \frac{(2800\pi)(5280)}{12 \cdot 60 \cdot 60} \frac{\text{ft}}{\text{sec}} \end{aligned}$$
So the linear velocity is approximately 1075.1 feet per second.

Section 1.5

Progress Check 1.21

1. $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ and $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$.

2. $\cos\left(\frac{7\pi}{6}\right) - \frac{\sqrt{3}}{2}$ and $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$.

3. $\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$.

Progress Check 1.22

1. $\cos\left(\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$ and $\sin\left(\dfrac{3\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$.

2. $\cos\left(\dfrac{5\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$ and $\sin\left(\dfrac{5\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$.

3. $\cos\left(\dfrac{7\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$ and $\sin\left(\dfrac{7\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$.

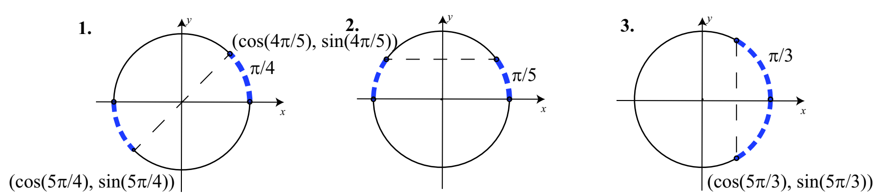
Progress Check 1.23

As shown in the following diagram:

1. The reference arc is $\dfrac{5\pi}{4} - \pi = \dfrac{\pi}{4}$.

2. The reference arc is $\pi - \dfrac{4\pi}{5} = \dfrac{\pi}{5}$.

3. The reference arc is $2\pi - \dfrac{5\pi}{3} = \dfrac{\pi}{3}$.



Progress Check 1.24

1. $\cos\left(\dfrac{2\pi}{3}\right) = -\dfrac{1}{2}$ and $\sin\left(\dfrac{2\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$.

2. $\cos\left(\dfrac{4\pi}{3}\right) = -\dfrac{1}{2}$ and $\sin\left(\dfrac{4\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$.

3. $\cos\left(\dfrac{5\pi}{3}\right) = \dfrac{1}{2}$ and $\sin\left(\dfrac{5\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$.

Progress Check 1.25

1. The terminal point of $t = -\dfrac{\pi}{6}$ is in the fourth quadrant and the reference arc for $t = -\dfrac{\pi}{6}$ is $\hat{t} = \dfrac{\pi}{6}$. So
$$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \text{ and } \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}.$$
2. The terminal point of $t = -\dfrac{2\pi}{3}$ is in the third quadrant and the reference arc for $t = -\dfrac{2\pi}{3}$ is $\hat{t} = \dfrac{\pi}{3}$. So
$$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2} \text{ and } \sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}.$$
3. The terminal point of $t = -\dfrac{5\pi}{4}$ is in the second quadrant and the reference arc for $t = -\dfrac{5\pi}{4}$ is $\hat{t} = \dfrac{\pi}{4}$. So
$$\cos\left(-\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} \text{ and } \sin\left(-\frac{5\pi}{4}\right) = \frac{\sqrt{2}}{2}.$$

Progress Check 1.27

1. We know that $\pi - t$ is in the second quadrant with reference arc t. So
$$\cos(\pi - t) = -\cos(t) = -\frac{\sqrt{5}}{3}.$$
2. The arc $\pi + t$ is in the third quadrant with reference arc t. So
$$\sin(\pi + t) = -\sin(t) = -\frac{2}{3}.$$
3. The arc $\pi + t$ is in the third quadrant with reference arc t. So
$$\cos(\pi + t) = -\cos(t) = -\frac{\sqrt{5}}{3}.$$
4. The arc $2\pi - t$ is in the fourth quadrant with reference arc t. So
$$\sin(2\pi - t) = -\sin(t) = -\frac{2}{3}.$$

Section 1.6

Progress Check 1.28

1. Since $\tan(t) = \dfrac{\sin(t)}{\cos(t)}$, $\tan(t)$ positive when both $\sin(t)$ and $\cos(t)$ have the same sign. So $\tan(t) > 0$ in the first and third quadrants.

2. We see that $\tan(t)$ negative when$\sin(t)$ and $\cos(t)$ have opposite signs. So $\tan(t) < 0$ in the second and fourth quadrants.

3. $\tan(t)$ will be zero when $\sin(t) = 0$ and $\cos(t) \neq 0$. So $\tan(t) = 0$ when the terminal point of t is on the x-axis. That is, $\tan(t) = 0$ when $t = k\pi$ for some integer k.

4. Following is a completed version of Table 1.4.

t	$\cos(t)$	$\sin(t)$	$\tan(t)$
0	1	0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{4}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	0	1	undefined

Progress Check1.29

1.

$$\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1.$$

$$\tan\left(\frac{5\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}.$$

2. We first use the Pythagorean Identity.

$$\begin{aligned}\cos^2(t) + \sin^2(t) &= 1\\ \cos^2(t) + \left(\frac{1}{3}\right)^2 &= 1\\ \cos^2(t) &= \frac{8}{9}\end{aligned}$$

Since $\sin(t) > 0$ and $\tan(t) < 0$, we conclude that the terminal point of t must be in the second quadrant, and hence, $\cos(t) < 0$. Therefore,

$$\begin{aligned}\cos(t) &= -\frac{\sqrt{8}}{3}\\ \tan(t) &= \frac{\frac{1}{3}}{-\frac{\sqrt{8}}{3}} = -\frac{1}{\sqrt{8}}\end{aligned}$$

Progress Check 1.30

1.

$$\begin{aligned}\sec\left(\frac{7\pi}{4}\right) &= \frac{1}{\cos\left(\frac{7\pi}{4}\right)}\\ &= \frac{1}{\cos\left(\frac{\pi}{4}\right)}\\ &= \frac{2}{\sqrt{2}} = \sqrt{2}\end{aligned}$$

2.

$$\begin{aligned}\csc\left(\frac{-\pi}{4}\right) &= \frac{1}{\sin\left(\frac{-\pi}{4}\right)}\\ &= \frac{1}{\sin\left(-\frac{\pi}{4}\right)}\\ &= -\frac{2}{\sqrt{2}} = -\sqrt{2}\end{aligned}$$

3. $\tan\left(\frac{7\pi}{8}\right) \approx -0.4142$

4.

$$\begin{aligned}\cot\left(\frac{4\pi}{3}\right) &= \cot\left(\frac{\pi}{3}\right)\\ &= \frac{1}{\tan\left(\frac{\pi}{3}\right)}\\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

5. $\csc(5) = \dfrac{1}{\sin(5)} \approx -1.0428$

Progress Check 1.31

1. If $\cos(x) = \dfrac{1}{3}$ and $\sin(x) < 0$, we use the Pythagorean Identity to determine that $\sin(x) = -\dfrac{\sqrt{8}}{3}$. We can then determine that

$$\tan(x) = -\sqrt{8} \qquad \csc(x) = -\frac{3}{\sqrt{8}} \qquad \cot(x) = -\frac{1}{\sqrt{8}}$$

2. If $\sin(x) = -0.7$ and $\tan(x) > 0$, we can use the Pythagorean Identity to obtain

$$\begin{aligned} \cos^2(x) + (-0.7)^2 &= 1 \\ \cos^2(x) &= 0.51 \end{aligned}$$

Since we are also given that $\tan(x) > 0$, we know that the terminal point of x is in the third quadrant. Therefore, $\cos(x) < 0$ and $\cos(x) = -\sqrt{0.51}$. Hence,

$$\begin{aligned} \tan(x) &= \frac{-0.7}{-\sqrt{0.51}} \\ \cot(x) &= \frac{\sqrt{0.51}}{0.7} \end{aligned}$$

3. We can use the definition of $\tan(x)$ to obtain

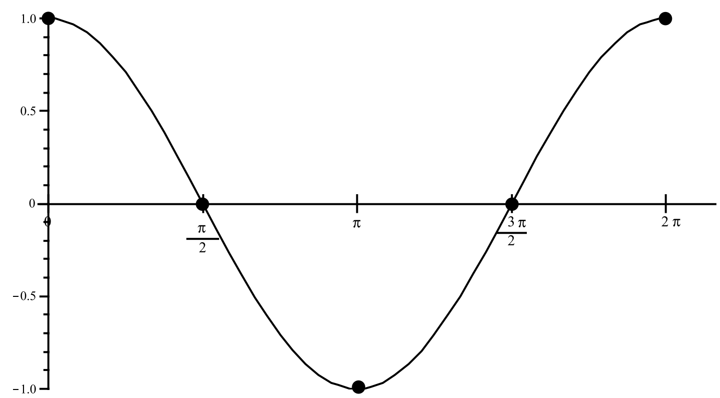
$$\begin{aligned} (\tan(x))(\cos(x)) &= \frac{\sin(x)}{\cos(x)} \cdot \cos(x) \\ &= \sin(x) \end{aligned}$$

So $\tan(x)\cos(x) = \sin(x)$, but it should be noted that this equation is only valid for those values of x for which $\tan(x)$ is defined. That is, this equation is only valid if x is not an integer multiple of π.

Section 2.1

Progress Check 2.1

Not all of the points are plotted, but the following is a graph of one complete period of $y = \cos(t)$ for $0 \leq t \leq 2\pi$.



Progress Check 2.2

1. The difference is that the graph in Figure 2.2 shows three complete periods of $y = \cos(t)$ over the interval $[-2\pi, 4\pi]$.

2. The graph of $y = \cos(t)$ has t-intercepts at $t = -\dfrac{3\pi}{2}$, $t = -\dfrac{\pi}{2}$, $t = \dfrac{\pi}{2}$, $t = \dfrac{3\pi}{2}$, $t = \dfrac{5\pi}{2}$, and $t = \dfrac{7\pi}{2}$.

3. The maximum value of $y = \cos(t)$ is 1. The graph attains this maximum at $t = -2\pi$, $t = 0$, $t = 2\pi$, and $t = 4\pi$.

4. The minimum value of $y = \cos(t)$ is -1. The graph attains this minimum at $t = -\pi$, $t = \pi$, and $t = 3\pi$.

Progress Check 2.4

- The graph of $y = \sin(t)$ has t-intercepts of $t = 0$, $t = \pi$, and $t = 2\pi$ in the interval $[0, 2\pi]$.

- If we add the period of 2π to each of these t-intercepts and subtract the period of 2π from each of these t-intercepts, we see that the graph of $y = \sin(t)$ has t-intercepts of $t = -2\pi$, $t = -\pi$, $t = 0$, $t = \pi$, $t = 2\pi$, $t = 3\pi$, and $t = 4\pi$ in the interval $[-2\pi, 4\pi]$.

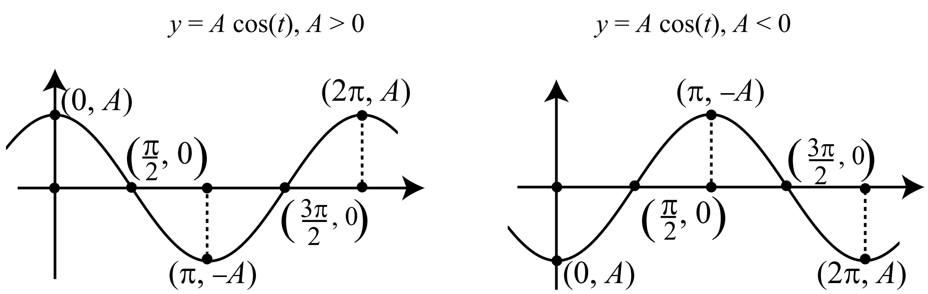
We can determine other t-intercepts of $y = \sin(t)$ by repeatedly adding or subtracting the period of 2π. For example, there is a t-intercept at:

- $t = 3\pi + 2\pi = 5\pi$;

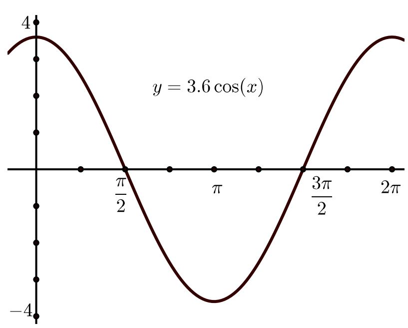
- $t = 5\pi + 2\pi = 7\pi$.

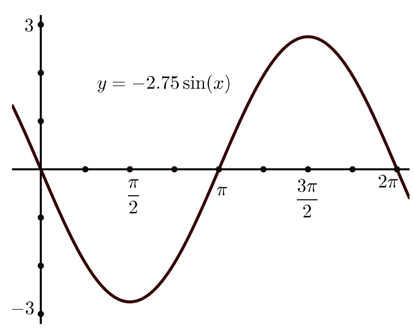
However, if we look more carefully at the graph of $y = \sin(t)$, we see that the t-intercepts are spaced π units apart. This means that we can say that $t = 0 + k\pi$, where k is some integer, is a t-intercept of $y = \sin(t)$.

Progress Check 2.6



Progress Check 2.7

1. 

2. 

Section 2.2

Progress Check 2.9

1. (**a**) For $y = 3\cos\left(\dfrac{1}{3}t\right)$, the amplitude is 3 and the period is $\dfrac{2\pi}{\dfrac{1}{3}} = 6\pi$.

(**b**) For $y = -2\sin\left(\dfrac{\pi}{2}t\right)$, the amplitude is 2 and the period is $\dfrac{2\pi}{\frac{\pi}{2}} = 4$.

2. From the graph, the amplitude is 2.5 and the period is 2. Using a cosine function, we have $A = 2.5$ and $\dfrac{2\pi}{B} = 2$. Solving for B gives $B = \pi$. So an equation is $y = 2.5\cos(\pi t)$.

Progress Check 2.11

1. (**a**) For $y = 3.2\left(\sin(t - \dfrac{\pi}{3}\right)$, the amplitude is 3.2 and the phase shift is $\dfrac{\pi}{3}$.

(**b**) For $y = 4\cos\left(t + \dfrac{\pi}{6}\right)$, notice that $y = 4\cos\left(t - \left(-\dfrac{\pi}{6}\right)\right)$. So the amplitude is 4 and the phase shift is $-\dfrac{\pi}{6}$.

2. There are several possible equations for this sinusoid. Some of these equations are:

$$y = 3\sin\left(t + \frac{3\pi}{4}\right) \qquad y = 3\cos\left(t + \frac{\pi}{4}\right)$$
$$y = -3\sin\left(t - \frac{\pi}{4}\right) \qquad y = -3\cos\left(t - \frac{3\pi}{4}\right)$$

A graphing utility can be used to verify that any of these equations produce the given graph.

Progress Check 2.14

1. The amplitude is 6.3.

2. The period is $\dfrac{2\pi}{50\pi} = \dfrac{1}{25}$.

3. We write $y = 6.3\cos(50\pi(t - (-0.01))) + 2$ and see that the phase shift is -0.01 or 0.01 units to the left.

4. The vertical shift is 2.

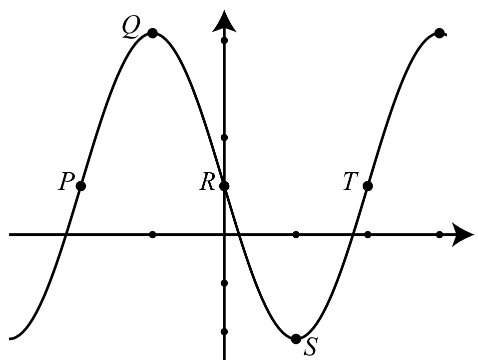
5. Because we are using a cosine and the phase shift is -0.01, we can use -0.01 as the t-coordinate of Q. The y-coordinate will be the vertical shift plus the amplitude. So the y-coordinate is 8.3. Point Q has coordinates $(-0.01, 8.3)$.

6. We now use the fact that the horizontal distance between P and Q is one-quarter of a period. Since the period is $\dfrac{1}{25} = 0.04$, we see that one-quarter of a period is 0.01. The point P also lies on the center line, which is $y = 2$. So the coordinates of P are $(-0.02, 2)$.

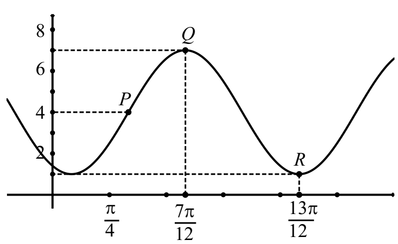
We now use the fact that the horizontal distance between Q and R is one-quarter of a period. The point R is on the center line of the sinusoid and so R has coordinates $(0, 2)$.

The point S is a low point on the sinusoid. So its y-coordinate will be D minus the amplitude, which is $2 - 6.3 = -4.3$. Using the fact that the horizontal distance from R to S is one-quarter of a period, the coordinates of S are $(0.01, -4.3)$. Since the point T is on the center line and the horizontal distance from S to T is one-quarter of a period, the coordinates of T are $(0.03, 2)$.

7. We will use a viewing window that is one-quarter of a period to the left of P and one-quarter of a period to the right of T. So we will use $-0.03 \le t \le 0.03$. Since the maximum value is 8.3 and the minimum value is -4.3, we will use $-5 \le y \le 9$.



Progress Check 2.15



1. The coordinates of Q are $\left(\dfrac{7\pi}{12}, 7\right)$ and the coordinates of R are $\left(\dfrac{13\pi}{12}, 1\right)$. So two times the amplitude is $7 - 1 = 6$ and the amplitude is 3.

2. We add the amplitude to the lowest y-value to determine D. This gives $D = 1 + 3 = 4$ and the center line is $y = 4$.

3. The horizontal distance between Q and R is $\dfrac{13\pi}{12} - \dfrac{7\pi}{12} = \dfrac{6\pi}{12}$. So we see that one-half of a period is $\dfrac{\pi}{2}$ and the period is π. So $B = \dfrac{2\pi}{\pi} = 2$.

4. For $y = A\cos(B(t - C)) + D$, we can use the point Q to determine a phase shift of $\dfrac{7\pi}{12}$. So an equation for this sinusoid is

$$y = 3\cos\left(2\left(t - \frac{7\pi}{12}\right)\right) + 4.$$

5. The point P is on the center line and so the horizontal distance between P and Q is one-quarter of a period. So this horizontal distance is $\dfrac{\pi}{4}$ and the t-coordinate of P is

$$\frac{7\pi}{12} - \frac{\pi}{4} = \frac{4\pi}{12} = \frac{\pi}{3}.$$

This can be the phase shift for $y = A\sin(B(t - C')) + D$. So another equation for this sinusoid is

$$y = 3\sin\left(2\left(t - \frac{\pi}{3}\right)\right) + 4.$$

Section 2.3

Progress Check 2.16

1. The maximum value of $V(t)$ is 140 ml and the minimum value of $V(t)$ is 70 ml. So the difference $(140 - 70 = 70)$ is twice the amplitude. Hence, the amplitude is 35 and we will use $A = 35$. The center line will then be 35 units below the maximum. That is, $D = 140 - 35 = 105$.

2. Since there are 50 beats per minute, the period is $\dfrac{1}{50}$ of a minute. Since we are using seconds for time, the period is $\dfrac{60}{50}$ seconds or $\dfrac{6}{5}$ sec. We can determine B by solving the equation

$$\frac{2\pi}{B} = \frac{6}{5}$$

for B. This gives $B = \dfrac{10\pi}{6} = \dfrac{5\pi}{3}$. Our function is

$$V(t) = 35\cos\left(\frac{5\pi}{3}t\right) + 105.$$

Progress Check 2.18

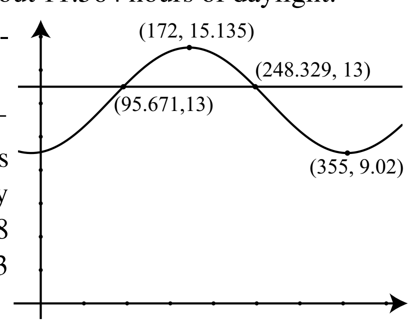
1. Since we have the coordinates for a high and low point, we first do the following computations:

   - $2(\text{amp}) = 15.35 - 9.02 = 6.33$. Hence, the amplitude is 3.165.
   - $D = 9.02 + 3.165 = 12.185$.
   - $\frac{1}{2}\text{period} = 355 - 172 = 183$. So the period is 366. Please note that we usually say that there are 365 days in a year. So it would also be reasonable to use a period of 365 days. Using a period of 366 days, we find that
   $$\frac{2\pi}{B} = 366,$$
   and hence $B = \dfrac{\pi}{183}$.

   We must now decide whether to use a sine function or a cosine function to get the phase shift. Since we have the coordinates of a high point, we will use a cosine function. For this, the phase shift will be 172. So our function is
   $$y = 3.165\cos\left(\frac{\pi}{183}(t - 172)\right) + 12.185.$$
   We can check this by verifying that when $t = 155$, $y = 15.135$ and that when $t = 355$, $y = 9.02$.

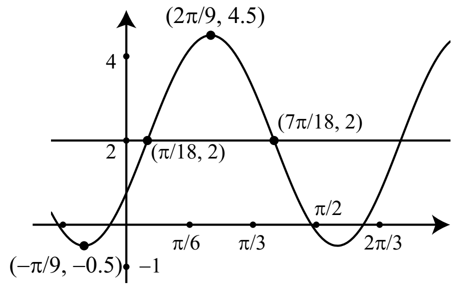
   (**a**) March 10 is day number 69. So we use $t = 69$ and get
   $$y = 3.165\cos\left(\frac{\pi}{183}(69 - 172)+\right) + 12.125 \approx 11.5642.$$
   So on March 10, 2014, there were about 11.564 hours of daylight.

   (**b**) We use a graphing utility to approximate the points of intersection of $y = 3.165\cos\left(\frac{\pi}{183}(69 - 172)+\right) + 12.125$ and $y = 13$. The results are shown to the right. So on day 96 (April 6, 2014) and on day 248 (September 5), there were about 13 hours of daylight.

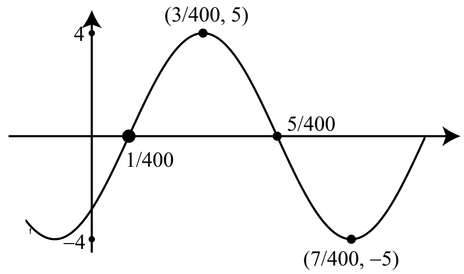


Progress Check 2.20

1.
   (a) The amplitude is 2.5.
   The period is $\dfrac{2\pi}{3}$.
   The phase shift is $-\dfrac{\pi}{9}$.
   The vertical shift is 2.



   (b) The amplitude is 4.
   The period is $\dfrac{1}{50}$.
   The phase shift is $\dfrac{1}{400}$.
   The vertical shift is 0.



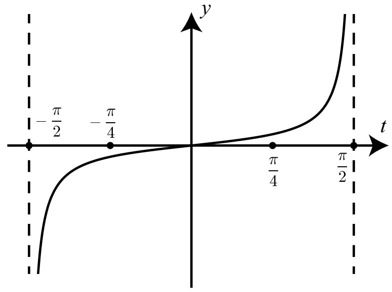
2.

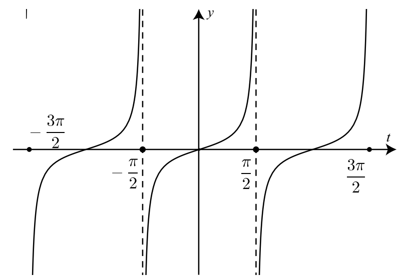
first equation		second equation
5.22	amplitude	5.153
12	period	12.30
3.7	phase shift	3.58
12.28	vertical shift	12.174

Section 2.4

Progress Check 2.21

The graphs for (1) and (2) are shown below.





3. In both graphs, the graph just to the left of $t = \dfrac{\pi}{2}$ and just to the right of $t = -\dfrac{\pi}{2}$ is consistent with the information in Table 2.4. The graph on the right is also consistent with the information in this table on both sides of $t = \dfrac{\pi}{2}$ and $t = -\dfrac{\pi}{2}$.

4. The range of the tangent function is the set of all real numbers.

5. Based on the graph in (2), the period of the tangent function appears to be π. The period is actually equal to π, and more information about this is given in Exercise (1).

Progress Check 2.23

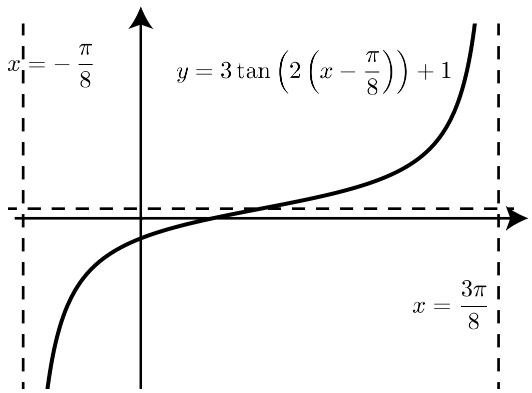
The equation for the function is $y = 3\tan\left(2\left(x - \dfrac{\pi}{8}\right)\right) + 1$.

1. The period of this function is $\dfrac{\pi}{2}$.

2. The effect of the parameter 3 is to vertically stretch the graph of the tangent function.

3. The effect of the parameter $\dfrac{\pi}{8}$ is to shift the graph of $y = 3\tan(2(x)) + 1$ to the right by $\dfrac{\pi}{8}$ units.

4. Following is a graph of one period of this function using $-\dfrac{\pi}{8} < x \leq \dfrac{3\pi}{8}$ and $-20 \leq y \leq 20$. The vertical asymptotes at $x = -\dfrac{\pi}{8}$ and $x = \dfrac{3\pi}{8}$ are shown as well as the horizontal line $y = 1$.



Progress Check 2.24

1. The secant function is the reciprocal of the cosine function. That is, $\sec(t) = \dfrac{1}{\cos(t)}$.

2. The domain of the secant function is the set of all real numbers t for which $t \neq \dfrac{\pi}{2} + k\pi$ for every integer k.

3. The graph of the secant function will have a vertical asymptote at those values of t that are not in the domain. So there will be a vertical asymptote when $t = \dfrac{\pi}{2} + k\pi$ for some integer k.

4. Since $\sec(t) = \dfrac{1}{\cos(t)}$, and the period of the cosine function is 2π, we conclude that the period of the secant function is also 2π.

Progress Check 2.26

1. All of the graphs are consistent.

2. Since $\sec(x) = \dfrac{1}{\cos(x)}$, we see that $\sec(x) > 0$ if and only if $\cos(x) > 0$. So the graph of $y = \sec(x)$ is above the x-axis if and only if the graph of $y = \cos(x)$ is above the x-axis.

3. Since $\sec(x) = \dfrac{1}{\cos(x)}$, we see that $\sec(x) < 0$ if and only if $\cos(x) < 0$. So the graph of $y = \sec(x)$ is below the x-axis if and only if the graph of $y = \cos(x)$ is below the x-axis.

4. The key is that $\sec(x) = \dfrac{1}{\cos(x)}$. Since $-1 \leq \cos(x) \leq 1$, we conclude that $\sec(x) \geq 1$ when $\cos(x) > 0$ and $\sec(x) \leq -1$ when $\cos(x) < 0$. Since the graph of the secant function has vertical asymptotes, we see that the range of the secant function consists of all real numbers y for which $y \geq 1$ or $y \leq -1$. This can also be seen on the graph of $y = \sec(x)$.

Section 2.5

Progress Check 2.28

1. $\arcsin\left(-\dfrac{\sqrt{3}}{2}\right) = -\dfrac{\pi}{3}$ since $\sin\left(-\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$ and $-\dfrac{\pi}{2} \leq -\dfrac{\pi}{3} \leq \dfrac{\pi}{2}$.

2. $\sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$ since $\sin\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$ and $-\dfrac{\pi}{2} \leq \dfrac{\pi}{6} \leq \dfrac{\pi}{2}$.

3. $\arcsin(-1) = -\dfrac{\pi}{2}$ since $\sin\left(-\dfrac{\pi}{2}\right) = -1$ and $-\dfrac{\pi}{2} \leq -\dfrac{\pi}{2} \leq \dfrac{\pi}{2}$.

4. $\arcsin\left(-\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\pi}{4}$ since $\sin\left(-\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{2} \leq -\dfrac{\pi}{4} \leq \dfrac{\pi}{2}$.

Progress Check 2.29

1. Since $\sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$, we see that $\sin\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right) = \sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$.

2. $\arcsin\left(\sin\left(\dfrac{\pi}{4}\right)\right) = \arcsin\left(\dfrac{\sqrt{2}}{2}\right)$. In addition, $\arcsin\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$ since $\sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{2} \leq \dfrac{\pi}{4} \leq \dfrac{\pi}{2}$. So we see that

$$\arcsin\left(\sin\left(\frac{\pi}{4}\right)\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

3. We do not know an exact value for $\sin^{-1}\left(\dfrac{2}{5}\right)$. So we let $t = \sin^{-1}\left(\dfrac{2}{5}\right)$.

We then know that $\sin(t) = \dfrac{2}{5}$ and $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$. So

$$\sin\left(\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(t) = \frac{2}{5}.$$

4. $\arcsin\left(\sin\left(\dfrac{3\pi}{4}\right)\right) = \arcsin\left(\dfrac{\sqrt{2}}{2}\right)$. In addition, $\arcsin\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$ since $\sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{2} \leq \dfrac{\pi}{4} \leq \dfrac{\pi}{2}$. So we see that

$$\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

Progress Check 2.31

1. Since $\cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$, we see that $\cos\left(\cos^{-1}\left(\dfrac{1}{2}\right)\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$.

2. $\arccos\left(\cos\left(\dfrac{\pi}{4}\right)\right) = \arccos\left(\dfrac{\sqrt{2}}{2}\right)$. In addition, $\arccos\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$ since $\cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$ and $0 \leq \dfrac{\pi}{4} \leq \pi$. So we see that

$$\arccos\left(\cos\left(\frac{\pi}{4}\right)\right) = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

3. $\arccos\left(\cos\left(-\dfrac{\pi}{4}\right)\right) = \arccos\left(\dfrac{\sqrt{2}}{2}\right)$. In addition, $\arccos\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$ since $\cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$ and $0 \leq \dfrac{3\pi}{4} \leq \pi$. So we see that

$$\arccos\left(\cos\left(-\frac{\pi}{4}\right)\right) = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

4. $\tan^{-1}\left(\tan\left(\dfrac{5\pi}{4}\right)\right) = \tan^{-1}(1)$. In addition, $\tan^{-1}(1) = \dfrac{\pi}{4}$ since $\tan\left(\dfrac{\pi}{4}\right) = 1$ and $-\dfrac{\pi}{2} < \dfrac{\pi}{4} < \dfrac{\pi}{2}$. So we see that

$$\tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right) = \tan^{-1}(1) = \frac{\pi}{4}.$$

Progress Check 2.32

1. $y = \arccos(1) = 0$
2. $y = \tan^{-1}\left(\sqrt{3}\right) = \dfrac{\pi}{3}$
3. $y = \arctan(-1) = -\dfrac{\pi}{4}$
4. $y = \cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\pi}{4}$
5. $\sin\left(\arccos\left(\dfrac{1}{2}\right)\right) = \dfrac{\sqrt{3}}{2}$
6. $\tan\left(\arcsin\left(-\dfrac{\sqrt{3}}{2}\right)\right) = -\sqrt{3}$
7. $\arccos\left(\sin\left(\dfrac{\pi}{6}\right)\right) = \dfrac{\pi}{3}$

Progress Check 2.33

1. Let $t = \arccos\left(\dfrac{1}{3}\right)$. We then know that

$$\cos(t) = \frac{1}{3} \quad \text{and} \quad 0 \leq t \leq \pi.$$

Using the Pytagorean Identity, we see that $\left(\dfrac{1}{3}\right)^2 + \sin^2(t) = 1$ and this implies that $\sin^2(t) = \dfrac{8}{9}$. Since $0 \leq t \leq \pi$, t is in the second quadrant and in both of these quadrants, $\sin(t) > 0$. So, $\sin(t) = \frac{\sqrt{8}}{3}$. That is,

$$\sin\left(\arccos\left(\frac{1}{3}\right)\right) = \frac{\sqrt{8}}{3}.$$

2. For $\cos\left(\arcsin\left(-\dfrac{4}{7}\right)\right)$, we let $t = \arcsin\left(-\dfrac{4}{7}\right)$. This means that

$$\sin(t) = -\frac{4}{7} \quad \text{and} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}.$$

We can use the Pythagorean Identity to obtain $\cos^2(t) + \left(-\dfrac{4}{7}\right)^2 = 1$. This gives $\cos^2(t) = \dfrac{33}{49}$. We also have the restriction $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$ and we know $\sin(t) < 0$. This means that t must be in QIV and so $\cos(t) > 0$. Hence, $\cos(t) = \dfrac{\sqrt{33}}{7}$. That is,

$$\cos\left(\arcsin\left(-\frac{4}{7}\right)\right) = \frac{\sqrt{33}}{7}.$$

Note: You can use your calculator to check this work. Use your calculator to approximate both $\cos\left(\arcsin\left(-\frac{4}{7}\right)\right)$ and $\frac{\sqrt{33}}{7}$. Both results should be 0.8206518066.

Section 2.6

Progress Check 2.34

Any solution of the equation $\sin(x) = -0.6$ may be approximated with one of the following:

$$x \approx -0.64350 + k(2\pi) \quad \text{or} \quad x \approx -2.49809 + k(2\pi).$$

Progress Check 2.36

We first rewrite the equation $4\cos(x) + 3 = 2$ as follows:

$$\begin{aligned} 4\cos(x) + 3 &= 2 \\ 4\cos(x) &= -1 \\ \cos(x) &= -\frac{1}{4} \end{aligned}$$

So in the interval $[-\pi, \pi]$, the solutions are $x_1 = \arccos\left(-\frac{1}{4}\right)$ and $x_2 = -\arccos\left(-\frac{1}{4}\right)$. So any solution of the equation $4\cos(x) + 3 = 2$ is of the form

$$x = \arccos\left(-\frac{1}{4}\right) + k(2\pi) \quad \text{or} \quad x = -\arccos\left(-\frac{1}{4}\right) + k(2\pi).$$

Progress Check 2.38

We first use algebra to rewrite the equation $2\sin(x) + 1.2 = 2.5$ in the form

$$\sin(x) = 0.65.$$

So in the interval $[-\pi, \pi]$, the solutions are $x_1 = \arcsin(0.65)$ and $x_2 = \pi - \arcsin(0.65)$. So any solution of the equation $2\sin(x) + 1.2 = 2.5$ is of the form

$$x = \arcsin(0.65) + k(2\pi) \quad \text{or} \quad x = \pi - \arcsin(0.65) + k(2\pi).$$

Progress Check 2.39

1.

$$\begin{aligned} 3\cos(2x+1)+6 &= 5 \\ 3\cos(2x+1) &= -1 \\ \cos(2x+1) &= -\frac{1}{3} \end{aligned}$$

2. $t = \cos^{-1}\left(-\dfrac{1}{3}\right)$ or $t = -\cos^{-1}\left(-\dfrac{1}{3}\right)$.

3.

$$\begin{aligned} 2x+1 &= \cos^{-1}\left(-\frac{1}{3}\right) & \qquad 2x+1 &= -\cos^{-1}\left(-\frac{1}{3}\right) \\ 2x &= \cos^{-1}\left(-\frac{1}{3}\right)-1 & 2x &= -\cos^{-1}\left(-\frac{1}{3}\right)-1 \\ x &= \frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right)-\frac{1}{2} & x &= -\frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right)-\frac{1}{2} \end{aligned}$$

4. The period of the function $y = \cos(2x+1)$ is π. So the following formulas can be used to generate the solutions for the equation.

$$x = \left(\frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right)-\frac{1}{2}\right)+k\pi \quad \text{or} \quad x = \left(-\frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right)-\frac{1}{2}\right)+k\pi,$$

where k is some integer. Notice that we added an integer multiple of the period, which is π, to the solutions in (3).

Progress Check 2.40

We first write the equation $4\tan(x)+1 = 10$ in the form $\tan(x) = \dfrac{9}{4}$. So the only solution of the equation in the interval $\left(-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}\right)$ is

$$x = \arctan\left(\frac{9}{4}\right).$$

Since the period of the tangent function is π, any solution of this equation can be written in the form

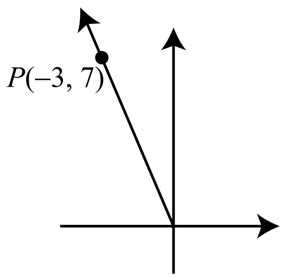
$$x = \arctan\left(\frac{9}{4}\right)+k\pi,$$

where k is some integer.

Section 3.1

Progress Check 3.1

1.



2. $r = \sqrt{(-3)^2 + 7^2} = \sqrt{58}$

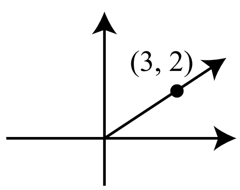
3.

$$\cos(\theta) = -\frac{3}{\sqrt{58}} \qquad \tan(\theta) = -\frac{7}{3} \qquad \sec(\theta) = -\frac{\sqrt{58}}{3}$$

$$\sin(\theta) = \frac{7}{\sqrt{58}} \qquad \cot(\theta) = -\frac{3}{7} \qquad \csc(\theta) = \frac{\sqrt{58}}{7}$$

Progress Check 3.2

1.



2. Since $\tan(\alpha) = \dfrac{2}{3}$, we can conclude that the point $(3, 2)$ lies on the terminal side of α.

3. Since $(3, 2)$ is on the terminal side of α, we can use $x = 3$, $y = 2$, and $r = \sqrt{3^2 + 2^2} = \sqrt{13}$. So

$$\cos(\theta) = \frac{2}{\sqrt{13}} \qquad \tan(\theta) = \frac{2}{3} \qquad \sec(\theta) = \frac{\sqrt{13}}{2}$$

$$\sin(\theta) = \frac{3}{\sqrt{13}} \qquad \cot(\theta) = \frac{3}{2} \qquad \csc(\theta) = \frac{\sqrt{13}}{3}$$

Progress Check 3.3

The completed work should look something like the following:

$$\begin{aligned}\cos^2(\theta) + \sin^2(\theta) &= \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 \\ &= \frac{x^2}{r^2} + \frac{y^2}{r^2} \\ &= \frac{x^2 + y^2}{r^2} \\ &= \frac{r^2}{r^2} \\ &= 1\end{aligned}$$

Progress Check 3.4

1. Using the Pythagorean Identity, we see that $\cos^2(\theta) + \left(\dfrac{1}{3}\right)^2 = 1$ and so $\cos^2(\theta) = \dfrac{8}{9}$. Since $\dfrac{\pi}{2} < \theta < \pi$, $\cos(\theta) < 0$. Hence, $\cos(\theta) = -\dfrac{\sqrt{8}}{3}$.

2. $\tan(\theta) = \dfrac{\frac{1}{3}}{-\frac{\sqrt{8}}{3}} = -\dfrac{1}{\sqrt{8}}$.

3. $\cot(\theta) = -\sqrt{8}$, $\csc(\theta) = 3$, and $\sec(\theta) = -\dfrac{3}{\sqrt{8}}$.

Progress Check 3.5

2. $\tan^{-1}(-2.5) \approx -68.199°$.

3. $\theta \approx -68.199° + 180° \approx 111.801°$.

Section 3.2

Progress Check 3.9

We let α be the angle opposite the side of length 5 feet and let β be the angle adjacent to that side. We then see that

$$\begin{aligned} \sin(\alpha) &= \frac{5}{17} & \cos(\beta) &= \frac{5}{17} \\ \alpha &= \arcsin\left(\frac{5}{17}\right) & \beta &= \arccos\left(\frac{5}{17}\right) \\ \alpha &\approx 17.1046^\circ & \beta &= 72.8954^\circ \end{aligned}$$

As a check, we notice that $\alpha + \beta = 90^\circ$. We can use the Pythagorean theorem to determine the third side, which using our notation, is b. So

$$5^2 + b^2 = 17^2,$$

and so we see that $b = \sqrt{264} \approx 16.2481$ feet.

Progress Check 3.11

With a rise of 1 foot for every 12 feet of run, we see if we let θ be the angle of elevation, then

$$\begin{aligned} \tan(\theta) &= \frac{1}{12} \\ \theta &= \arctan\left(\frac{1}{12}\right) \\ \theta &\approx 4.7636^\circ \end{aligned}$$

The length of the ramp will be the hypotenuse of the right triangle. So if we let h be the length of the hypotenuse, then

$$\begin{aligned} \sin(\theta) &= \frac{7.5}{h} \\ h &= \frac{7.5}{\sin(\theta)} \\ h &\approx 90.3120 \end{aligned}$$

The length of the hypotenuse is approximately 90.3 feet. We can check our result by determining the length of the third side, which is $7.5 \cdot 12$ or 90 feet and then verifying the result of the Pythagorean theorem. We can verify that

$$7.5^2 + 90^2 \approx 90.3120^2.$$

Progress Check 3.12

1. $h = x\tan(\alpha)$. So
$$\tan(\beta) = \frac{x\tan(\alpha)}{d+x}. \tag{3}$$

2. $\tan(\beta)(d+x) = x\tan(\alpha)$.

3. We can proceed to solve for x as follows:
$$\begin{aligned} d\tan(\beta) + x\tan(\beta) &= x\tan(\alpha) \\ d\tan(\beta) &= x\tan(\alpha) - x\tan(\beta) \\ d\tan(\beta) &= x(\tan(\alpha) - \tan(\beta)) \\ \frac{d\tan(\beta)}{\tan(\alpha) - \tan(\beta)} &= x \end{aligned}$$
So we see that $x = \dfrac{22.75\tan(34.7°)}{\tan(43.2°) - \tan(34.7°)} \approx 63.872$. Using this value for x, we obtain $h = x\tan(43.2°) \approx 59.980$. So the top of the flagpole is about 59.98 feet above the ground.

4. There are several ways to check this result. One is to use the values for d, h, and x and the inverse tangent function to determine the values for α and β. If we use approximate values for d, h, and x, these checks may not be exact. For example,
$$\begin{aligned} \alpha &= \arctan\left(\frac{h}{x}\right) \approx \arctan\left(\frac{59.98}{63.872}\right) \approx 43.2° \\ \beta &= \arctan\left(\frac{h}{d+x}\right) \approx \arctan\left(\frac{59.980}{22.75 + 63.872}\right) \approx 34.7° \end{aligned}$$
Another method to check the results is to use the sine of α or β to determine the length of the hypotenuse of one of the right triangles and then check using the Pythagorean Theorem.

Section 3.3

Progress Check 3.14

We first note that the third angle in the triangle is 30° since the sum of the two given angles is 150°. We let x be the length of the side opposite the 15° angle and let y be the length of the side opposite the $135°$ angle. We then see that

$$\begin{aligned} \frac{x}{\sin(15°)} &= \frac{71}{\sin(30°)} \\ x &= \frac{71\sin(15°)}{\sin(30°)} \\ x &\approx 36.752 \end{aligned} \qquad \begin{aligned} \frac{y}{\sin(135°)} &= \frac{71}{\sin(30°)} \\ y &= \frac{71\sin(135°)}{\sin(30°)} \\ y &\approx 100.409 \end{aligned}$$

So the length of the side opposite the $15°$ angle is about 36.75 inches, and the length of the side opposite the $135°$ angle is about 100.41 inches.

Progress Check 3.15

1. The side opposite the angle of $40°$ has length 1.7 feet. So we get

$$\begin{aligned} \frac{\sin(\theta)}{2} &= \frac{\sin(40°)}{1.7} \\ \sin(\theta) &= \frac{2\sin(40°)}{1.7} \approx 0.75622 \end{aligned}$$

2. We see that

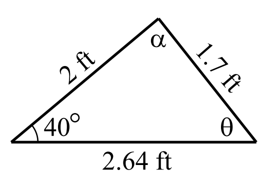
$$\theta_1 = \sin^{-1}\left(\frac{2\sin(40°)}{1.7}\right) \approx 49.132°.$$

3. $\theta_2 = 180° - \theta_1 \approx 130.868°$. Using reference angles instead of reference arcs, θ_1 is the reference angle for θ_2, which is in the second quadrant. Hence, $\sin(\theta_2) = \sin(\theta_1)$.

4. The third angle α can be determined using the sum of the angles of a triangle.

$$\begin{aligned} \alpha + \theta_1 + 40° &= 180° \\ \alpha &\approx 180° - 40° - 49.132° \\ \alpha &\approx 90.868° \end{aligned}$$

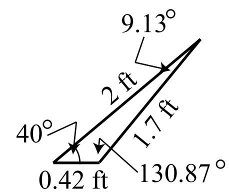
We use the Law of Sines to determine the length x of the side opposite α. The resulting triangle is shown on the right.



$$\begin{aligned} \frac{x}{\sin(\alpha)} &= \frac{1.7}{\sin(40°)} \\ x &= \frac{1.7\sin(\alpha)}{\sin(40°)} \\ x &\approx 2.644 \text{ ft} \end{aligned}$$

5. Using the same procedure that we did in part (4), we obtain

$$\theta_2 \approx 130.868°$$
$$\alpha_2 \approx 9.132°$$
$$x_2 \approx 0.420 \text{ ft}$$



The triangle is shown on the right.

Progress Check 3.16

1. The side opposite the angle of 40° has length 3 feet. So we get

$$\frac{\sin(\theta)}{2} = \frac{\sin(40°)}{3}$$
$$\sin(\theta) = \frac{2\sin(40°)}{3} \approx 0.42853$$

2. We see that
$$\theta_1 = \sin^{-1}\left(\frac{2\sin(40°)}{3}\right) \approx 25.374°.$$

3. $\theta_2 = 180° - \theta_1 \approx 154.626°$. Using reference angles instead of reference arcs, θ_1 is the reference angle for θ_2, which is in the second quadrant. Hence, $\sin(\theta_2) = \sin(\theta_1)$.

4. The third angle α can be determined using the sum of the angles of a triangle.

$$\alpha + \theta_1 + 40° = 180°$$
$$\alpha \approx 180° - 40° - 25.374°$$
$$\alpha \approx 114.626°$$

We use the Law of Sines to determine the length x of the side opposite α. The resulting triangle is shown on the right.

$$\frac{x}{\sin(\alpha)} = \frac{3}{\sin(40°)}$$
$$x = \frac{3\sin(\alpha)}{\sin(40°)}$$
$$x \approx 4.243 \text{ ft}$$

5. Using the same procedure that we did in part (4), we obtain

$$\theta_2 \approx 154.626^\circ$$
$$40^\circ + \theta_2 = 194.626\circ$$

This is not possible since the sum of the angles of a triangle is 180°. So there is no triangle where the angle the angle opposite the side of length 2 is θ_2.

Progress Check 3.17

1. Using the Law of Cosines, we obtain

$$\begin{aligned} c^2 &= 3.5^2 + 2.5^2 - 2(3.5)(2.5)\cos\left(60^\circ\right) \\ &= 9.75 \end{aligned}$$

So $c = \sqrt{9.75} \approx 3.12250$ ft.

2. Using the Law of Sines, we obtain

$$\begin{aligned} \frac{\sin(\alpha)}{2.5} &= \frac{\sin(60^\circ)}{c} \\ \sin(\alpha) &= \frac{2.5\sin(60^\circ)}{c} \approx 0.69338 \end{aligned}$$

From this, we get $\alpha \approx 43.898^\circ$ or $\alpha \approx 136.102^\circ$. However, since the given angle in 60°, the second value is not possible since $136.102^\circ + 60^\circ < 180^\circ$. So $\alpha \approx 43.898^\circ$.

3. Since the sum of the angles of a triangle must be 180°, we have

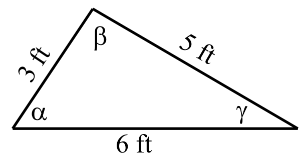
$$\begin{aligned} 60^\circ + 43.898^\circ + \beta &= 180^\circ \\ \beta &\approx 76.102^\circ \end{aligned}$$

4. With the values we have determined, we can check our work by showing that

$$\frac{\sin(60^\circ)}{c} = \frac{\sin(\alpha)}{2.5} = \frac{\sin(\beta)}{3.5} \approx 0.27735.$$

Progress Check 3.18

The first step is to draw a reasonably accurate diagram and label the angles. We will use the diagram on the right.



Using the Law of Cosines, we obtain

$$\begin{aligned} 5^2 &= 3^2 + 6^2 - 2(3)(6)\cos(\alpha) \\ \cos(\alpha) &= \frac{20}{36} \\ \alpha &\approx 56.251^\circ \end{aligned} \qquad \begin{aligned} 6^2 &= 3^2 + 5^2 - 2(3)(5)\cos(\beta) \\ \cos(\beta) &= \frac{-2}{30} \\ \beta &\approx 98.823^\circ \end{aligned}$$

$$\begin{aligned} 3^2 &= 5^2 + 6^2 - 2(5)(6)\cos(\gamma) \\ \cos(\gamma) &= \frac{52}{60} \\ \gamma &\approx 29.926^\circ \end{aligned}$$

We check these results by verifying that $\alpha + \beta + \gamma = 180^\circ$.

Section 3.4

Progress Check 3.20

We first note that $\angle BAC = 180^\circ - 94.2^\circ - 48.5^\circ$ and so $\angle BAC = 37.3^\circ$. We can then use the Law of Sines to determine the length from A to B as follows:

$$\begin{aligned} \frac{AB}{\sin(48.5^\circ)} &= \frac{98.5}{\sin(37.3^\circ)} \\ AB &= \frac{98.5\sin(48.5^\circ)}{\sin(37.3^\circ)} \\ AB &\approx 121.7 \end{aligned}$$

The bridge from point B to point A will be approximately 121.7 feet long.

Progress Check 3.21

Using the right triangle, we see that $\sin(26.5°) = \dfrac{h}{5}$. So $h = 5\sin(26.5°)$, and the area of the triangle is

$$\begin{aligned} A &= \frac{1}{2}(7)\left[5\sin\left(26.5°\right)\right] \\ &= \frac{35}{2}\sin\left(26.5°\right) \approx 7.8085 \end{aligned}$$

The area of the triangle is approximately 7.8085 square meters.

Progress Check 3.22

Using the right triangle, we see that $\sin(\theta) = \dfrac{h}{a}$. So $h = a\sin(\theta)$, and the area of the triangle is

$$\begin{aligned} A &= \frac{1}{2}b\left(a\sin(\theta)\right) \\ &= \frac{1}{2}ab\sin(\theta) \end{aligned}$$

Progress Check 3.23

1. Using the Law of Cosines, we see that

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos(\gamma) \\ 2ab\cos(\gamma) &= a^2 + b^2 - c^2 \\ \cos(\gamma) &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}$$

2. We see that

$$\sin^2(\gamma) = 1 - \cos^2(\gamma).$$

Since γ is between $0°$ and $180°$, we know that $\sin(\gamma) > 0$ and so

$$\sin(\gamma) = \sqrt{1 - \left(\frac{a^2 + b^2 - c^2}{2ab}\right)^2}.$$

3. Substituting the equation in part (2) into the formula $A = \frac{1}{2}ab\sin(\gamma)$, we obtain

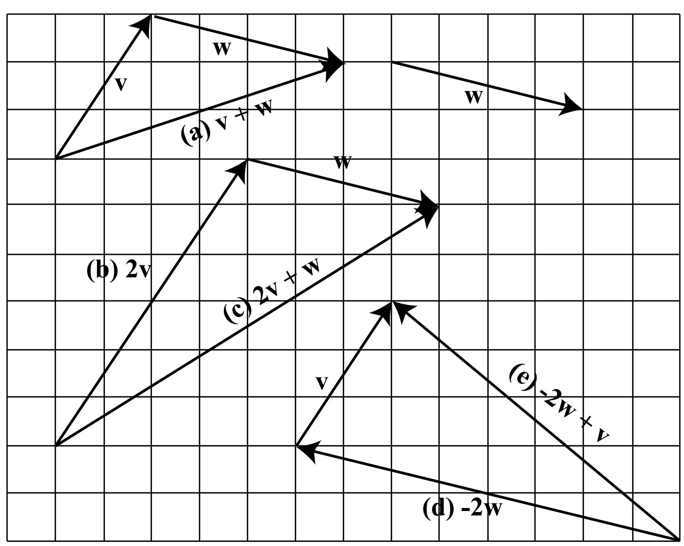
$$\begin{aligned} A &= \frac{1}{2}ab\sin(\gamma) \\ &= \frac{1}{2}ab\sqrt{1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2} \end{aligned}$$

Section 3.5

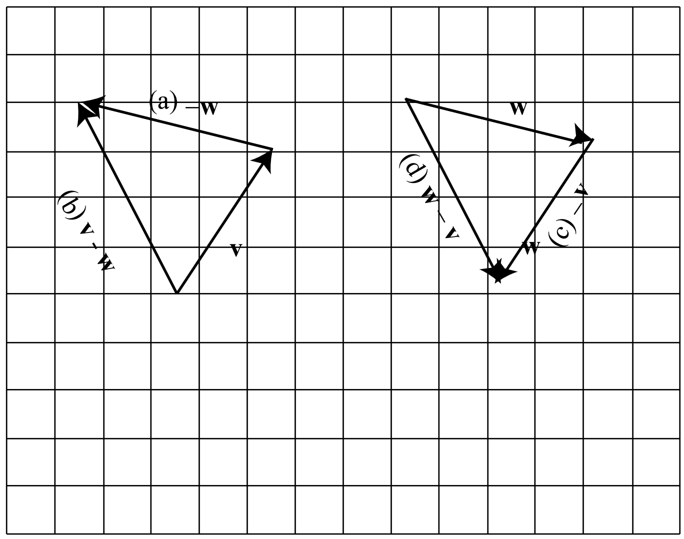
Progress Check 3.24

The vector **w** is the only vector that is equal to the vector **v**. Vector **u** has the same direction as **v** but a different magnitude. Vector **a** has the same magnitude as **v** but a different direction (note that the direction of **a** is the opposite direction of **v**). Vector **b** has a different direction and a different magnitude than **v**.

Progress Check 3.25



Progress Check 3.26



Progress Check 3.27

1. $\angle ABC = 180° - \theta = 127°$.

2. Using the Law of Cosines, we see that

$$\begin{aligned} |\mathbf{a}+\mathbf{b}|^2 &= |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2|\mathbf{a}| \cdot |\mathbf{b}| \cos(\angle ABC) \\ &= 80^2 + 60^2 - 2 \cdot 60 \cdot 80 \cos\left(127°\right) \\ &= 10000 - 9600 \cos\left(127°\right) \\ &\approx 15777.42422 \end{aligned}$$

So we see that $|\mathbf{a}+\mathbf{b}| \approx 125.61$.

3. The angle between the vectors **a** and $\mathbf{a}+\mathbf{b}$ is $\angle CAB$. In $\triangle ABC$, we know that $\angle ABC = 127°$, and so $\angle CAB$ must be an acute angle. We will use the Law of Sines to determine this angle.

$$\begin{aligned} \frac{\sin(\angle CAB)}{|\mathbf{b}|} &= \frac{\sin(\angle ABC)}{|\mathbf{a}+\mathbf{b}|} \\ \sin(\angle CAB) &= \frac{60 \sin\left(127°\right)}{|\mathbf{a}+\mathbf{b}|} \\ \sin(\angle CAB) &\approx 0.38148341 \end{aligned}$$

So the angle between the vectors **a** and $\mathbf{a}+\mathbf{b}$ is approximately $22.43°$.

Progress Check 3.29

Using the Law of Sines, we see that

$$\begin{aligned}
\frac{|\mathbf{a}|}{\sin(20°)} &= \frac{100}{\sin(140°)} \\
|\mathbf{a}| &= \frac{100\sin(20°)}{\sin(140°)} \\
|\mathbf{a}| &\approx 53.21
\end{aligned}$$

The magnitude of the vector **a** (and the vector **b**) is approximately 53.21 pounds.

Progress Check 3.30

Using the notation in Figure 3.28, we obtain the following:

$$\begin{aligned}
\frac{|\mathbf{b}|}{|\mathbf{w}|} &= \cos\left(12°\right) & \qquad \frac{|\mathbf{a}|}{|\mathbf{w}|} &= \sin\left(12°\right) \\
|\mathbf{b}| &= |\mathbf{w}|\cos\left(12°\right) & |\mathbf{a}| &= |\mathbf{w}|\sin\left(12°\right) \\
|\mathbf{b}| &\approx 244.54 & |\mathbf{a}| &\approx 51.98
\end{aligned}$$

The object exerts a force of about 244.54 pounds perpendicular to the plane and the force of gravity down the plane on the object is about 51.98 pounds. So in order to keep the object stationary, a force of about 51.98 pounds up the plane must be applied to the object.

Section 3.6

Progress Check 3.31

1. $\mathbf{v} = 7\mathbf{i} + (-3)\mathbf{j}$. So $|\mathbf{v}| = \sqrt{7^2 + (-3)^2} = \sqrt{58}$. In addition,

$$\cos(\theta) = \frac{7}{\sqrt{58}} \quad \text{and} \quad \sin(\theta) = \frac{-3}{\sqrt{58}}.$$

So the terminal side of θ is in the fourth quadrant, and we can write

$$\theta = 360° - \arccos\left(\frac{7}{\sqrt{58}}\right).$$

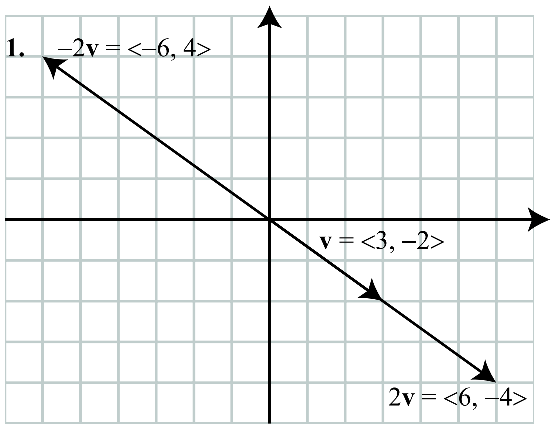
So $\theta \approx 336.80°$.

2. We are given $|\mathbf{w}| = 20$ and the direction angle θ of $\mathbf{w}$ is $200°$. So if we write $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j}$, then

$$\begin{aligned} w_1 &= 20\cos\left(200°\right) & \qquad w_2 &= 20\sin\left(200°\right) \\ &\approx -1.794 & &\approx -6.840 \end{aligned}$$

Progress Check 3.32

1.



2. For a vector $\mathbf{a} = \langle a_1, a_2 \rangle$ and a scalar c, we define the scalar multiple $c\mathbf{a}$ to be

$$c\mathbf{a} = \langle ca_1, ca_2 \rangle.$$

Progress Check 3.33

Let $\mathbf{u} = \langle 1, -2 \rangle$, $\mathbf{v} = \langle 0, 4 \rangle$, and $\mathbf{w} = \langle -5, 7 \rangle$.

1. $2\mathbf{u} - 3\mathbf{v} = \langle 2, -4 \rangle - \langle 0, 12 \rangle = \langle 2, -16 \rangle$.

2. $|2\mathbf{u} - 3\mathbf{v}| = \sqrt{2^2 + (-16)^2} = \sqrt{260}$. So now let θ be the direction angle of $2\mathbf{u} - 3\mathbf{v}$. Then

$$\cos(\theta) = \frac{2}{\sqrt{260}} \quad \text{and} \quad \sin(\theta) = \frac{-16}{\sqrt{260}}.$$

So the terminal side of θ is in the fourth quadrant. We see that $\arcsin\left(\frac{-16}{\sqrt{260}}\right) \approx -82.87°$. Since the direction angle θ must satisfy $0 \le \theta < 360°$, we see that $\theta \approx -82.87° + 360° \approx 277.13°$.

3. $\mathbf{u} + 2\mathbf{v} - 7\mathbf{w} = \langle 1, -2 \rangle + \langle 0, 8 \rangle - \langle -35, 49 \rangle = \langle 36, -43 \rangle$.

Progress Check 3.34

1. If θ is the angle between $\mathbf{u} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = -5\mathbf{i} + 2\mathbf{j}$, then

$$\cos(\theta) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \frac{-13}{\sqrt{10}\sqrt{29}}$$
$$\theta = \cos^{-1}\left(\frac{-13}{\sqrt{10}\sqrt{29}}\right)$$

so $\theta \approx 139.764°$.

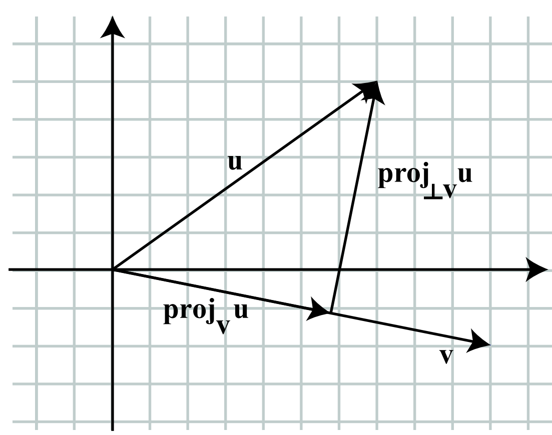
2. If $\mathbf{v} = \langle a, b\rangle$ is perpendicular to $\mathbf{u} = \langle 1, 3\rangle$, then the angle θ between them is $90°$ and so $\cos(\theta) = 0$. So we must have $\mathbf{u}\cdot\mathbf{v} = 0$ and this means that $a + 3b = 0$. So any vector $\mathbf{v} = \langle a, b\rangle$ where $a = -3b$ will be perpendicular to $\mathbf{v}$, and there are infinitely many such vectors. One vector perpendicular to $\mathbf{u}$ is $\langle -3, 1\rangle$.

Progress Check 3.35

Let $\mathbf{u} = \langle 7, 5\rangle$ and $\mathbf{v} = \langle 10, -2\rangle$. Then

$$\begin{aligned}\mathbf{proj_v u} &= \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} = \frac{60}{104}\mathbf{v} \\ &= \left\langle \frac{600}{104}, \frac{-120}{104}\right\rangle \\ &\approx \langle 5.769, -1.154\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{proj_{\perp v} u} &= \mathbf{u} - \mathbf{proj_v u} \\ &= \langle 7, 5\rangle - \left\langle \frac{600}{104}, \frac{-120}{104}\right\rangle \\ &= \left\langle \frac{128}{104}, \frac{640}{104}\right\rangle \\ &\approx \langle 1.231, 6.154\rangle\end{aligned}$$



Section 4.1

Progress Check 4.4

1. The graphs of both sides of the equation indicate that this is an indentity.
2. The graphs of both sides of the equation indicate that this is not an indentity. For example, if we let $x = \dfrac{\pi}{2}$, then

$$\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) = 0 \cdot 1 = 0 \qquad \text{and} \qquad 2\sin\left(\frac{\pi}{2}\right) = 2 \cdot 1 = 2.$$

.

Section 4.2

Progress Check 4.6

We divide both sides of the equation $4\cos(x) = 2\sqrt{2}$ by 4 to get $\cos(x) = \dfrac{\sqrt{2}}{2}$. So

$$x = \frac{\pi}{4} + k(2\pi) \quad \text{or} \quad x = \frac{7\pi}{4} + k(2\pi),$$

where k is an integer.

Progress Check 4.7

1. We divide both sides of the equation $5\sin(x) = 2$ by 5 to get $\sin(x) = 0.4$. So

$$x = \sin^{-1}(0.4) + k(2\pi) \quad \text{or} \quad x = (\pi - \sin^{-1}(0.4)) + k(2\pi),$$

where k is an integer.

2. We use $\alpha = 40°$ and $\dfrac{c_a}{c_w} = 1.33$ in the Law of Refraction.

$$\begin{aligned}
\frac{\sin(40°)}{\sin(\beta)} &= 1.33 \\
\sin(\beta) &= \frac{\sin(40°)}{1.33} \approx 0.483299 \\
\beta &\approx 28.90°
\end{aligned}$$

The angle of refraction is approximately 28.90°.

Progress Check 4.9

We will use the identity $\cos^2(x) = 1 - \sin^{(}x)$. So we have

$$\begin{aligned}\sin^2(x) &= 3\left(1 - \sin^2(x)\right)\\ \sin^2(x) &= \frac{3}{4}\end{aligned}$$

So we have $\sin(x) = \dfrac{\sqrt{3}}{2}$ or $\sin(x) = -\dfrac{\sqrt{3}}{2}$. For the first equation, we see that

$$x = \frac{\pi}{3} + 2\pi k \quad \text{or} \quad x = \frac{\pi}{3} + 2\pi k,$$

where k is an integer, and for the second equation, we have

$$x = \frac{4\pi}{3} + 2\pi k \quad \text{or} \quad x = \frac{5\pi}{3} + 2\pi k,$$

where k is an integer. The graphs of $y = \sin^2(x)$ and $y = 3\cos^2(x)$ will show 4 points of intersection on the interval $[0, 2\pi]$.

Progress Check 4.11

We write the equation as $\sin^2(x) - 4\sin(x) + 3 = 0$ and factor the right side to get $(\sin(x) - 3)(\sin(x) - 1) = 0$. So we see that $\sin(x) - 3 = 0$ or $\sin(x) - 1 = 0$. However, the equation $\sin(x) - 3 = 0$ is equivalent to $\sin(x) = 3$, and this equation has no solution. We write $\sin(x) - 1 = 0$ as $\sin(x) = 1$ and so the solutions are

$$x = \frac{\pi}{2} + 2\pi k,$$

where k is an integer.

Section 4.3

Progress Check 4.13

1. We first note that $\dfrac{7\pi}{12} = \dfrac{9\pi}{12} - \dfrac{2\pi}{6} = \dfrac{3\pi}{4} - \dfrac{\pi}{6}$.

$$\begin{aligned}
\cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) \\
&= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) \\
&= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\
&= \frac{-\sqrt{6}+\sqrt{2}}{4}.
\end{aligned}$$

2.

$$\begin{aligned}
\cos\left(\frac{5\pi}{12}\right) &= \cos\left(\frac{\pi}{6} - \left(-\frac{\pi}{4}\right)\right) \\
&= \cos\left(\frac{\pi}{6}\right)\cos\left(-\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{6}\right)\sin\left(-\frac{\pi}{4}\right) \\
&= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) \\
&= \frac{\sqrt{6}-\sqrt{2}}{4}.
\end{aligned}$$

Progress Check 4.14

1. $\cos(\pi + x) = \cos(\pi)\cos(x) - \sin(\pi)(\sin(x) = -\cos(x)$. The graphs of $y = \cos(\pi + x)$ and $y = \cos(x)$ are indentical.

2. $\cos\left(\dfrac{\pi}{2} - x\right) = \cos\left(\dfrac{\pi}{2}\right)\cos(x) + \sin(\left(\dfrac{\pi}{2}\right)\sin(x) = 0 \cdot \cos(x) + 1 \cdot \sin(x)$.

   So we see that $\cos\left(\dfrac{\pi}{2} - x\right) = \sin(x)$

Progress Check 4.15

We will use the identity $\tan(y) = \dfrac{\sin(y)}{\cos(y)}$.

$$\begin{aligned}\tan\left(\frac{\pi}{2} - x\right) &= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right)} \\ &= \frac{\cos(x)}{\sin(x)} \\ &= \cot(x)\end{aligned}$$

Progress Check 4.16

1. We note that $\dfrac{\pi}{12} = \dfrac{\pi}{3} - \dfrac{\pi}{4}$.

$$\begin{aligned}\sin\left(\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

2. We note that $\dfrac{5\pi}{12} = \dfrac{\pi}{4} + \dfrac{\pi}{6}$.

$$\begin{aligned}\sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) \\ &= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

Progress Check 4.18

We first use the Sine Sum Identity to rewrite the equation as $\sin(x + 1) = 0.2$. If we let $t = x + 1$, we see that for $0 \le t < 2\pi$,

$$t = \arcsin(0.2) \quad \text{or} \quad t = (\pi - \arcsin(0.2)).$$

So we have $x + 1 = \arcsin(0.2)$ or $x + 1 = \pi - \arcsin(0.2)$. Since the period of the functions we are working with is 2π, we see that

$$x = (-1 + \arcsin(0.2)) + k(2\pi) \text{ or } x = (-1 + \pi - \arcsin(0.2)) + k(2\pi),$$

where k is an integer.

Section 4.4

Progress Check 4.19

We are assuming that $\cos(\theta) = \dfrac{5}{13}$ and $\dfrac{3\pi}{2} \leq \theta \leq 2\pi$. To determine $\cos(2\theta)$ and $\sin(2\theta)$, we will use the double angle indentities.

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \qquad \sin(2\theta) = 2\cos(\theta)\sin(\theta).$$

To use these identities, we also need to know $\sin(\theta)$. So we use the Pythagorean identity.

$$\begin{aligned} \cos^2(\theta) + \sin^2(\theta) &= 1 \\ \sin^2(\theta) &= 1 - \cos^2(\theta) \\ &= 1 - \left(\frac{5}{13}\right)^2 \\ &= \frac{144}{169} \end{aligned}$$

Since $\dfrac{3\pi}{2} \leq \theta \leq 2\pi$, we see that $\sin(\theta) < 0$ and so $\sin(\theta) = -\dfrac{12}{13}$. Hence,

$$\begin{aligned} \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= \left(\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2 \\ &= -\frac{119}{169} \end{aligned} \qquad \begin{aligned} \sin(2\theta) &= 2\cos(\theta)\sin(\theta) \\ &= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right) \\ &= -\frac{120}{169} \end{aligned}$$

Progress Check 4.20

We will prove alternate forms of the double angle identity for cosine.

$$\begin{aligned} \cos(2A) &= \cos^2(A) - \sin^2(A) \\ &= \left(1 - \sin^2(A)\right) - \sin^2(A) \\ &= 1 - \sin^2(A) - \sin^2(A) \\ &= 1 - 2\sin^2(A) \end{aligned} \qquad \begin{aligned} \cos(2A) &= \cos^2(A) - \sin^2(A) \\ &= \cos^2(A) - \left(1 - \cos^2(A)\right) \\ &= \cos^2(A) - 1 + \cos^2(A) \\ &= 2\cos^2(A) - 1 \end{aligned}$$

Progress Check 4.22

We will approximate the smallest positive solution in degrees, to two decimal places, to the range equation

$$45000\sin(2\theta) = 1000.$$

Dividing both sides of the equation by 45000, we obtain

$$\sin(2\theta) = \frac{1000}{45000} = \frac{1}{45}.$$

So

$$\begin{aligned} 2\theta &= \arcsin\left(\frac{1}{45}\right) \\ \theta &= \frac{1}{2}\arcsin\left(\frac{1}{45}\right) \end{aligned}$$

Using a calculator in degree mode, we obtain $\theta \approx 0.64°$.

Progress Check 4.24

1. We use the double angle identity $\cos(2\theta) = 1 - 2\sin^2(\theta)$ to obtain

$$\begin{aligned} 1 - 2\sin^2(\theta) &= \sin(\theta) \\ 1 - 2\sin^2(\theta) - \sin(\theta) &= 0 \\ 2\sin^2(\theta) + \sin(\theta) - 1 &= 0 \end{aligned}$$

2. Factoring gives $(2\sin(\theta) - 1)(\sin(\theta) + 1) = 0$. Setting each factor equal to 0 and solving for $\sin(\theta)$, we obtain

$$\sin(\theta) = \frac{1}{2} \quad \text{or} \quad \sin(\theta) = -1.$$

So we have

$$\theta = \frac{\pi}{6} + k(2\pi) \ \text{ or } \ \theta = \frac{5\pi}{6} + k(2\pi) \ \text{ or } \ \theta = -\frac{pi}{2} + k(2pi),$$

where k is an integer.

Progress Check 4.26

To determine the exact value of $\cos\left(\frac{\pi}{8}\right)$, we use the Half Angle Identity for cosine with $A = \frac{\pi}{4}$.

$$\begin{aligned}
\cos\left(\frac{\pi}{8}\right) &= \pm\sqrt{\frac{1+\cos\left(\frac{\pi}{4}\right)}{2}} \\
&= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}
\end{aligned}$$

Since $\frac{\pi}{8}$ is in the first quadrant, we will use the positive square root. We can also rewrite the expression under the square root sign to obtain

$$\begin{aligned}
\cos\left(\frac{\pi}{8}\right) &= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} \\
&= \sqrt{\frac{2+\sqrt{2}}{4}} \\
&= \frac{\sqrt{2+\sqrt{2}}}{2}
\end{aligned}$$

This result can be checked using a calculator.

Section 4.5

Progress Check 4.27

To determine the exact value of $\sin(52.5°)\sin(7.5°)$, we will use the Product-to-Sum identity

$$\sin(A)\sin(B) = \left(\frac{1}{2}\right)[\cos(A-B) - \cos(A+B)].$$

So we see that

$$\begin{aligned}
\sin(52.5°)\sin(7.5°) &= \left(\frac{1}{2}\right)\left[\cos(45°) - \cos(60°)\right] \\
&= \left(\frac{1}{2}\right)\left[\frac{\sqrt{2}}{2} - \frac{1}{2}\right] \\
&= \frac{\sqrt{2} - 1}{4}
\end{aligned}$$

Progress Check 4.28

To determine the exact value of $\cos(112.5°) + \cos(67.5°)$, we will use the Sum-to-Product Identity

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

So we see that

$$\begin{aligned}
\cos(112.5°) + \cos(67.5°) &= 2\cos\left(\frac{180°}{2}\right)\cos\left(\frac{45°}{2}\right) \\
&= \cos(90°)\cos(22.5°) \\
&= 0 \cdot \cos(22.5°) \\
&= 0
\end{aligned}$$

Section 5.1

Progress Check 5.1

1. **(a)** $(2 + 3i) + (7 - 4i) = 9 - i$

(b) $(4 - 2i)(3 + i) = (4 - 2i)3 + (4 - 2i)i = 14 - 2i$

(c) $(2 + i)i - (3 + 4i) = (2i - 1) - 3 - 4i = -4 - 2i$

2. We use the quadratice formula to solve the equation and obtain $x = \dfrac{1 \pm \sqrt{-7}}{2}$. We can then write $\sqrt{-7} = i\sqrt{7}$. So the two solutions of the quadratic equation are:

$$x = \frac{1 + i\sqrt{7}}{2} \qquad\qquad x = \frac{1 - i\sqrt{7}}{2}$$
$$x = \frac{1}{2} + \frac{\sqrt{7}}{2}i \qquad\qquad x = \frac{1}{2} - \frac{\sqrt{7}}{2}i$$

Progress Check 5.5

1. Using our formula with $a = 5$, $b = -1$, $c = 3$, and $d = 4$ gives us

$$\frac{5 - i}{3 + 4i} = \frac{15 - 4}{15} + \frac{-3 - 20}{25}i = \frac{11}{25} - \frac{23}{25}i.$$

As a check, we see that

$$\begin{aligned}\left(\frac{11}{25} - \frac{23}{25}i\right)(3 + 4i) &= \left(\frac{33}{25} - \frac{69}{25}i\right) + \frac{44}{25}i - \frac{92}{25}i^2 \\ &= \left(\frac{33}{25} + \frac{92}{25}\right) + \left(-\frac{69}{25}i + \frac{44}{25}i\right) \\ &= 5 - i\end{aligned}$$

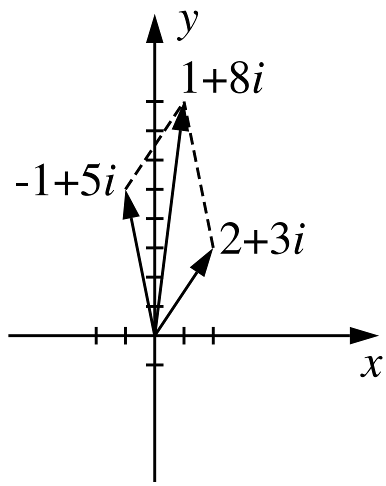
2. We can solve for x by dividing both sides of the equation by $3 + 4i$ to see that

$$x = \frac{5 - i}{3 + 4i} = \frac{11}{25} - \frac{23}{25}i.$$

Progress Check 5.2

1. The sum is $w + z = (2 - 1) + (3 + 5)i = 1 + 8i$.

2. A representation of the complex sum using vectors is shown in the figure below.



Progress Check 5.3

1. Using the definition of the conjugate of a complex number we find that $\overline{w} = 2 - 3i$ and $\overline{z} = -1 - 5i$.

2. Using the definition of the norm of a complex number we find that $|w| = \sqrt{2^2 + 3^3} = \sqrt{13}$ and $|z| = \sqrt{(-1)^2 + 5^2} = \sqrt{26}$.

3. Using the definition of the product of complex numbers we find that

$$\begin{aligned} w\overline{w} &= (2 + 3i)(2 - 3i) = 4 + 9 = 13 \\ z\overline{z} &= (-1 + 5i)(-1 - 5i) = 1 + 25 = 26. \end{aligned}$$

4. Let $z = a + 0i = a$ for some $a \in \mathbb{R}$. Then $\overline{z} = a - 0i = a$. Thus, $\overline{z} = z$ when $z \in \mathbb{R}$.

Section 5.2

Progress Check 5.6

1. Note that $|w| = \sqrt{4^2 + (4\sqrt{3})^2} = 4\sqrt{4} = 8$ and the argument of w is $\arctan\left(\frac{4\sqrt{3}}{4}\right) = \arctan\sqrt{3} = \frac{\pi}{3}$. So

$$w = 8\left(\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\right).$$

Also, $|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$ and the argument of z is $\arctan\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$. So

$$\begin{aligned} z &= \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\right) \\ &= \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\right). \end{aligned}$$

2. Recall that $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. So

$$3\left(\cos\left(\frac{\pi}{6}\right) + i\,\sin\left(\frac{\pi}{6}\right)\right) = 3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i.$$

So $a = \frac{3\sqrt{3}}{2}$ and $b = \frac{3}{2}$.

Progress Check 5.8

1. Since $|w| = 3$ and $|z| = 2$, we see that

$$|wz| = |w||z| = (3)(2) = 6.$$

2. The argument of w is $\dfrac{5\pi}{3}$ and the argument of z is $-\dfrac{\pi}{4}$, we see that the argument of wz is

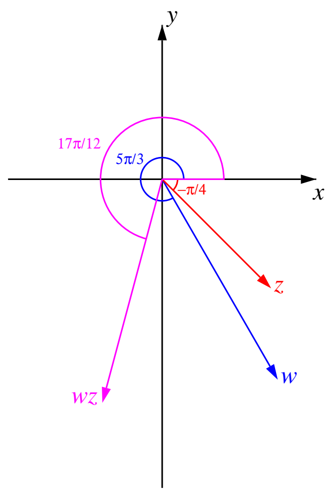
$$\frac{5\pi}{3} - \frac{\pi}{4} = \frac{20\pi - 3\pi}{12} = \frac{17\pi}{12}.$$

3. The terminal side of an angle of $\dfrac{17\pi}{12} = \pi + \dfrac{5\pi}{12}$ radians is in the third quadrant.

4. We know the magnitude and argument of wz, so the polar form of wz is

$$wz = 6\left[\cos\left(\frac{17\pi}{12}\right) + \sin\left(\frac{17\pi}{12}\right)\right].$$

5. Following is a picture of w, z, and wz that illustrates the action of the complex product.



Progress Check 5.9

1. Since $|w| = 3$ and $|z| = 2$, we see that

$$\left|\frac{w}{z}\right| = \frac{|w|}{|z|} = \frac{3}{2}.$$

2. The argument of w is $\dfrac{5\pi}{3}$ and the argument of z is $-\dfrac{\pi}{4}$, we see that the argument of $\dfrac{w}{z}$ is

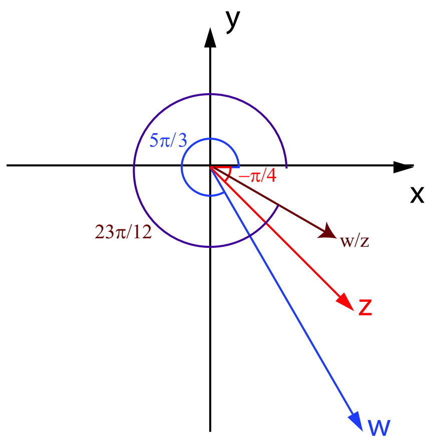
$$\frac{5\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{20\pi + 3\pi}{12} = \frac{23\pi}{12}.$$

3. The terminal side of an angle of $\dfrac{23\pi}{12} = 2\pi - \dfrac{\pi}{12}$ radians is in the fourth quadrant.

4. We know the magnitude and argument of wz, so the polar form of wz is

$$\frac{w}{z} = \frac{3}{2}\left[\cos\left(\frac{23\pi}{12}\right) + \sin\left(\frac{23\pi}{12}\right)\right].$$

5. Following is a picture of w, z, and wz that illustrates the action of the complex product.



Section 5.3

Progress Check 5.10

In polar form,

$$1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\right).$$

So

$$\begin{aligned}(1-i)^{10} &= \left(\sqrt{2}\right)^{10}\left(\cos\left(-\frac{10\pi}{4}\right) + \sin\left(-\frac{10\pi}{4}\right)\right)\\ &= 32\left(\cos\left(-\frac{5\pi}{2}\right) + \sin\left(-\frac{5\pi}{2}\right)\right)\\ &= 32(0-i)\\ &= -32i.\end{aligned}$$

Progress Check 5.13

1. We find the solutions to the equation $z^4 = 1$. Let $\omega = \cos\left(\frac{2\pi}{4}\right) + i\sin\left(\frac{2\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$. Then

   - $\omega^0 = 1$,
   - $\omega = i$,
   - $\omega^2 = \cos\left(\frac{2\pi}{2}\right) + i\sin\left(\frac{2\pi}{2}\right) = -1$

- $\omega^3 = \cos\left(\frac{3\pi}{2}\right) + i\,\sin\left(\frac{3\pi}{2}\right) = -i$.

So the four fourth roots of unity are 1, i, -1, and $-i$.

2. We find the solutions to the equation $z^6 = 1$. Let $\omega = \cos\left(\frac{2\pi}{6}\right) + i\,\sin\left(\frac{2\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) + i\,\sin\left(\frac{\pi}{3}\right)$. Then

- $\omega^0 = 1$,
- $\omega = \frac{1}{2} + \sqrt{3}2i$,
- $\omega^2 = \cos\left(\frac{2\pi}{3}\right) + i\,\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \sqrt{3}2i$,
- $\omega^3 = \cos\left(\frac{3\pi}{3}\right) + i\,\sin\left(\frac{3\pi}{3}\right) = -1$,
- $\omega^4 = \cos\left(\frac{4\pi}{3}\right) + i\,\sin\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - \sqrt{3}2i$,
- $\omega^5 = \cos\left(\frac{5\pi}{3}\right) + i\,\sin\left(\frac{5\pi}{3}\right) = \frac{1}{2} - \sqrt{3}2i$.

So the six fifth roots of unity are 1, $\frac{1}{2} + \sqrt{3}2i$, $-\frac{1}{2} + \sqrt{3}2i$, -1, $-\frac{1}{2} - \sqrt{3}2i$, and $\frac{1}{2} - \sqrt{3}2i$.

Progress Check 5.15

Since $-256 = 256\,[\cos(\pi) + i\,\sin(\pi)]$ we see that the fourth roots of -256 are

$$\begin{aligned}
x_0 &= \sqrt[4]{256}\left[\cos\left(\frac{\pi + 2\pi(0)}{4}\right) + i\,\sin\left(\frac{\pi + 2\pi(0)}{4}\right)\right] \\
&= 4\cos\left(\frac{\pi}{4}\right) + i\,\sin\left(\frac{\pi}{4}\right) \\
&= 4\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right] \\
&= 2\sqrt{2} + 2i\sqrt{2},
\end{aligned}$$

$$\begin{aligned}
x_1 &= \sqrt[4]{256}\left[\cos\left(\frac{\pi + 2\pi(1)}{4}\right) + i\,\sin\left(\frac{\pi + 2\pi(1)}{4}\right)\right] \\
&= 4\cos\left(\frac{3\pi}{4}\right) + i\,\sin\left(3\frac{\pi}{4}\right) \\
&= 4\left[-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right] \\
&= -2\sqrt{2} + 2i\sqrt{2},
\end{aligned}$$

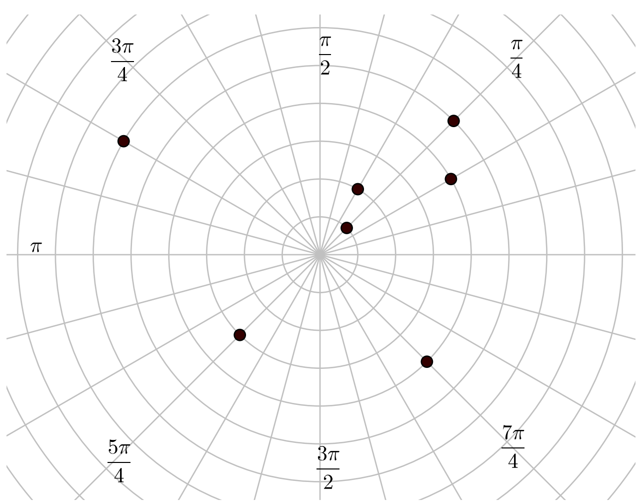
$$\begin{aligned}
x_2 &= \sqrt[4]{256}\left[\cos\left(\frac{\pi + 2\pi(2)}{4}\right) + i\,\sin\left(\frac{\pi + 2\pi(2)}{4}\right)\right] \\
&= 4\cos\left(\frac{5\pi}{4}\right) + i\,\sin\left(5\frac{\pi}{4}\right) \\
&= 4\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right] \\
&= -2\sqrt{2} - 2i\sqrt{2},
\end{aligned}$$

and

$$\begin{aligned}
x_3 &= \sqrt[4]{256}\left[\cos\left(\frac{\pi + 2\pi(3)}{4}\right) + i\,\sin\left(\frac{\pi + 2\pi(3)}{4}\right)\right] \\
&= 4\cos\left(\frac{7\pi}{4}\right) + i\,\sin\left(7\frac{\pi}{4}\right) \\
&= 4\left[\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right] \\
&= 2\sqrt{2} - 2i\sqrt{2}.
\end{aligned}$$

Section 5.4

Progress Check 5.16



Progress Check 5.17

The left column shows some sets of polar coordinates with a positive value for r and the right column shows some sets of polar coordinates with a negative value for r.

$$\begin{array}{cc}
(3, 470^\circ) & (-3, 290^\circ) \\
(3, 830^\circ) & (-3, 650^\circ) \\
(3, -250^\circ) & (-3, -70^\circ) \\
(3, -510^\circ) & (-3, -430^\circ) \\
(3, 1190^\circ) & (-3, 1010^\circ)
\end{array}$$

Progress Check 5.18

For each point, we use the equations $x = r\cos(\theta)$ and $y = r\sin(\theta)$. In each of these cases, we can determine the exact values for x and y.

	Polar Coordinates	Rectangular Coordinates
1.	$\left(3, \frac{\pi}{3}\right)$	$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$
2.	$\left(5, \frac{11\pi}{6}\right)$	$\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$
3.	$\left(-5, \frac{3\pi}{4}\right)$	$\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

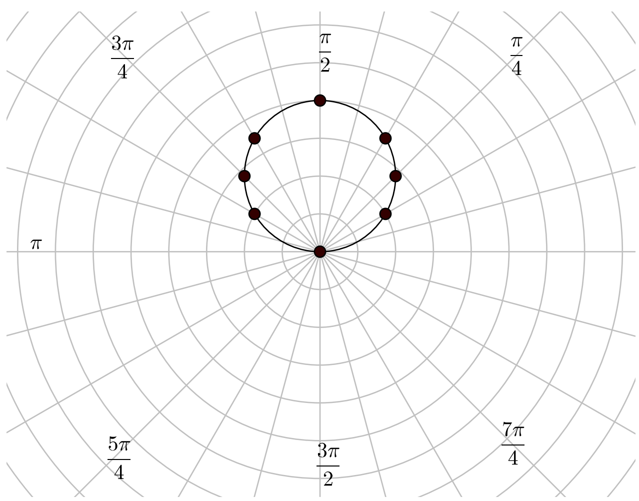
Progress Check 5.20

1. For the point $(6, 6\sqrt{3})$, $r^2 = 6^2 + \left(6\sqrt{3}\right)^2 = 144$ and so $r = 12$. Since the point is in the first quadrant, we can use $\tan(\theta) = \sqrt{3}$ or $\cos(\theta) = \frac{1}{2}$ to conclude that $\theta = \frac{\pi}{3}$. So the polar coordinates are $\left(12, \frac{\pi}{3}\right)$.

2. For the point $(0, -4)$, $r^2 = 0^2 + (-4)^2 = 16$ and so $r = 4$. Since the point is on the y-axis, we can use $\cos(\theta) = 0$ and $\sin(\theta) - 1$ to conclude that $\theta = \frac{3\pi}{2}$. So the polar coordinates are $\left(4, \frac{3\pi}{2}\right)$.

3. For the point $(-4, 5\sqrt{3})$, $r^2 = (-4)^2 + 5^2 = 41$ and so $r = \sqrt{41}$. Since the point is in the second quadrant, we can use $\tan(\theta) = -1.25$ to conclude that the reference angle is $\hat{\theta} = tan^{-1}(-1.25)$. We cannot determine an exact value for θ and so we can say that the polar coordinates are $\left(\sqrt{41}, \pi - \tan^{-1}(1.25)\right)$. We can also approximate the angle and see that the approximate polar coordinates are $\left(\sqrt{41}, 2.24554\right)$. Note: There are other ways to write the angle θ. It is also true that $\theta = \pi - \cos^{-1}\left(\dfrac{4}{\sqrt{21}}\right) = \cos^{-1}\left(\dfrac{-4}{\sqrt{21}}\right)$.

Progress Check 5.21



Progress Check 5.22

1. $r^2 = 4r\sin(\theta)$.

2. $x^2 + y^2 = 4y$.

Progress Check 5.23

$$\begin{aligned} r^2 &= 6r\sin(\theta) & \qquad x^2 - 6x + 9 + y^2 &= 9 \\ x^2 + y^2 &= 6x & (x-3)^2 + y^2 &= 3^2 \end{aligned}$$

So the graph of $r = 3\cos(\theta)$ is a circle with radius 3 and center at $(3, 0)$.