Appendix B

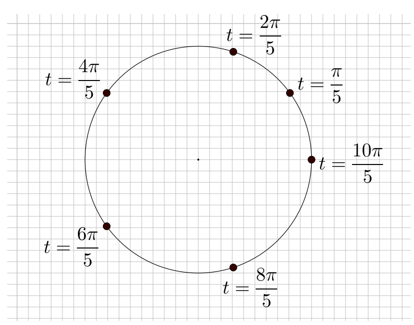
Answers and Hints for Selected Exercises

Section 1.1

1. **(b)**

t	point
1	$(0.54, 0.84)$
5	$(0.28, -0.96)$
9	$(-0.91, 0.41)$

2.



4.

	(a)	**(b)**	**(d)**	**(i)**	**(j)**	**(l)**	**(m)**
t	$\frac{7\pi}{4}$	$-\frac{7\pi}{4}$	$-\frac{3\pi}{5}$	2.5	-2.5	$3 + 2\pi$	$3 - \pi$
Quadrant	IV	I	III	II	III	II	IV

5. **(a)** We substitute $x = \dfrac{1}{3}$ into the equation $x^2 + y^2 = 1$. Solving for y, we obtain $y = \pm\dfrac{\sqrt{8}}{3}$. So the points are $\left(\dfrac{1}{3}, \dfrac{\sqrt{8}}{3}\right)$ and $\left(\dfrac{1}{3}, -\dfrac{\sqrt{8}}{3}\right)$.

(b) We substitute $y = -\dfrac{1}{2}$ into the equation $x^2 + y^2 = 1$. Solving for x, we obtain $x = \pm\dfrac{\sqrt{3}}{2}$. So the points are $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$ and $\left(-\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$.

Section 1.2

1. **(a)** For a real number t, the value of $\cos(t)$ is defined to be the <u>x</u>-coordinate of the <u>terminal</u> point of an arc t whose initial point is <u>$(1, 0)$</u> on the <u>unit circle</u> whose equation is $x^2 + y^2 = 1$.

(b) The domain of the cosine function is <u>the set of all real numbers</u>.

(c) The maximum value of $\cos(t)$ is <u>1</u> and this occurs at $t =$ <u>0</u> for $0 \le t < 2\pi$. The minimum value of $\cos(t)$ is <u>-1</u> and this occurs at $t =$ <u>π</u> for $0 \le t < 2\pi$.

(d) The range of the cosine function is <u>the closed interval $[-1, 1]$</u>.

4. **(a)** $\cos(t) = \dfrac{4}{5}$ or $\cos(t) = -\dfrac{4}{5}$.

(c) $\sin(t) = -\dfrac{\sqrt{5}}{3}$.

5. **(a)** $0 < \cos^2(t) < \dfrac{1}{9}$.

(b) $-\dfrac{1}{9} < -\cos^2(t) < 0$ and so $\dfrac{8}{9} < 1 - \cos^2(t) < 1$

(c) $\dfrac{8}{9} < \sin^2(t) < 1$

(d) $\dfrac{\sqrt{8}}{3} < \sin(t) < 1$

Section 1.3

1. **(a)** $\dfrac{1}{12}\pi \approx 0.2618$

(e) $-\dfrac{2}{9}\pi \approx -0.6981$

(b) $\dfrac{29}{90}\pi \approx 1.0123$

2. **(a)** $67.5°$

(d) $57.2958°$

(b) $231.4286°$

4. **(a)** $\cos(10°) \approx 0.9848$, $\sin(10°) \approx 0.1736$

(d) $\cos(-10°) \approx 0.9848$, $\sin(-10°) \approx -0.1736$

Section 1.4

1. **(a)** The arc length is 4π feet, which is equal to $\dfrac{1}{3}$ of the circumference of the circle.

(b) The arc length is 200 miles.

(c) The arc length is 26π meters.

(d) The arc length is $\dfrac{1520}{180}\pi$ feet ≈ 26.53 feet.

2. **(a)** $\theta = \dfrac{3\pi}{5}$ radians.

(b) $\theta = \dfrac{18}{5}$ radians = 3.6 radians.

3. **(a)** $\theta = 108°$.

(b) $\theta = \left(\dfrac{648}{\pi}\right)^{\circ} \approx 206.26°$.

5. Earth travels through an angle of $\dfrac{2\pi}{365.25}$ radians in one day. Earth travels a distance of about 1.599 million miles in one day.

8. **(b)** $v = 720\pi \dfrac{\text{in}}{\text{min}} \approx 2261.95 \dfrac{\text{in}}{\text{min}}$.

9. **(b)** $v = 3600\pi \dfrac{\text{cm}}{\text{min}} \approx 11309.73 \dfrac{\text{cm}}{\text{min}}$.

Section 1.5

1. **(a)** $t = \dfrac{\pi}{3}$, $\cos(t) = \dfrac{1}{2}$, $\sin(t) = \dfrac{\sqrt{3}}{2}$.

(b) $t = \dfrac{\pi}{2}$, $\cos(t) = 0$, $\sin(t) = 1$.

(c) $t = \dfrac{\pi}{4}$, $\cos(t) = \dfrac{\sqrt{2}}{2}$, $\sin(t) = \dfrac{\sqrt{2}}{2}$.

(d) $t = \dfrac{\pi}{6}$, $\cos(t) = \dfrac{\sqrt{3}}{2}$, $\sin(t) = \dfrac{1}{2}$.

2. **(a)** $\cos^2\left(\dfrac{\pi}{6}\right) = \left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{3}{4}$.

(b) $2\sin^2\left(\dfrac{\pi}{4}\right) + \cos(\pi) = 0$.

3. **(a)** The reference arc is $\dfrac{\pi}{3}$.

(b) The reference arc is $\dfrac{3\pi}{8}$.

(d) The reference arc is $\dfrac{\pi}{3}$.

4. **(a)** The reference arc is $\dfrac{\pi}{6}$; $\cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$; $\sin\left(\dfrac{5\pi}{6}\right) = -\dfrac{1}{2}$.

(d) The reference arc is $\dfrac{\pi}{3}$; $\cos\left(-\dfrac{2\pi}{3}\right) = -\dfrac{1}{2}$; $\sin\left(-\dfrac{2\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$.

6. **(a)** $\cos(t) = \dfrac{\sqrt{24}}{5}$.

(d) $\sin(\pi + t) = -\dfrac{1}{5}$.

Section 1.6

1.

t	$\cot(t)$	$\sec(t)$	$\csc(t)$
0	undefined	1	undefined
$\frac{\pi}{6}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$\frac{\pi}{4}$	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
$\frac{\pi}{2}$	0	undefined	1

3. **(a)** The terminal point is in the fourth quadrant.

(b) The terminal point is in the third quadrant.

4. $\cos(t) = -\dfrac{\sqrt{8}}{3}$ $\quad\tan(t) = -\dfrac{1}{\sqrt{8}}$ $\quad\sec(t) = -\dfrac{3}{\sqrt{8}}$

$\csc(t) = 3$ $\quad\cot(t) = -\sqrt{8}$

8. **(a)** $t = \dfrac{5\pi}{4}$ **(b)** $t = \dfrac{\pi}{2}$

Section 2.1

1. **(a)** $C(\pi, -1)$ $\quad R(\pi, 0)$

(b) $B\left(\dfrac{\pi}{3}, \dfrac{1}{2}\right)$ $\quad Q\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}}{2}\right)$

2. **(a)** $y = 3\sin(x)$ **(b)** $y = 2\cos(x)$

3. **(a)** t-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$ $\quad y$-intercept: $(0, 0)$

The maximum value is 1. Maximum value occurs at the points $\left(-\dfrac{3\pi}{2}, 1\right)$ and $\left(\dfrac{\pi}{2}, 1\right)$.

The minimum value is -1. Minimum value occurs at the points $\left(-\dfrac{\pi}{2}, -1\right)$ and $\left(\dfrac{3\pi}{2}, -1\right)$.

(b) t-intercepts: $-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\quad$ y-intercept: $(0, 2)$
The maximum value is 2. Maximum value occurs at the points $(0, 2)$ and $(2\pi, 2)$.
The minimum value is -2. Minimum value occurs at the points $(-\pi, -2)$, $(\pi, -2)$, and $(3\pi, -2)$.

Section 2.2

1. **(a)** $y = 2\sin(\pi x)$. The amplitude is 2; the period is 2; the phase shift is 0; and the vertical shift is 0.

$A(0, 0)$	$B\left(\dfrac{1}{2}, 2\right)$	$C(1, 0)$
$E\left(\dfrac{3}{2}, -2\right)$	$F(2, 0)$	$G\left(\dfrac{5}{2}, 2\right)$

(c) $y = 3\sin\left(x - \dfrac{\pi}{4}\right)$. The amplitude is 3; the period is 2π; the phase shift is $\dfrac{\pi}{4}$; and the vertical shift is 0.

$A\left(\dfrac{\pi}{4}, 0\right)$	$B\left(\dfrac{3\pi}{4}, 3\right)$	$C\left(\dfrac{5\pi}{4}, 0\right)$
$E\left(\dfrac{7\pi}{4}, -3\right)$	$F\left(\dfrac{9\pi}{4}, 0\right)$	$G\left(\dfrac{11\pi}{4}, 3\right)$

(g) $y = 4\sin\left(2\left(x - \dfrac{\pi}{4}\right)\right) + 1$. The amplitude is 4; the period is π; the phase shift is $\dfrac{\pi}{4}$; and the vertical shift is 1.

$A\left(\dfrac{\pi}{4}, 1\right)$	$B\left(\dfrac{\pi}{2}, 5\right)$	$C\left(\dfrac{3\pi}{4}, 1\right)$
$E\ (\pi, -3)$	$F\left(\dfrac{5\pi}{4}, 1\right)$	$G\left(\dfrac{3\pi}{2}, 5\right)$

2. **(a)** The amplitude is 2; the period is $\dfrac{2\pi}{3}$; and there is no vertical shift.

- For $y = A\sin(B(x - C)) + D$, there is no phase shift and so $C = 0$. So
$$y = 2\sin(3x).$$
- For $y = A\cos(B(x - C)) + D$, the phase shift is $\dfrac{\pi}{6}$ and so $C = \dfrac{\pi}{6}$. So
$$y = 2\cos\left(3\left(x - \frac{\pi}{6}\right)\right).$$

(d) The amplitude is 8; the period is 2; and the vertical shift is 1.

- For $y = A\sin(B(x - C)) + D$, the phase shift is $-\dfrac{1}{6}$ and so $C = -\dfrac{1}{6}$. So
$$y = 8\sin\left(\pi\left(x + \frac{1}{6}\right)\right).$$
- For $y = A\cos(B(x - C)) + D$, the phase shift is $\dfrac{1}{3}$ and so $C = \dfrac{1}{3}$. So
$$y = 8\cos\left(\pi\left(x - \frac{1}{3}\right)\right) + 1.$$

Section 2.3

1. **(a)** We write $y = 4\sin\left(\pi x - \dfrac{\pi}{8}\right) = 4\sin\left(\pi\left(x - \dfrac{1}{8}\right)\right)$. So the amplitude is 4, the period is 2, the phase shift is $\dfrac{1}{8}$, and there is no vertical shift.

- Some high points on the graph: $\left(\dfrac{5}{8}, 4\right)$, $\left(\dfrac{21}{8}, 4\right)$.
- Some low points on the graph: $\left(\dfrac{13}{8}, -4\right)$, $\left(\dfrac{29}{8}, -4\right)$.
- Graph crosses the center line at: $\left(\dfrac{1}{8}, 0\right)$, $\left(\dfrac{9}{8}, 0\right)$, $\left(\dfrac{17}{8}, 0\right)$.

(b) We write $y = 5\cos\left(4x + \dfrac{\pi}{2}\right) + 2 = 5\cos\left(4\left(x + \dfrac{\pi}{8}\right)\right) + 2$. So the amplitude is 5, the period is $\dfrac{\pi}{2}$, the phase shift is $-\dfrac{\pi}{8}$, and the vertical shift is 2.

- Some high points on the graph: $\left(-\frac{\pi}{8}, 7\right)$, $\left(\frac{3\pi}{8}, 7\right)$.
- Some low points on the graph: $\left(\frac{\pi}{8}, -3\right)$, $\left(\frac{5\pi}{8}, -3\right)$.
- Graph crosses the center line at: $(0, 2)$, $\left(\frac{\pi}{4}, 2\right)$, $\left(\frac{\pi}{2}, 2\right)$.

2. **(b)** The maximum value is 150 ml, and the minimum value is 81 ml. So we can use $A = \dfrac{150 - 81}{2} = 34.5$ and $D = \dfrac{150 + 81}{2} = 115.5$.

(c) The period is $\dfrac{1}{75}$ min.

Section 2.4

1. **(a)** $\tan(t + 2\pi) = \dfrac{\sin(t + 2\pi)}{\cos(t + 2\pi)} = \dfrac{\sin(t)}{\cos(t)} = \tan(t)$.

3. **(a)** $\csc(-t) = \dfrac{1}{\sin(-t)} = \dfrac{1}{-\sin(t)} = -\dfrac{1}{\sin(t)} = -\csc(t)$.

Section 2.5

1. **(a)** $t = \arcsin\left(\dfrac{\sqrt{2}}{2}\right)$ means $\sin(t) = \dfrac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$. Since $\sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$, we see that $t = \arcsin\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$.

(b) $t = \arcsin\left(-\dfrac{\sqrt{2}}{2}\right)$ means $\sin(t) = -\dfrac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$. Since $\sin\left(\dfrac{-\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$, we see that $t = \arcsin\left(\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\pi}{4}$.

(d) $t = \arccos\left(-\dfrac{\sqrt{2}}{2}\right)$ means $\cos(t) = -\dfrac{\sqrt{2}}{2}$ and $0 \leq t \leq \pi$. Since $\cos\left(\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$, we see that $t = \arccos\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\pi}{4}$.

(f) $y = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ means $\tan(y) = -\frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Since $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$, we see that $y = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

(h) $t = \arctan(0) = 0$.

(j) $y = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

2. **(a)** $\sin\left(\sin^{-1}(1)\right) = \sin\left(\frac{\pi}{2}\right) = 1$

(b) $\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

(e) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

(f) $\arcsin\left(\sin\left(\frac{2\pi}{3}\right)\right) = \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

(i) $\arctan\left(\tan\left(\frac{3\pi}{4}\right)\right) = \arctan(-1) = -\frac{\pi}{4}$

3. **(a)** Let $t = \arcsin\left(\frac{2}{5}\right)$. Then $\sin(t) = \frac{2}{5}$ and $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$, and

$$\begin{aligned}
\cos^2(t) + \sin(t)^2 &= 1 \\
\cos^2(t) + \frac{4}{25} &= 1 \\
\cos^2(t) &= \frac{21}{25}
\end{aligned}$$

Since $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$, we know that $\cos(t) \geq 0$. Hence, $\cos(t) = \frac{\sqrt{21}}{5}$ and $\cos\left(\arcsin\left(\frac{2}{5}\right)\right) = \frac{\sqrt{21}}{5}$.

(b) $\sin\left(\arccos\left(-\frac{2}{3}\right)\right) = \frac{\sqrt{5}}{3}$.

(c) $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{\sqrt{8}}$.

Section 2.6

1. **(a)** $x = 0.848 + k(2\pi)$ or $x = 2.294 + k(2\pi)$, where k is an integer.

   (d) $x = -0.848 + k(2\pi)$ or $x = -2.294 + k(2\pi)$, where k is an integer.

2. **(a)** $x = \sin^{-1}(0.75) + k(2\pi)$ or $x = \left(\pi - \sin^{-1}(0.75)\right) + k(2\pi)$, where k is an integer.

   (d) $x = \arcsin(-0.75) + k(2\pi)$ or $x = (\pi - \arcsin(-0.75)) + k(2\pi)$, where k is an integer.

3. **(a)** $x = \sin^{-1}(0.4) + k(2\pi)$ or $x = \left(\pi - \sin^{-1}(0.4)\right) + k(2\pi)$, where k is an integer.

   (b) $x = \cos^{-1)}\left(\dfrac{4}{5}\right) + k(2\pi)$ or $x = -\cos^{-1)}\left(\dfrac{4}{5}\right) + k(2\pi)$, where k is an integer.

4. **(a)** The period for the trigonometric function is π. We first solve the equation $4\sin(t) = 3$ with $-\pi \le t \le \pi$ and obtain $t = \sin^{-1}(0.75) + k(2\pi)$ or $t = \left(\pi - \sin^{-1}(0.75)\right) + k(2\pi)$. We then use the substitution $t = 2x$ to obtain

   $x = \dfrac{1}{2}\sin^{-1}(0.75) + k(\pi)$ or $x = \dfrac{1}{2}\left(\pi - \sin^{-1}(0.75)\right) + k(\pi)$, where k is an integer.

   (d) The period for the trigonometric function is 2. We first solve the equation $\sin(t) = 0.2$ with $-\pi \le t \le \pi$ and obtain $t = \sin^{-1}(0.2) + k(2\pi)$ or $t = \left(\pi - \sin^{-1}(0.2)\right) + k(2\pi)$. We now use the substitution $t = \pi x - \dfrac{\pi}{4}$ to obtain

   $x = \left(\dfrac{1}{\pi}\sin^{-1}(0.2) + \dfrac{1}{4}\right) + 2k$ or $x = \left(-\dfrac{1}{\pi}\sin^{-1}(0.2) + \dfrac{5}{4}\right) + 2k$, where k is an integer.

Section 3.1

1. **(a)** We see that $r = \sqrt{3^2 + 3^2} = \sqrt{18}$. So

$$\cos(\theta) = \frac{3}{\sqrt{18}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \qquad \sin(\theta) = \frac{3}{\sqrt{18}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$\tan(\theta) = \frac{3}{3} = 1 \qquad \cot(\theta) = \frac{3}{3} = 1$$
$$\sec(\theta) = \frac{\sqrt{18}}{3} = \sqrt{2} \qquad \csc(\theta) = \frac{\sqrt{18}}{3} = \sqrt{2}$$

(b) We see that $r = \sqrt{5^2 + 8^2} = \sqrt{89}$. So

$$\cos(\theta) = \frac{5}{\sqrt{89}} \qquad \sin(\theta) = \frac{8}{\sqrt{89}}$$
$$\tan(\theta) = \frac{8}{5} \qquad \cot(\theta) = \frac{5}{8} = 1$$
$$\sec(\theta) = \frac{\sqrt{89}}{5} \qquad \csc(\theta) = \frac{\sqrt{89}}{8}$$

(e) We see that $r = \sqrt{(-1)^2 + (-4)^2} = \sqrt{17}$. So

$$\cos(\theta) = \frac{-1}{\sqrt{17}} \qquad \sin(\theta) = \frac{-4}{\sqrt{17}}$$
$$\tan(\theta) = 4 \qquad \cot(\theta) = \frac{1}{4}$$
$$\sec(\theta) = -\sqrt{17} - \frac{\sqrt{26}}{4} \qquad \csc(\theta) = -\frac{\sqrt{17}}{4}$$

2. **(b)** We first use the Pythagorean Identity and obtain $\sin^2(\beta) = \frac{5}{9}$. Since the terminal side of β is in the second quadrant, $\sin(\beta) = \frac{\sqrt{5}}{3}$. In addition,

$$\tan(\beta) = -\frac{\sqrt{5}}{2} \qquad \cot(\beta) = -\frac{2}{\sqrt{5}}$$
$$\sec(\beta) = -\frac{3}{2} \qquad \csc(\beta) = \frac{3}{\sqrt{5}}$$

3. **(c)** Since the terminal side of θ is in the second quadrant, θ is not the inverse sine of $\frac{2}{3}$. So we let $\alpha = \arcsin\left(\frac{2}{3}\right)$. Using α as the reference

angle, we then see that

$$\theta = 180° - \arcsin\left(\frac{2}{3}\right) \approx 138.190°.$$

(e) $\theta = \arccos\left(-\frac{1}{4}\right) \approx 104.478°$, or use $\alpha = \arccos\left(\frac{1}{4}\right)$ for the reference angle.

$$\theta = 180° - \arccos\left(\frac{1}{4}\right) \approx 104.478°.$$

4. **(c)** $\theta = \pi - \arcsin\left(\frac{2}{3}\right) \approx 2.142.$

(e) $\theta = \pi - \arccos\left(\frac{1}{4}\right) \approx 1.823.$

5. **(b)** Let $\theta = \cos^{-1}\left(\frac{2}{3}\right)$. Then $\cos(\theta) = \frac{2}{3}$ and $0 \leq \theta \leq \pi$. So $\sin(\theta) > 0$ and $\sin^2(\theta) = 1 - \cos^2(\theta) = \frac{5}{9}$. So

$$\begin{aligned}\tan\left(\cos^{-1}\left(\frac{2}{3}\right)\right) &= \tan(\theta)\\ &= \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}\\ &= \frac{\sqrt{5}}{2}\end{aligned}$$

Using a calculator, we obtain

$$\tan\left(\cos^{-1}\left(\frac{2}{3}\right)\right) \approx 1.11803 \quad \text{and} \quad \frac{\sqrt{5}}{2} \approx 1.11803.$$

Section 3.2

1. **(a)** $x = 6\tan(47°) \approx 6.434$. **(b)** $x = 3.1\cos(67°) \approx 1.211$.

   (c) $x = \tan^{-1}\left(\dfrac{7}{4.9}\right) \approx 55.008°$.

   (d) $x = \sin^{-1}\left(\dfrac{7}{9.5}\right) \approx 47.463°$.

4. The other acute angle is $64°48'$.

   - The side opposite the $27°12'$ angle is $4\tan(27°12') \approx 2.056$ feet.
   - The hypotenuse is $\dfrac{4}{\cos(27°12')} \approx 4.497$ feet.

   Note that the Pythagorean Theorem can be used to check the results by showing that $4^2 + 2.056^2 \approx 4.497^2$. The check will not be exact because the 2.056 and 4.497 are approximations of the exact values.

7. We first note that $\theta = 40°$. We use the following two equations to determine x.

$$\tan(\alpha) = \frac{h}{c+x} \qquad \tan(\theta) = \frac{h}{x}$$

   Substituting $h = x\tan(\theta)$ into the first equation and solving for x gives

$$x = \frac{c\tan(\alpha)}{\tan(\theta) - \tan(\alpha)} \approx 8.190.$$

   We can then use right triangles to obtain $h \approx 6.872$ ft, $a \approx 10.691$ ft, and $b \approx 17.588$ ft.

Section 3.3

1. The third angle is $65°$. The side opposite the $42°$ angle is 4.548 feet long. The side opposite the $65°$ angle is 6.160 feet long.

3. There are two triangles that satisfy these conditions. The sine of the angle opposite the 5 inch side is approximately 0.9717997.

5. The angle opposite the 9 foot long side is $95.739°$. The angle opposite the 7 foot long side is $50.704°$. The angle opposite the 5 foot long side is $33.557°$.

Section 3.4

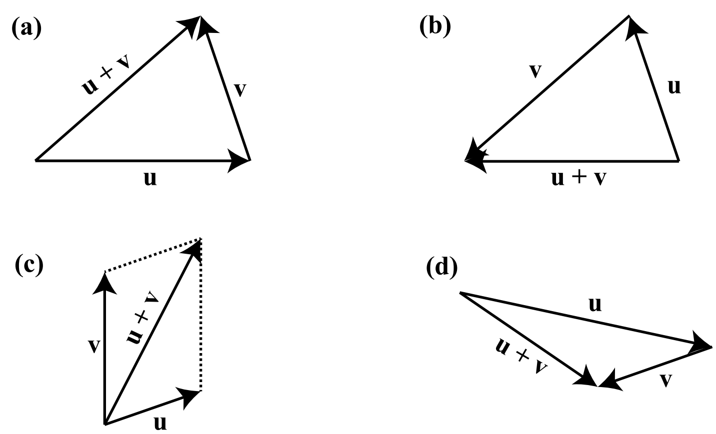
1. The ski lift is about 1887.50 feet long.
2. **(a)** The boat is about 67.8 miles from Chicago.

   (b) $\gamma \approx 142.4°$. So the boat should turn through an angle of about $180° - 142.4° = 37.6°$.

   (c) The direct trip from Muskegon to Chicago would take $\dfrac{121}{15}$ hours or about 8.07 hours. By going off-course, the trip now will take $\dfrac{127.8}{15}$ hours or about 8.52 hours.

Section 3.5

1.



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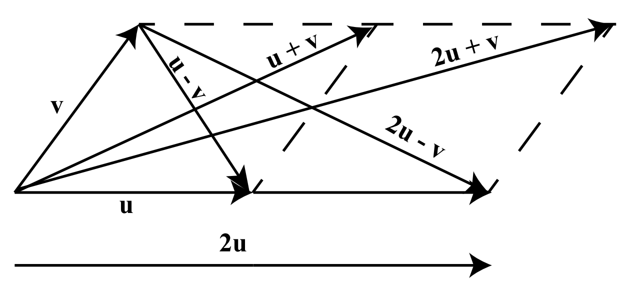


3. The angle between the vectors $\mathbf{a}$ and $\mathbf{a}+\mathbf{b}$ is approximately $9.075°$. In addition, $|\mathbf{b}| \approx 4.416$.

Section 3.6

1. **(a)** $|\mathbf{v}| = \sqrt{34}$. The direction angle is approximately $59.036°$.

   (b) $|\mathbf{w}| = \sqrt{45}$. The direction angle is approximately $116.565°$.

2. **(a)** $\mathbf{v} = 12\cos(50°) + 12\sin(50°) \approx 7.713\mathbf{i} + 9.193\mathbf{j}$.

   (b) $\mathbf{u} = \sqrt{20}\cos(125°) + \sqrt{20}\sin(125°) \approx -2.565\mathbf{i} + 3.663\mathbf{j}$.

3. **(a)** $5\mathbf{u} - \mathbf{v} = 11\mathbf{i} + 10\mathbf{j}$.

   (c) $\mathbf{u} + \mathbf{v} + \mathbf{w} = 5\mathbf{i} + 6\mathbf{j}$.

4. **(a)** $\mathbf{v}\cdot\mathbf{w} = -4$.

   (b) $\mathbf{a}\cdot\mathbf{b} = 9\sqrt{3}$.

5. **(a)** The angle between $\mathbf{v}$ and $\mathbf{w}$ is $\cos^{-1}\left(\dfrac{-4}{\sqrt{29}\sqrt{13}}\right) \approx 101.89°$.

6. **(a)** $\mathbf{proj}_{\mathbf{v}}\mathbf{w} = \dfrac{\mathbf{w}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v} = \dfrac{-4}{29}(2\mathbf{i} + 5\mathbf{j}) = -\dfrac{8}{29}\mathbf{i} - \dfrac{20}{29}\mathbf{j}$.

   $\mathbf{proj}_{\perp\mathbf{v}}\mathbf{w} = \mathbf{w} - \mathbf{proj}_{\mathbf{v}}\mathbf{w} = \dfrac{95}{29}\mathbf{i} - \dfrac{38}{29}\mathbf{j}$

Section 4.1

1. **(a)**

$$\begin{aligned}\cos(x)\tan(x) &= \cos(x)\frac{\sin(x)}{\cos(x)}\\ &= \sin(x)\end{aligned}$$

(b)

$$\begin{aligned}\frac{\cot(x)}{(x)} &= \frac{\dfrac{\cos(x)}{\sin(x)}}{\dfrac{1}{\sin(x)}}\\ &= \frac{\cos(x)}{\sin(x)} \times \frac{\sin(x)}{1}\\ &= \cos(x)\end{aligned}$$

(e) A graph will show that this is not an identity. In particular, we see that

$$\sec^2\left(\frac{\pi}{4}\right) + \csc^2\left(\frac{\pi}{4}\right) = \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 4$$

Section 4.2

1. **(a)** $x = \dfrac{\pi}{6} + k(2\pi)$ or $x = \dfrac{5\pi}{6} + k(2\pi)$, where k is an integer.

(b) $x = \dfrac{2\pi}{3} + k(2\pi)$ or $x = \dfrac{4\pi}{3} + k(2\pi)$, where k is an integer.

(d) $x = \cos^{-1}\left(\dfrac{3}{4}\right) + k(2\pi)$ or $x = \cos^{-1}\left(-\dfrac{3}{4}\right) + k(2\pi)$, where k is an integer.

(f) $x = k\pi$, where k is an integer.

2. $\theta = \sin^{-1}\left(\dfrac{2}{3}\right) \approx 41.81°$ is one solution of the equation $\sin(\theta) + \dfrac{1}{3} = 1$ with $0 \leq \theta \leq 360°$. There is another solution (in the second quadrant) for this equation with $0 \leq \theta \leq 360°$.

Section 4.3

1. **(a)** $\cos(-10° - 35°) = \cos(-45°) = \dfrac{\sqrt{2}}{2}$.

(b) $\cos\left(\dfrac{7\pi}{9} + \dfrac{2\pi}{9}\right) = \cos(\pi) = -1$.

2. We first use the Pythagorean Identity to determine $\cos(A)$ and $\sin(B)$. From this, we get

$$\cos(A) = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin(B) = -\frac{\sqrt{7}}{4}.$$

(a)

$$\begin{aligned}\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B)\\ &= \frac{\sqrt{3}}{2}\cdot\frac{3}{4} - \frac{1}{2}\cdot\left(-\frac{\sqrt{7}}{4}\right)\\ &= \frac{3\sqrt{3}+\sqrt{7}}{8}\end{aligned}$$

3. **(a)** $\cos(15°) = \cos(45° - 30°) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

(d) We can use $345° = 300° + 45°$ and first evaluate $\cos(345°)$. This gives $\cos(345°) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ and $\sec(345°) = \dfrac{4}{\sqrt{6}+\sqrt{2}}$. We could have also used the fact that $\cos(345°) = \cos(15°)$ and the result in part (a).

5. **(a)**

$$\begin{aligned}\cot\left(\frac{\pi}{2}-x\right) &= \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\\ &= \frac{\sin(x)}{\cos(x)}\\ &= \tan(x)\end{aligned}$$

9. **(c)** It can be shown that $\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right) = \sin(x)+\cos(x)$. So the graph of $f(x) = \sin(x)+\cos(x)$ has an amplitude of $\sqrt{2}$, a phase shift of $-\dfrac{\pi}{4}$, and a period of 2π.

Section 4.4

1. Use the Pythagorean Identity to obtain $\sin^2(\theta) = \dfrac{5}{9}$. Since $\sin(\theta) < 0$, we see that $\sin(\theta) = -\dfrac{\sqrt{5}}{3}$. Now use appropriate double angle identities to get

$$\sin(2\theta) = -\frac{4\sqrt{5}}{9} \qquad\qquad \cos(2\theta) = -\frac{1}{9}$$

Then use $\tan(2\theta) = \dfrac{\sin(2\theta)}{\cos(2\theta)} = 4\sqrt{5}$.

2. **(a)** $x = \dfrac{\pi}{4} + k\pi$, where k is an integer.

3. **(a)** This is an identity. Start with the left side of the equation and use $\cot(t) = \dfrac{\cos(t)}{\sin(t)}$ and $\sin(2t) = 2\sin(t)\cos(t)$.

6. **(a)** $\sin(22.5°) = \sqrt{\dfrac{1 - \frac{\sqrt{2}}{2}}{2}} = \dfrac{1}{2}\sqrt{2 - \sqrt{2}}$.

(c) $\tan(22.5°) = \sqrt{\dfrac{2 - \sqrt{2}}{2 + \sqrt{2}}} = \sqrt{3 - 2\sqrt{2}}$.

(h) $\cos(195°) = -\sqrt{\dfrac{1 + \frac{\sqrt{3}}{2}}{2}} = -\dfrac{1}{2}\sqrt{2 + \sqrt{3}}$.

7. **(a)** $\sin\left(\dfrac{3\pi}{8}\right) = \sqrt{\dfrac{1 + \frac{\sqrt{2}}{2}}{2}} = \dfrac{1}{2}\sqrt{2 + \sqrt{2}}$.

(c) $\tan\left(\dfrac{3\pi}{8}\right) = \sqrt{\dfrac{2 + \sqrt{2}}{2 - \sqrt{2}}} = \sqrt{3 + 2\sqrt{2}}$.

(h) $\cos\left(\dfrac{11\pi}{12}\right) = -\sqrt{\dfrac{1 + \frac{\sqrt{3}}{2}}{2}} = -\dfrac{1}{2}\sqrt{2 + \sqrt{3}}$.

8. **(a)** We note that since $\dfrac{3\pi}{2} \le x \le 2\pi$, $\dfrac{3\pi}{4} \le \dfrac{x}{2} \le \pi$.

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1 - \frac{2}{3}}{2}} = \frac{1}{\sqrt{6}}.$$

Section 4.5

1. **(a)** $\sin(37.5°)\cos(7.5°) = \dfrac{1}{2}[\sin(45°) + \sin(30°)] = \dfrac{\sqrt{2} + 1}{4}$

(e) $\cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) = \frac{1}{2}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{3}\right)\right] = \frac{2-\sqrt{3}}{4}$

2. **(a)** $\sin(50°) + \sin(10°) = 2\sin(30°)\cos(20°) = \cos(20°)$

(e) $\cos\left(\frac{7\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right) = 2\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

3. **(a)**

$$\begin{aligned} \sin(2x) + \sin(x) &= 0 \\ 2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right) &= 0 \end{aligned}$$

So $\sin\left(\frac{3x}{2}\right) = 0$ or $\cos\left(\frac{x}{2}\right) = 0$. This gives

$$x = k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + k(2\pi) \quad \text{or} \quad x = \frac{4\pi}{3} + k(2\pi),$$

where k is an integer.

Section 5.1

1. **(a)** $(4+i) + (3-3i) = 7-2i$

(b) $5(2-i) + i(3-2i) = 12-2i$

(c) $(4+2i)(5-3i) = 26-2i$

(d) $(2+3i)(1+i) + (4-3i) = 3+2i$

2. **(a)** $x = \frac{3}{2} + \frac{\sqrt{11}}{2}i$, $x = \frac{3}{2} - \frac{\sqrt{11}}{2}i$.

3. **(a)** $w + z = 8 - 2i$. **(b)** $w + z = -3 + 6i$.

4. **(a)** $\overline{z} = 5 + 2i$, $|z| = \sqrt{29}$, $z\overline{z} = 29$.

(b) $\overline{z} = -3i$, $|z| = 3$, $z\overline{z} = 9$.

5. **(a)** $\frac{5+i}{3+2i} = \frac{17}{13} - \frac{7}{13}i$. **(b)** $\frac{3+3i}{i} = 3 - 3i$.

Section 5.2

1. **(a)** $3+3i = \sqrt{18}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$

(e) $4\sqrt{3}+4i = 8\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$

2. **(a)** $5\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)=5i$

(b) $2.5\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)=1.25\sqrt{2}+1.25\sqrt{2}i$

3. **(a)** $wz = 10\left(\cos\left(\frac{6\pi}{12}\right)+i\sin\left(\frac{6\pi}{12}\right)\right)=10i$

(b) $wz = 6.9\left(\cos\left(\frac{19\pi}{12}\right)+i\sin\left(\frac{19\pi}{12}\right)\right)$

4. **(a)** $\frac{w}{z}=\frac{5}{2}\left(\cos\left(\frac{-4\pi}{12}\right)+i\sin\left(\frac{-4\pi}{12}\right)\right)=\frac{5}{4}-\frac{5\sqrt{3}}{4}i$

(b) $\frac{w}{z}=\frac{23}{30}\left(\cos\left(\frac{-11\pi}{12}\right)+i\sin\left(\frac{-11\pi}{12}\right)\right)$

Section 5.3

1. **(a)** $(2+2i)^6=\left[\sqrt{8}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)\right]^6=512i$

(b) $\left(\sqrt{3}+i\right)^8=\left[2\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)\right]^8=-128-128\sqrt{3}i$

2. **(a)** Write $16i = 16\left(\cos\left(\frac{\pi}{2}\right)+i\sin(\left(\frac{\pi}{2}\right)\right)$. The two square roots of $16i$ are

$$4\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)=2\sqrt{2}+2i\sqrt{2}$$
$$4\left(\cos\left(\frac{5\pi}{4}\right)+i\sin\left(\frac{5\pi}{4}\right)\right)=-2\sqrt{2}-2i\sqrt{2}$$

(c) The three cube roots of $5\left(\cos\left(\frac{3\pi}{4}\right) + i\,\sin\left(\frac{3\pi}{4}\right)\right)$ are

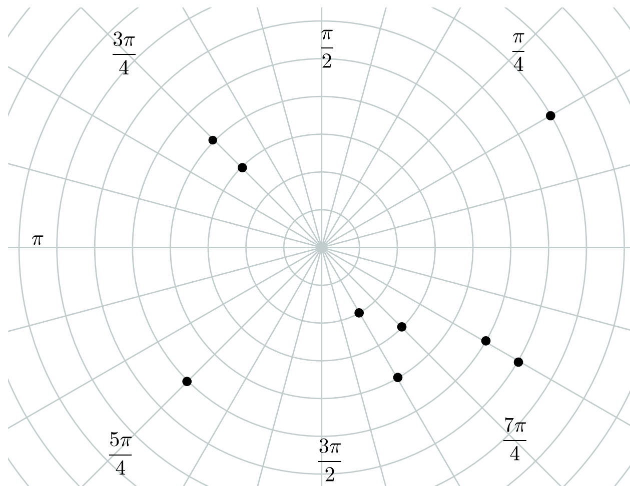
$$\sqrt[3]{5}\left(\cos\left(\frac{\pi}{4}\right) + i\,\sin\left(\frac{\pi}{4}\right)\right) = \sqrt[3]{5}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$$

$$\sqrt[3]{5}\left(\cos\left(\frac{11\pi}{12}\right) + i\,\sin\left(\frac{11\pi}{12}\right)\right)$$

$$\sqrt[3]{5}\left(\cos\left(\frac{19\pi}{12}\right) + i\,\sin\left(\frac{19\pi}{12}\right)\right)$$

Section 5.4

1.



2. **(a)** Some correct answers are: $(5, 390°)$, $(5, -330°)$, and $(-5, 210°)$.

(b) Some correct answers are: $(4, 460°)$, $(4, -260°)$, and $(-4, 280°)$.

3. **(a)** Some correct answers are: $\left(5, \frac{13\pi}{6}\right)$, $\left(5, -\frac{11\pi}{6}\right)$, and $\left(-5, \frac{7\pi}{6}\right)$.

(b) Some correct answers are: $\left(4, \frac{23\pi}{9}\right)$, $\left(4, -\frac{13\pi}{9}\right)$, and $\left(-4, \frac{14\pi}{9}\right)$.

4. **(a)** $(-5, 5\sqrt{3})$.

(c) $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$

5. **(a)** $\left(5, \frac{5\pi}{6}\right)$.

(b) $\left(\sqrt{34}, \tan^{-1}\left(\frac{5}{3}\right)\right) \approx \left(\sqrt{34}, 1.030\right)$

6. **(a)** $x^2 + y^2 = 25$

(b) $y = \frac{\sqrt{3}}{3}x$

(d) $x^2 + y^2 = \sqrt{x^2 + y^2} - y$

7. **(b)** $r\sin(\theta) = 4$ or $r = \dfrac{4}{\sin(\theta)}$

(e) $r\cos(\theta) + r\sin(\theta) = 4$ or $r = \dfrac{4}{\cos(\theta) + \sin(\theta)}$