

Appendix C

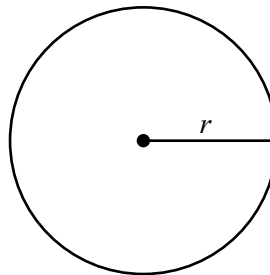
Some Geometric Facts about Triangles and Parallelograms

This appendix contains some formulas and results from geometry that are important in the study of trigonometry.

Circles

For a circle with radius r :

- **Circumference:** $C = 2\pi r$
- **Area:** $A = \pi r^2$



Triangles

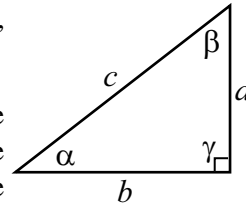
- The sum of the measures of the three angles of a triangle is 180° .
- A triangle in which each angle has a measure of less than 90° is called an **acute triangle**.
- A triangle that has an angle whose measure is greater than 90° is called an **obtuse triangle**.
- A triangle that contains an angle whose measure is 90° is called a **right triangle**. The side of a right triangle that is opposite the right angle is called

the **hypotenuse**, and the other two sides are called the **legs**.

- An **isosceles triangle** is a triangle in which two sides of the triangle have equal length. In this case, the two angles across from the two sides of equal length have equal measure.
- An **equilateral triangle** is a triangle in which all three sides have the same length. Each angle of an equilateral triangle has a measure of 60° .

Right Triangles

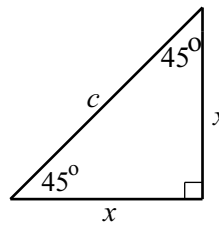
- The sum of the measures of the two acute angles of a right triangle is 90° . In the diagram on the right, $\alpha + \beta = 90^\circ$.
- **The Pythagorean Theorem.** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In the diagram on the right, $c^2 = a^2 + b^2$.



Special Right Triangles

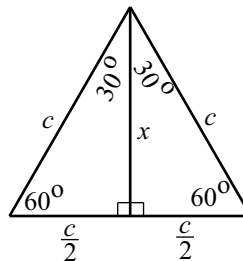
- A **right triangle in which both acute angles are 45°** . For this type of right triangle, the lengths of the two legs are equal. So if c is the length of the hypotenuse and x is the length of each of the legs, then by the Pythagorean Theorem, $c^2 = x^2 + x^2$. Solving this equation for x , we obtain

$$\begin{aligned} 2x^2 &= c^2 \\ x^2 &= \frac{c^2}{2} \\ x &= \sqrt{\frac{c^2}{2}} \\ x &= \frac{c}{\sqrt{2}} = \frac{\sqrt{2}}{2}c \end{aligned}$$



- A right triangle with acute angles of 30° and 60° .

We start with an equilateral triangle with sides of length c . By drawing an angle bisector at one of the vertices, we create two congruent right triangles with acute angles of 30° and 60° .

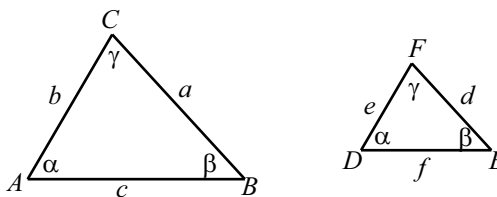


This means that the third side of each of these right triangles will have a length of $\frac{c}{2}$. If the length of the altitude is x , then using the Pythagorean Theorem, we obtain

$$\begin{aligned} c^2 &= x^2 + \left(\frac{c}{2}\right)^2 \\ x^2 &= c^2 - \frac{c^2}{4} \\ x^2 &= \frac{3c^2}{4} \\ x &= \sqrt{\frac{3c^2}{4}} = \frac{\sqrt{3}}{2}c \end{aligned}$$

Similar Triangles

Two triangles are **similar** if the three angles of one triangle are equal in measure to the three angles of the other triangle. The following diagram shows similar triangles $\triangle ABC$ and $\triangle DEF$. We write $\triangle ABC \sim \triangle DEF$.

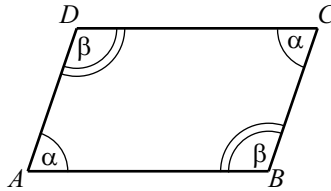


The sides of similar triangles do not have to have the same length but they will be proportional. Using the notation in the diagram, this means that

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}.$$

Parallelograms

We use some properties of parallelograms in the study of vectors in Section 3.5. A **parallelogram** is a quadrilateral with two pairs of parallel sides. We will use the diagram on the right to describe some properties of parallelograms.



- Opposite sides are equal in length. In the diagram, this means that

$$AB = DC \text{ and } AD = BC.$$

- As shown in the diagram, opposite angles are equal. That is,

$$\angle DAB = \angle BCD \text{ and } \angle ABC = \angle CDA.$$

- The sum of two adjacent angles is 180° . In the diagram, this means that

$$\alpha + \beta = 180^\circ.$$