

Appendix B

Answers and Hints for Selected Exercises

Section 1.1

1. (b)

t	point
1	(0.54, 0.84)
5	(0.28, -0.96)
9	(-0.91, 0.41)

4.

	(a)	(b)	(d)	(i)	(j)	(l)	(m)
t	$\frac{7\pi}{4}$	$-\frac{7\pi}{4}$	$-\frac{3\pi}{5}$	2.5	-2.5	$3 + 2\pi$	$3 - \pi$
Quadrant	IV	I	III	II	III	II	IV

5. (a) We substitute $x = \frac{1}{3}$ into the equation $x^2 + y^2 = 1$. Solving for y , we obtain $y = \pm \frac{\sqrt{8}}{3}$. So the points are $\left(\frac{1}{3}, \frac{\sqrt{8}}{3}\right)$ and $\left(\frac{1}{3}, -\frac{\sqrt{8}}{3}\right)$.

(b) We substitute $y = -\frac{1}{2}$ into the equation $x^2 + y^2 = 1$. Solving for x , we obtain $x = \pm \frac{\sqrt{3}}{2}$. So the points are $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ and

$$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right).$$

Section 1.2

1. (a) For a real number t , the value of $\cos(t)$ is defined to be the x -coordinate of the terminal point of an arc t whose initial point is $(1, 0)$ on the unit circle whose equation is $x^2 + y^2 = 1$.
- (b) The domain of the cosine function is the set of all real numbers.
- (c) The maximum value of $\cos(t)$ is 1 and this occurs at $t =$ 0 for $0 \leq t < 2\pi$. The minimum value of $\cos(t)$ is -1 and this occurs at $t =$ π for $0 \leq t < 2\pi$.
- (d) The range of the cosine function is the closed interval $[-1, 1]$.
4. (a) $\cos(t) = \frac{4}{5}$ or $\cos(t) = -\frac{4}{5}$.
- (c) $\sin(t) = -\frac{\sqrt{5}}{3}$.
5. (a) $0 < \cos^2(t) < \frac{1}{9}$.
- (b) $-\frac{1}{9} < -\cos^2(t) < 0$ and so $\frac{8}{9} < 1 - \cos^2(t) < 1$
- (c) $\frac{8}{9} < \sin^2(t) < 1$
- (d) $\frac{\sqrt{8}}{3} < \sin(t) < 1$

Section 1.3

1. (a) $\frac{1}{12}\pi \approx 0.2618$ (e) $-\frac{2}{9}\pi \approx -0.6981$
- (b) $\frac{29}{90}\pi \approx 1.0123$



2. (a) 67.5° (d) 57.2958°
(b) 231.4286°
4. (a) $\cos(10^\circ) \approx 0.9848$, $\sin(10^\circ) \approx 0.1736$
(d) $\cos(-10^\circ) \approx 0.9848$, $\sin(-10^\circ) \approx -0.1736$
-

Section 1.4

1. (a) The arc length is 4π feet, which is equal to $\frac{1}{3}$ of the circumference of the circle.
(b) The arc length is 200 miles.
(c) The arc length is 26π meters.
2. (a) $\theta = \frac{3\pi}{5}$ radians.
(b) $\theta = \frac{18}{5}$ radians = 3.6 radians.
3. (a) $\theta = 108^\circ$.
(b) $\theta = \left(\frac{648}{\pi}\right)^\circ \approx 206.26^\circ$.
8. (b) $v = 720\pi \frac{\text{in}}{\text{min}} \approx 2261.95 \frac{\text{in}}{\text{min}}$.
9. (b) $v = 3600\pi \frac{\text{cm}}{\text{min}} \approx 11309.73 \frac{\text{cm}}{\text{min}}$.
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Section 1.5

1. (a) $t = \frac{\pi}{3}$, $\cos(t) = \frac{1}{2}$, $\sin(t) = \frac{\sqrt{3}}{2}$.
(b) $t = \frac{\pi}{2}$, $\cos(t) = 0$, $\sin(t) = 1$.
(c) $t = \frac{\pi}{4}$, $\cos(t) = \frac{\sqrt{2}}{2}$, $\sin(t) = \frac{\sqrt{2}}{2}$.
(d) $t = \frac{\pi}{6}$, $\cos(t) = \frac{\sqrt{3}}{2}$, $\sin(t) = \frac{1}{2}$.

2. (a) $\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{2}$.
 (b) $2 \sin^2\left(\frac{\pi}{4}\right) + \cos(\pi) = 0$.
3. (a) The reference arc is $\frac{\pi}{3}$.
 (b) The reference arc is $\frac{3\pi}{8}$.
 (d) The reference arc is $\frac{\pi}{3}$.
4. (a) The reference arc is $\frac{\pi}{6}$; $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$; $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$.
 (d) The reference arc is $\frac{\pi}{3}$; $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$; $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.
6. (a) $\cos(t) = \frac{\sqrt{24}}{5}$. (d) $\sin(\pi + t) = -\frac{1}{5}$.

Section 1.6

1.

t	$\cot(t)$	$\sec(t)$	$\csc(t)$
0	undefined	1	undefined
$\frac{\pi}{6}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$\frac{\pi}{4}$	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
$\frac{\pi}{2}$	0	undefined	1

3. (a) The terminal point is in the fourth quadrant.
 (b) The terminal point is in the third quadrant.

4.



$$\cos(t) = -\frac{\sqrt{8}}{3}$$

$$\tan(t) = -\frac{1}{\sqrt{8}}$$

$$\sec(t) = -\frac{3}{\sqrt{8}}$$

$$\csc(t) = 3$$

$$\cot(t) = -\sqrt{8}$$

8. (a) $t = \frac{5\pi}{4}$

(b) $t = \frac{\pi}{2}$

Section 2.1

1. (a) $C(\pi, -1)$ $R(\pi, 0)$

(b) $B\left(\frac{\pi}{3}, \frac{1}{2}\right)$ $Q\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

2. (a) $y = 3 \sin(x)$

(b) $y = 2 \cos(x)$

3. (a) t -intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$ y -intercept: $(0, 0)$

The maximum value is 1. Maximum value occurs at the points $\left(-\frac{3\pi}{2}, 1\right)$ and $\left(\frac{\pi}{2}, 1\right)$.

The minimum value is -1 . Minimum value occurs at the points $\left(-\frac{\pi}{2}, -1\right)$ and $\left(\frac{3\pi}{2}, -1\right)$.

(b) t -intercepts: $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ y -intercept: $(0, 2)$

The maximum value is 2. Maximum value occurs at the points $(0, 2)$ and $(2\pi, 2)$.

The minimum value is -2 . Minimum value occurs at the points $(-\pi, -2)$, $(\pi, -2)$, and $(3\pi, -2)$.

Section 2.2

1. (a) $y = 2 \sin(\pi x)$. The amplitude is 2; the period is 2; the phase shift is 0; and the vertical shift is 0.

$$\begin{array}{lll} A(0, 0) & B\left(\frac{\pi}{2}, 2\right) & C(\pi, 0) \\ E\left(\frac{3\pi}{2}, -2\right) & F(2\pi, 0) & G\left(\frac{5\pi}{2}, 2\right) \end{array}$$

- (c) $y = 3 \sin\left(x - \frac{\pi}{4}\right)$. The amplitude is 3; the period is 2π ; the phase shift is $\frac{\pi}{4}$; and the vertical shift is 0.

$$\begin{array}{lll} A\left(\frac{\pi}{4}, 0\right) & B\left(\frac{3\pi}{4}, 3\right) & C\left(\frac{5\pi}{4}, 0\right) \\ E\left(\frac{7\pi}{4}, -3\right) & F\left(\frac{9\pi}{4}, 0\right) & G\left(\frac{11\pi}{4}, 3\right) \end{array}$$

- (g) $y = 4 \sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$. The amplitude is 4; the period is π ; the phase shift is $\frac{\pi}{4}$; and the vertical shift is 1.

$$\begin{array}{lll} A\left(\frac{\pi}{4}, 1\right) & B\left(\frac{\pi}{2}, 5\right) & C\left(\frac{3\pi}{4}, 1\right) \\ E(\pi, -3) & F\left(\frac{5\pi}{4}, 1\right) & G\left(\frac{3\pi}{2}, 5\right) \end{array}$$

2. (a) The amplitude is 2; the period is $\frac{2\pi}{3}$; and there is no vertical shift.
- For $y = A \sin(B(x - C)) + D$, there is no phase shift and so $C = 0$. So

$$y = 2 \sin(3x).$$
 - For $y = A \cos(B(x - C)) + D$, the phase shift is $\frac{\pi}{6}$ and so $C = \frac{\pi}{6}$. So

$$y = 2 \cos\left(3\left(x - \frac{\pi}{6}\right)\right).$$

- (d) The amplitude is 8; the period is 2; and the vertical shift is 1.



- For $y = A \sin(B(x - C)) + D$, the phase shift is $-\frac{1}{6}$ and so $C = -\frac{1}{6}$. So

$$y = 8 \sin\left(\pi\left(x + \frac{1}{6}\right)\right).$$

- For $y = A \cos(B(x - C)) + D$, the phase shift is $\frac{1}{3}$ and so $C = \frac{1}{3}$. So

$$y = 8 \cos\left(\pi\left(x - \frac{1}{3}\right)\right) + 1.$$

Section 2.3

- (a) We write $y = 4 \sin\left(\pi x - \frac{\pi}{8}\right) = 4 \sin\left(\pi\left(x - \frac{1}{8}\right)\right)$. So the amplitude is 4, the period is 2, the phase shift is $\frac{1}{8}$, and there is no vertical shift.

 - Some high points on the graph: $\left(\frac{5}{8}, 4\right), \left(\frac{21}{8}, 4\right)$.
 - Some low points on the graph: $\left(\frac{13}{8}, -4\right), \left(\frac{29}{8}, -4\right)$.
 - Graph crosses the center line at: $\left(\frac{1}{8}, 0\right), \left(\frac{9}{8}, 0\right), \left(\frac{17}{8}, 0\right)$.
 - (b) We write $y = 5 \cos\left(4x + \frac{\pi}{2}\right) + 2 = 5 \cos\left(4\left(x + \frac{\pi}{8}\right)\right) + 2$. So the amplitude is 5, the period is $\frac{\pi}{2}$, the phase shift is $-\frac{\pi}{8}$, and the vertical shift is 2.

 - Some high points on the graph: $\left(-\frac{\pi}{8}, 7\right), \left(\frac{3\pi}{8}, 7\right)$.
 - Some low points on the graph: $\left(\frac{\pi}{8}, -3\right), \left(\frac{5\pi}{8}, -3\right)$.
 - Graph crosses the center line at: $(0, 2), \left(\frac{\pi}{4}, 2\right), \left(\frac{\pi}{2}, 2\right)$.
- (b) The maximum value is 150 ml, and the minimum value is 81 ml. So we can use $A = \frac{150 - 81}{2} = 34.5$ and $D = \frac{150 + 81}{2} = 115.5$.

(c) The period is $\frac{1}{75}$ min.

Section 2.4

1. (a) $\tan(t + 2\pi) = \frac{\sin(t + 2\pi)}{\cos(t + 2\pi)} = \frac{\sin(t)}{\cos(t)} = \tan(t).$

3. (a) $\csc(-t) = \frac{1}{\sin(-t)} = \frac{1}{-\sin(t)} = -\frac{1}{\sin(t)} = -\csc(t).$

Section 2.5

1. (a) $t = \arcsin\left(\frac{\sqrt{2}}{2}\right)$ means $\sin(t) = \frac{\sqrt{2}}{2}$ and $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$. Since $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, we see that $t = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.

(b) $t = \arcsin\left(-\frac{\sqrt{2}}{2}\right)$ means $\sin(t) = -\frac{\sqrt{2}}{2}$ and $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$. Since $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, we see that $t = \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$.

(d) $t = \arccos\left(-\frac{\sqrt{2}}{2}\right)$ means $\cos(t) = -\frac{\sqrt{2}}{2}$ and $0 \leq t \leq \pi$. Since $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, we see that $t = \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$.

(f) $y = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ means $\tan(y) = -\frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Since $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$, we see that $y = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

(h) $t = \arctan(0) = 0.$

(j) $y = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}.$



2. (a) $\sin(\sin^{-1}(1)) = \sin\left(\frac{\pi}{2}\right) = 1$
- (b) $\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
- (c) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$
- (d) $\arcsin\left(\sin\left(\frac{2\pi}{3}\right)\right) = \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
- (e) $\arctan\left(\tan\left(\frac{3\pi}{4}\right)\right) = \arctan(-1) = -\frac{\pi}{4}$
3. (a) Let $t = \arcsin\left(\frac{2}{5}\right)$. Then $\sin(t) = \frac{2}{5}$ and $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$, and

$$\cos^2(t) + \sin^2(t) = 1$$

$$\cos^2(t) + \frac{4}{25} = 1$$

$$\cos^2(t) = \frac{21}{25}$$

Since $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$, we know that $\cos(t) \geq 0$. Hence, $\cos(t) = \frac{\sqrt{21}}{5}$

$$\text{and } \cos\left(\arcsin\left(\frac{2}{5}\right)\right) = \frac{\sqrt{21}}{5}.$$

(b) $\sin\left(\arccos\left(-\frac{2}{3}\right)\right) = \frac{\sqrt{5}}{3}.$

(c) $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{\sqrt{8}}.$

Section 2.6

1. (a) $x = 0.848 + k(2\pi)$ or $x = 2.294 + k(2\pi)$, where k is an integer.
- (b) $x = -0.848 + k(2\pi)$ or $x = -2.294 + k(2\pi)$, where k is an integer.
2. (a) $x = \sin^{-1}(0.75) + k(2\pi)$ or $x = (\pi - \sin^{-1}(0.75)) + k(2\pi)$, where k is an integer.



- (d) $x = \arcsin(-0.75) + k(2\pi)$ or $x = (\pi - \arcsin(-0.75)) + k(2\pi)$, where k is an integer.
3. (a) $x = \sin^{-1}(0.4) + k(2\pi)$ or $x = (\pi - \sin^{-1}(0.4)) + k(2\pi)$, where k is an integer.
- (b) $x = \cos^{-1}\left(\frac{4}{5}\right) + k(2\pi)$ or $x = -\cos^{-1}\left(\frac{4}{5}\right) + k(2\pi)$, where k is an integer.
4. (a) The period for the trigonometric function is π . We first solve the equation $4 \sin(t) = 3$ with $-\pi \leq t \leq \pi$ and obtain $t = \sin^{-1}(0.75) + k(2\pi)$ or $t = (\pi - \sin^{-1}(0.75)) + k(2\pi)$. We then use the substitution $t = 2x$ to obtain
- $$x = \frac{1}{2} \sin^{-1}(0.75) + k(\pi) \text{ or } x = \frac{1}{2} (\pi - \sin^{-1}(0.75)) + k(\pi), \text{ where } k \text{ is an integer.}$$
- (d) The period for the trigonometric function is 2. We first solve the equation $\sin(t) = 0.2$ with $-\pi \leq t \leq \pi$ and obtain $t = \sin^{-1}(0.2) + k(2\pi)$ or $t = (\pi - \sin^{-1}(0.2)) + k(2\pi)$. We now use the substitution $t = \pi x - \frac{\pi}{4}$ to obtain
- $$x = \left(\frac{1}{\pi} \sin^{-1}(0.2) + \frac{1}{4}\right) + 2k \text{ or } x = \left(-\frac{1}{\pi} \sin^{-1}(0.2) + \frac{5}{4}\right) + 2k, \text{ where } k \text{ is an integer.}$$
-

Section 3.1

1. (a) We see that $r = \sqrt{3^2 + 3^2} = \sqrt{18}$. So

$$\cos(\theta) = \frac{3}{\sqrt{18}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \sin(\theta) = \frac{3}{\sqrt{18}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan(\theta) = \frac{3}{3} = 1 \quad \cot(\theta) = \frac{3}{3} = 1$$

$$\sec(\theta) = \frac{\sqrt{18}}{3} = \sqrt{2} \quad \csc(\theta) = \frac{\sqrt{18}}{3} = \sqrt{2}$$



(b) We see that $r = \sqrt{5^2 + 8^2} = \sqrt{89}$. So

$$\begin{aligned}\cos(\theta) &= \frac{5}{\sqrt{89}} & \sin(\theta) &= \frac{8}{\sqrt{89}} \\ \tan(\theta) &= \frac{8}{5} & \cot(\theta) &= \frac{5}{8} = 1 \\ \sec(\theta) &= \frac{\sqrt{89}}{5} & \csc(\theta) &= \frac{\sqrt{89}}{8}\end{aligned}$$

(e) We see that $r = \sqrt{(-1)^2 + (-4)^2} = \sqrt{26}$. So

$$\begin{aligned}\cos(\theta) &= \frac{-4}{\sqrt{26}} & \sin(\theta) &= \frac{-1}{\sqrt{26}} \\ \tan(\theta) &= 4 & \cot(\theta) &= \frac{1}{4} = 1 \\ \sec(\theta) &= -\frac{\sqrt{26}}{4} & \csc(\theta) &= -\sqrt{26}\end{aligned}$$

2. (b) We first use the Pythagorean Identity and obtain $\sin^2(\beta) = \frac{5}{9}$. Since the terminal side of β is in the second quadrant, $\sin(\beta) = \frac{\sqrt{5}}{3}$. In addition,

$$\begin{aligned}\tan(\beta) &= -\frac{\sqrt{5}}{2} & \cot(\beta) &= -\frac{2}{\sqrt{5}} \\ \sec(\beta) &= -\frac{3}{2} & \csc(\beta) &= \frac{3}{\sqrt{5}}\end{aligned}$$

3. (c) Since the terminal side of θ is in the second quadrant, θ is not the inverse sine of $\frac{2}{3}$. So we let $\alpha = \arcsin\left(\frac{2}{3}\right)$. Using α as the reference angle, we then see that

$$\theta = 180^\circ - \arcsin\left(\frac{2}{3}\right) \approx 138.190^\circ.$$

- (e) $\theta = \arccos\left(-\frac{1}{4}\right) \approx 104.478^\circ$, or use $\alpha = \arccos\left(\frac{1}{4}\right)$ for the reference angle.

$$\theta = 180^\circ - \arccos\left(\frac{1}{4}\right) \approx 104.478^\circ.$$



4. (c) $\theta = \pi - \arcsin\left(\frac{2}{3}\right) \approx 2.142$.

(e) $\theta = \pi - \arccos\left(\frac{1}{4}\right) \approx 1.823$.

5. (b) Let $\theta = \cos^{-1}\left(\frac{2}{3}\right)$. Then $\cos(\theta) = \frac{2}{3}$ and $0 \leq \theta \leq \pi$. So $\sin(\theta) > 0$ and $\sin^2(\theta) = 1 - \cos^2(\theta) = \frac{5}{9}$. So

$$\begin{aligned} \tan\left(\cos^{-1}\left(\frac{2}{3}\right)\right) &= \tan(\theta) \\ &= \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} \\ &= \frac{\sqrt{5}}{2} \end{aligned}$$

Using a calculator, we obtain

$$\tan\left(\cos^{-1}\left(\frac{2}{3}\right)\right) \approx 1.11803 \quad \text{and} \quad \frac{\sqrt{5}}{2} \approx 1.11803.$$

Section 3.2

1. (a) $x = 6 \tan(47^\circ) \approx 6.434$. (b) $x = 3.1 \cos(67^\circ) \approx 1.211$.

(c) $x = \tan^{-1}\left(\frac{7}{4.9}\right) \approx 55.008^\circ$.

(d) $x = \sin^{-1}\left(\frac{7}{9.5}\right) \approx 47.463^\circ$.

4. The other acute angle is $64^\circ 48'$.

- The side opposite the $27^\circ 12'$ angle is $4 \tan(27^\circ 12') \approx 2.056$ feet.

- The hypotenuse is $\frac{4}{\cos(27^\circ 12')} \approx 4.497$ feet.



Note that the Pythagorean Theorem can be used to check the results by showing that $4^2 + 2.056^2 \approx 4.497^2$. The check will not be exact because the 2.056 and 4.497 are approximations of the exact values.

7. We first note that $\theta = 40^\circ$. We use the following two equations to determine x .

$$\tan(\alpha) = \frac{h}{c + x} \qquad \tan(\theta) = \frac{h}{x}$$

Substituting $h = x \tan(\theta)$ into the first equation and solving for x gives

$$x = \frac{c \tan(\alpha)}{\tan(\theta) - \tan(\alpha)} \approx 8.190.$$

We can then use right triangles to obtain $h \approx 6.872$ ft, $a \approx 10.691$ ft, and $b \approx 17.588$ ft.

Section 3.3

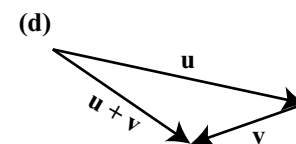
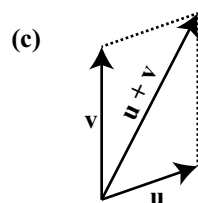
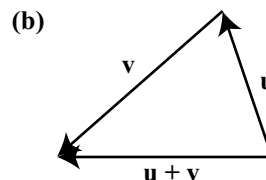
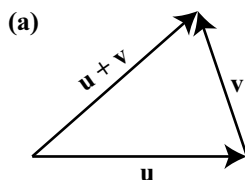
1. The third angle is 65° . The side opposite the 42° angle is 4.548 feet long. The side opposite the 65° angle is 6.160 feet long.
3. There are two triangles that satisfy these conditions. The sine of the angle opposite the 5 inch side is approximately 0.9717997.
5. The angle opposite the 9 foot long side is 95.739° . The angle opposite the 7 foot long side is 50.704° . The angle opposite the 5 foot long side is 33.557° .

Section 3.4

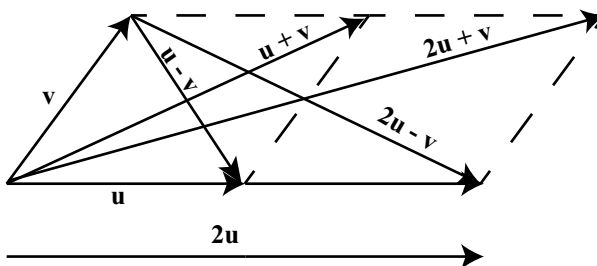
1. The ski lift is about 1887.50 feet long.
2. (a) The boat is about 67.8 miles from Chicago.
 (b) $\gamma \approx 142.4^\circ$. So the boat should turn through an angle of about $180^\circ - 142.4^\circ = 37.6^\circ$.
 (c) The direct trip from Muskegon to Chicago would take $\frac{121}{15}$ hours or about 8.07 hours. By going off-course, the trip now will take $\frac{127.8}{15}$ hours or about 8.52 hours.

Section 3.5

1.



2.



3. The angle between the vectors \mathbf{a} and $\mathbf{a} + \mathbf{b}$ is approximately 9.075° . In addition, $|\mathbf{b}| \approx 4.416$.

Section 3.6

1. (a) $|\mathbf{v}| = \sqrt{34}$. The direction angle is approximately 59.036° .
- (b) $|\mathbf{w}| = \sqrt{45}$. The direction angle is approximately 116.565° .
2. (a) $\mathbf{v} = 12 \cos(50^\circ) + 12 \sin(50^\circ) \approx 7.713\mathbf{i} + 9.193\mathbf{j}$.
- (b) $\mathbf{u} = \sqrt{20} \cos(125^\circ) + \sqrt{20} \sin(125^\circ) \approx -2.565\mathbf{i} + 3.663\mathbf{j}$.
3. (a) $5\mathbf{u} - \mathbf{v} = 11\mathbf{i} + 10\mathbf{j}$.



(c) $\mathbf{u} + \mathbf{v} + \mathbf{w} = 5\mathbf{i} + 6\mathbf{j}$.

4. (a) $\mathbf{v} \cdot \mathbf{w} = -4$.

(b) $\mathbf{a} \cdot \mathbf{b} = 9\sqrt{3}$.

5. (a) The angle between \mathbf{v} and \mathbf{w} is $\cos^{-1}\left(\frac{-4}{\sqrt{29}\sqrt{13}}\right) \approx 101.89^\circ$.

6. (a) $\text{proj}_{\mathbf{v}}\mathbf{w} = \frac{\mathbf{w} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\mathbf{v} = \frac{-4}{29}(2\mathbf{i} + 5\mathbf{j}) = -\frac{8}{29}\mathbf{i} - \frac{20}{29}\mathbf{j}$.

$$\text{proj}_{\perp\mathbf{v}}\mathbf{w} = \mathbf{w} - \text{proj}_{\mathbf{v}}\mathbf{w} = \frac{95}{29}\mathbf{i} - \frac{38}{29}\mathbf{j}$$

Section 4.1

(b)

1. (a)

$$\begin{aligned} \cos(x) \tan(x) &= \cos(x) \frac{\sin(x)}{\cos(x)} \\ &= \sin(x) \end{aligned}$$

$$\begin{aligned} \frac{\cot(x)}{(x)} &= \frac{\frac{\cos(x)}{\sin(x)}}{\frac{1}{\sin(x)}} \\ &= \frac{\cos(x)}{\sin(x)} \times \frac{\sin(x)}{1} \\ &= \cos(x) \end{aligned}$$

(e) A graph will show that this is not an identity. In particular, we see that

$$\sec^2\left(\frac{\pi}{4}\right) + \csc^2\left(\frac{\pi}{4}\right) = (\sqrt{2})^2 + (\sqrt{2})^2 = 4$$

Section 4.2

1. (a) $x = \frac{\pi}{6} + k(2\pi)$ or $x = \frac{5\pi}{6} + k(2\pi)$, where k is an integer.

(b) $x = \frac{2\pi}{3} + k(2\pi)$ or $x = \frac{4\pi}{3} + k(2\pi)$, where k is an integer.

(d) $x = \cos^{-1}\left(\frac{3}{4}\right) + k(2\pi)$ or $x = \cos^{-1}\left(-\frac{3}{4}\right) + k(2\pi)$, where k is an integer.



- (f) $x = k\pi$, where k is an integer.
2. $\theta = \sin^{-1}\left(\frac{2}{3}\right) \approx 41.81^\circ$ is one solution of the equation $\sin(\theta) + \frac{1}{3} = 1$ with $0 \leq \theta \leq 360^\circ$. There is another solution (in the second quadrant) for this equation with $0 \leq \theta \leq 360^\circ$.

Section 4.3

1. (a) $\cos(-10^\circ - 35^\circ) = \cos(-45^\circ) = \frac{\sqrt{2}}{2}$.
- (b) $\cos\left(\frac{7\pi}{9} + \frac{2\pi}{9}\right) = \cos(\pi) = -1$.
2. We first use the Pythagorean Identity to determine $\cos(A)$ and $\sin(B)$. From this, we get

$$\cos(A) = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin(B) = -\frac{\sqrt{7}}{4}.$$

(a)

$$\begin{aligned} \cos(A + B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \\ &= \frac{\sqrt{3}}{2} \cdot \frac{3}{4} - \frac{1}{2} \cdot \left(-\frac{\sqrt{7}}{4}\right) \\ &= \frac{3\sqrt{3} + \sqrt{7}}{8} \end{aligned}$$

3. (a) $\cos(15^\circ) = \cos(45^\circ - 30^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$.
- (d) We can use $345^\circ = 300^\circ + 45^\circ$ and first evaluate $\cos(345^\circ)$. This gives $\cos(345^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ and $\sec(345^\circ) = \frac{4}{\sqrt{6} + \sqrt{2}}$. We could have also used the fact that $\cos(345^\circ) = \cos(15^\circ)$ and the result in part (a).



5. (a)

$$\begin{aligned}\cot\left(\frac{\pi}{2} - x\right) &= \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} \\ &= \frac{\sin(x)}{\cos(x)} \\ &= \tan(x)\end{aligned}$$

Section 4.4

1. Use the Pythagorean Identity to obtain $\sin^2(\theta) = \frac{5}{9}$. Since $\sin(\theta) < 0$, we see that $\sin(\theta) = -\frac{\sqrt{5}}{3}$. Now use appropriate double angle identities to get

$$\sin(2\theta) = -\frac{4\sqrt{5}}{9} \qquad \cos(2\theta) = -\frac{1}{9}$$

Then use $\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = 4\sqrt{5}$.

2. (a) $x = \frac{\pi}{4} + k\pi$, where k is an integer.
3. (a) This is an identity. Start with the left side of the equation and use $\cot(t) = \frac{\cos(t)}{\sin(t)}$ and $\sin(2t) = 2\sin(t)\cos(t)$.

6. (a) $\sin(22.5^\circ) = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$.

(c) $\tan(22.5^\circ) = \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}} = \sqrt{3 - 2\sqrt{2}}$.

(h) $\cos(195^\circ) = -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$.

7. (a) $\sin\left(\frac{3\pi}{8}\right) = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}.$
- (c) $\tan\left(\frac{3\pi}{8}\right) = \sqrt{\frac{2 + \sqrt{2}}{2 - \sqrt{2}}} = \sqrt{3 + 2\sqrt{2}}.$
- (h) $\cos\left(\frac{11\pi}{12}\right) = -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\frac{1}{2}\sqrt{2 + \sqrt{3}}.$
8. (a) We note that since $\frac{3\pi}{2} \leq x \leq 2\pi$, $\frac{3\pi}{4} \leq \frac{x}{2} \leq \pi$.
- $$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1 - \frac{2}{3}}{2}} = \frac{1}{\sqrt{6}}.$$

Section 4.5

1. (a) $\sin(37.5^\circ) \cos(7.5^\circ) = \frac{1}{2} [\sin(45^\circ) + \sin(30^\circ)] = \frac{\sqrt{2} + 1}{4}$
- (e) $\cos\left(\frac{5\pi}{12}\right) \sin\left(\frac{\pi}{12}\right) = \frac{1}{2} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{3}\right) \right] = \frac{2 - \sqrt{3}}{4}$
2. (a) $\sin(50^\circ) + \sin(10^\circ) = 2 \sin(30^\circ) \cos(20^\circ) = \cos(20^\circ)$
- (e) $\cos\left(\frac{7\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right) = 2 \cos\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
3. (a)

$$\sin(2x) + \sin(x) = 0$$

$$2 \sin\left(\frac{3x}{2}\right) \cos\left(\frac{x}{2}\right) = 0$$

So $\sin\left(\frac{3x}{2}\right) = 0$ or $\cos\left(\frac{x}{2}\right) = 0$. This gives

$$x = k\pi \text{ or } x = \frac{2\pi}{3} + k(2\pi) \text{ or } x = \frac{4\pi}{3} + k(2\pi),$$

where k is an integer.

Section 5.1

1. (a) $(4 + i) + (3 - 3i) = 7 - 2i$
 (b) $5(2 - i) + i(3 - 2i) = 12 - 2i$
 (c) $(4 + 2i)(5 - 3i) = 26 - 2i$
 (d) $(2 + 3i)(1 + i) + (4 - 3i) = 3 + 2i$
2. (a) $x = \frac{3}{2} + \frac{\sqrt{11}}{2}i, x = \frac{3}{2} - \frac{\sqrt{11}}{2}i.$
3. (a) $w + z = 8 - 2i.$ (b) $w + z = -3 + 6i.$
4. (a) $\bar{z} = 5 + 2i, |z| = \sqrt{29}, z\bar{z} = 29.$
 (b) $\bar{z} = -3i, |z| = 3, z\bar{z} = 9.$
5. (a) $\frac{5 + i}{3 + 2i} = \frac{17}{13} - \frac{7}{13}i.$ (b) $\frac{3 + 3i}{i} = 3 - 3i.$

Section 5.2

1. (a) $3 + 3i = \sqrt{18} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$
 (e) $4\sqrt{3} + 4i = 8 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right)$
2. (a) $5 \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right) = 5i$
 (b) $2.5 \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right) = 1.25\sqrt{2} + 1.25\sqrt{2}i$
3. (a) $wz = 10 \left(\cos\left(\frac{6\pi}{12}\right) + i \sin\left(\frac{6\pi}{12}\right) \right) = 10i$
 (b) $wz = 6.9 \left(\cos\left(\frac{19\pi}{12}\right) + i \sin\left(\frac{19\pi}{12}\right) \right)$
4. (a) $\frac{w}{z} = \frac{5}{2} \left(\cos\left(\frac{-4\pi}{12}\right) + i \sin\left(\frac{-4\pi}{12}\right) \right) = \frac{5}{4} - \frac{5\sqrt{3}}{4}i$
 (b) $\frac{w}{z} = \frac{23}{30} \left(\cos\left(\frac{-11\pi}{12}\right) + i \sin\left(\frac{-11\pi}{12}\right) \right)$

Section 5.3

1. (a) $(2 + 2i)^6 = \left[\sqrt{8} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right) \right]^6 = 512i$

(b) $(\sqrt{3} + i)^8 = \left[2 \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right) \right]^8 = -128 - 128\sqrt{3}i$

2. (a) Write $16i = 16 \left(\cos \left(\frac{\pi}{2} \right) + i \sin \left(\frac{\pi}{2} \right) \right)$. The two square roots of $16i$ are

$$4 \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right) = 2\sqrt{2} + 2i\sqrt{2}$$

$$4 \left(\cos \left(\frac{5\pi}{4} \right) + i \sin \left(\frac{5\pi}{4} \right) \right) = -2\sqrt{2} - 2i\sqrt{2}$$

(c) The three cube roots of $5 \left(\cos \left(\frac{3\pi}{4} \right) + i \sin \left(\frac{3\pi}{4} \right) \right)$ are

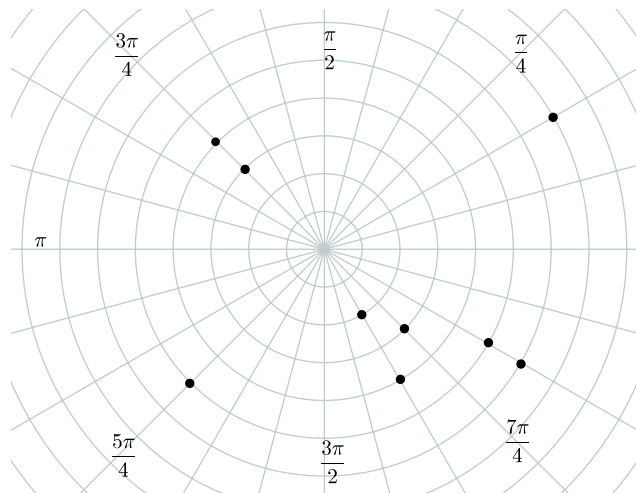
$$\sqrt[3]{5} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right) = \sqrt[3]{5} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)$$

$$\sqrt[3]{5} \left(\cos \left(\frac{11\pi}{12} \right) + i \sin \left(\frac{11\pi}{12} \right) \right)$$

$$\sqrt[3]{5} \left(\cos \left(\frac{19\pi}{12} \right) + i \sin \left(\frac{19\pi}{12} \right) \right)$$

Section 5.4

1.



2. (a) Some correct answers are: $(5, 390^\circ)$, $(5, -330^\circ)$, and $(-5, 210^\circ)$.
 (b) Some correct answers are: $(4, 460^\circ)$, $(4, -260^\circ)$, and $(-4, 280^\circ)$.
3. (a) Some correct answers are: $\left(5, \frac{13\pi}{6}\right)$, $\left(5, -\frac{11\pi}{6}\right)$, and $\left(-5, \frac{7\pi}{6}\right)$.
 (b) Some correct answers are: $\left(4, \frac{23\pi}{9}\right)$, $\left(4, -\frac{13\pi}{9}\right)$, and $\left(-4, \frac{14\pi}{9}\right)$.
4. (a) $(-5, 5\sqrt{3})$.
 (c) $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$
5. (a) $\left(5, \frac{5\pi}{6}\right)$.
 (b) $\left(\sqrt{34}, \tan^{-1}\left(\frac{5}{3}\right)\right) \approx (\sqrt{34}, 1.030)$
6. (a) $x^2 + y^2 = 25$
 (b) $y = \frac{\sqrt{3}}{3}x$
 (d) $x^2 + y^2 = \sqrt{x^2 + y^2} - y$

7. (b) $r \sin(\theta) = 4$ or $r = \frac{4}{\sin(\theta)}$

(e) $r \cos(\theta) + r \sin(\theta) = 4$ or $r = \frac{4}{\cos(\theta) + \sin(\theta)}$
