Summer 1995

A Comparison of Kentwood Public Schools' Algebra Standards and the UCSMP Algebra Objectives

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A COMPARISON OF
KENTWOOD PUBLIC SCHOOL'S ALGEBRA STANDARDS
AND THE UCSMP ALGEBRA OBJECTIVES

Carol A. Farrer
Summer, 1995

MASTERS THESIS
Submitted to the graduate faculty at
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ABSTRACT

This paper compares the Standards written by Kentwood Public Schools in Kentwood, Michigan for its algebra course and the objectives of the University of Chicago School Mathematics Project Algebra textbook. The paper includes an examination of national recommendations and publications which influenced the authors of the UCSMP text and the national and state publications which influenced Kentwood’s Mathematics Curriculum Committee. Lessons are provided for Kentwood’s algebra teachers for the few instances where the objectives of the textbook do not meet the district’s standards.
ACKNOWLEDGEMENTS

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CHAPTER ONE - THE PROBLEM

In Kentwood Public Schools the textbook used by algebra students, in the middle schools and in the high school, is the University of Chicago School Mathematics Project’s (UCSMP) Algebra (McConnell, et al., 1993). At the time it was selected it met the goals and objectives of the district’s algebra course. However, during the 1994/95 school year, the district adopted standards for its mathematics program, kindergarten through twelfth grade. There is now some concern that the existing textbook might not meet the new standards. Therefore, the purpose of this study is to compare the objectives of the UCSMP Algebra text (Appendix A) with Kentwood Public School’s Standards for its algebra classes (Appendix B) to determine whether the textbook incorporates all of the algebra Level Standards approved by the district.

Background

Since 1992 members of the Kentwood Mathematics Curriculum Committee have systematically studied the goals and objectives of the district's kindergarten through twelfth grade mathematics' program. Interest in this area was stimulated in part by state-mandated testing and in part by a realization that the rapidly growing district needed to clarify its outcomes in mathematics so all
teachers in all buildings would know the expectations. The goal was to keep standards as uniform as possible.

Although subject to periodic updating the final phase of this project was completed during the 1994-95 school year. A committee of representatives from kindergarten through twelfth grade reviewed existing Level Standards for each grade or subject area, revising and updating to reflect changing student needs. They then wrote suggested lesson plans and provided assessment pieces for each Level Standard.

It is important to know that district teachers have decided what is necessary for their students at each grade/level. It is also important to know the many factors on which those decisions were based. The influences range from national and state publications to local emphases.

In 1983, while Ronald Reagan was president, the National Committee on Excellence in Education published A Nation at Risk: The Imperative for Educational Reform which outlined concerns about public education at the time, including perceived shortcomings in the mathematics curriculum. Six years later, in 1989, the National Council of Teachers of Mathematics published the NCTM Standards. According to F. Joe Crosswhite in an article which appeared in School Science and Mathematics (Crosswhite, 1990), this publication was a major effort to initiate change in the
way mathematics was taught, including working with textbook companies to effect the change (Crosswhite, 1990). The intent was to address the points of dissatisfaction at the time and present curriculum content and teaching methods to achieve reform (Crosswhite, 1990).

It was not only national publications which influenced Kentwood staff as they wrote their Standards. The State of Michigan has published a list of performance objectives for mathematics (Michigan State Board of Education, 1991). Because state-mandated testing is based on those objectives, it was prudent for the committee to review that document. Care was also taken to consider what students could accomplish at each stage of development. For continuity, higher levels extended the skills achieved in previous years and incorporated new concepts needed as a basis for the next level.

In addition to all of the above factors, the committee tried to include skills needed by students when they enter the workplace, including problem-solving, reasoning, and communication skills (Owens, 1988). All of the ideas from national, state, and local sources were incorporated into the final product as much as possible. All teachers in the district were given a copy of the portions appropriate for their grade/level.
While this effort addressed the original concerns of producing a list of District Standards, some problems remain to be considered. After much research Kentwood began using the University of Chicago School Mathematics Project (UCSMP) textbook series for grades seven through twelve in 1990. Zalman Usiskin in *Educational Leadership* (1986/87) explained that the series was written originally with a grant from Amoco Foundation to the University of Chicago in 1983, with further funding coming from Carnegie Corporation of New York, the National Science Foundation, and the General Electric Foundation. Amoco authorized the production of a series that would provide solutions to the perceived problems in the mathematics curriculum in high school at that time. According to the authors of the series, the "UCSMP would not attempt to form its own set of recommendations, but would undertake the task of translating the existing recommendations into the reality of classrooms and schools" (McConnell, et al., 1993). Those same authors gave the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (1989) and the National Commission on Excellence in Education's *A Nation at Risk: The Imperative for Educational Reform* (1983) as two of the publications that influenced the textbook series.
Rationale

When it was adopted, the series "best fit" the district's mathematics' goals and objectives. It was not a perfect fit even then, but no one textbook can be expected to be perfect. Now, however, because of Kentwood's many-faceted approach to writing the district's Standards, there may be some areas that the staff felt were necessary, but that were not included in the text. These may be concepts deemed important by the state of Michigan or by the district itself.

So, the concern that the objectives of the text might not correlate completely with the Standards approved by Kentwood's teachers remains. It is necessary to examine both to determine if significant differences exist and to supplement with appropriate materials if necessary.

Purpose

Looking at the entire curriculum for the district would be too broad for this paper. Therefore, based on the magnitude and rationale of the problem, the purpose of this study is two-fold:

1. Compare Kentwood Public School's Standards for algebra with the objectives for the University of Chicago School Mathematics Project's Algebra textbook.
2. Provide supplementary units, including assessment, for those Standards not addressed at all or addressed in a cursory manner in the textbook.

Specifically, for each stated Kentwood Standard this study will cross-reference it with the objective(s) in the text which prepare(s) the student to accomplish the Standard. That is, if the student achieves the textbook’s objective(s), then the student will achieve the district’s Standard.

If there are some Standards which are not objectives of the UCSMP’s Algebra book, this study will include units to prepare students to accomplish those Standards. These may be compiled from existing material or may be original. In either case they will be accompanied by an assessment to discern student achievement. It is not expected that a large number of such units will be needed, based on the criteria for selecting the series in 1990. However, the importance of re-evaluating the chosen textbook cannot be overlooked.

In assessing the validity of this study one limitation must be noted. As a beginning to the research for this paper a letter was sent to Professor Zalman Usiskin, chairman of the University of Chicago School Mathematics Project, requesting statistics to verify that the stated objectives are achieved by students using the Algebra text.
He responded that those statistics were not collected. His response included assurance that the series was extensively field tested and inappropriate topics eliminated. A copy of the original letter and Professor Usiskin's response can be found in Appendix C. Because of the method of field testing described by Professor Usiskin in *Educational Leadership* (December, 1986/January, 1987), the positive results shown in a paper by J. Rivers when the UCSMP texts are compared to other series (1990), and the achievement shown by students using the books as documented in *Journal for Research in Mathematics Education* (Hirschhorn, 1993), this limitation is not sufficient to invalidate this study.

It is expected that this research will aid teachers of algebra in Kentwood to accomplish the stated Standards in that subject. In addition to clearly defining which Standards are found in the textbook, it will also provide convenient supplements where needed.

Time is a factor in this subject. Concepts taught in this class provide a basis for a wide range of mathematics' courses offered in high school, including geometry, advanced algebra, trigonometry, statistics, and probability. Therefore, a wide range of topics is covered of necessity. To do this effectively requires logical sequencing and student practice. Time becomes a major problem in completing the breadth of information needed.
Therefore, teachers may wonder if or when they can include any needed supplementary units. When the study is complete there may be some obvious topics currently being taught which, while interesting, are not felt to be necessary. Eliminating those would allow time for additional units. Although extraneous information is not a focus of this study, it could be a useful by-product.
CHAPTER TWO - LITERATURE REVIEW

As previously stated, the purpose of this paper is to compare the algebra Level Standards of Kentwood Public Schools, Kentwood, Michigan to the objectives in the UCSMP Algebra textbook. To do that effectively requires an overview of national, state, and local influences of the 1980s and 1990s that gave impetus and direction to the authors of the textbook as well as the teachers in the district. Chapter Two of this study will explore pertinent information from published works at the national level and the state of Michigan to give educators using this work an idea of the purpose behind the text and the district's Standards.

National Influences

In 1957, when the Soviets successfully launched Sputnik into space ahead of a United States' satellite, the American people became concerned about the state of mathematics and science education in this country. By the 1980s that concern returned, joined by perceived weaknesses in English, history, and other core areas. In response to the sense that the U.S. educational system was inferior to that of other nations Secretary of State T. H. Bell created the National Commission on Excellence in Education on August 26, 1981. Among its tasks were:
- assessing the quality of teaching and learning in our Nation's public and private schools, colleges, and universities;

- comparing American schools and colleges with those of other advanced nations;

- studying the relationship between college admissions requirements and student achievement in high school;

- identifying educational programs which result in notable student success in college;

- assessing the degree to which major social and educational changes in the last quarter century have affected student achievement; and

- defining problems which must be faced and overcome if we are successfully to pursue the course of excellence in education (National Commission on Excellence in Education, 1983).

The Commission's work, titled *A Nation at Risk: The Imperative for Education Reform*, included several reasons for completing the study. This paper notes only those related to mathematics, either directly or indirectly. When the scores of American students were compared with those of students in other industrialized countries on nineteen standardized tests, U.S. students were last seven times, but were never first or second. Standardized test
scores had fallen between 1963 and 1980 and SAT math scores were down around 40 points. Only one-third of 17 year olds could solve a multi-stepped math problem. Meanwhile, in the five years between 1975 and 1980, college students enrolled in remedial math classes rose 72% (National Commission on Excellence in Education, 1983).

Students had difficulty understanding the language of mathematics found in the textbooks and used by teachers when lecturing. Furthermore, they had difficulty reproducing that language in written work.

After an extensive study which included input from educational experts, administrators, teachers, students, parents, concerned citizens, and a review of existing literature on the condition of the nation’s educational system, the chairman of the committee published their findings and recommendations on April 29, 1983.

Although responses to surveys distributed by this committee and comments on letters were varied, some interesting patterns emerged. At that time many high schools offered a second year of algebra (usually to juniors), but only about 31% of students completed it. Calculus was completed by only 6%. Yet 75% of those surveyed felt that students planning to attend college should take four years of high school mathematics. After careful consideration, the committee recommended three
years of high school math for all students. It is intended that these years will include the study of algebra, geometry, statistics, probability, estimation, measurement and application (National Commission on Excellence in Education, 1983).

The committee was concerned by the fact that textbooks were not being written by experienced teachers and content was "watered-down" to accommodate students with lower reading levels. Controversy surrounding the appropriate method of teaching low-level readers was extensive. The feeling existed that the problem was the difference between the ability to read and the ability to write mathematically. The committee recommended a more challenging content, written by experienced educators. They encouraged schools to insist on documentation from the publishers about their textbook’s appropriateness (National Commission on Excellence in Education, 1983).

In 1987 the National Council of Teachers of Mathematics (NCTM) published a working draft of *Curriculum and Evaluation Standards for School Mathematics*, with the final draft published in 1989. Thomas A. Romberg, then chairman of the NCTM’s Commission on Standards, listed three purposes for the Standards in an article published in *Arithmetic Teacher*: 1) to ensure quality, 2) to indicate goals, and 3) to promote change. He stated that the
"information age" of calculators and computers had drastically changed the subject of mathematics, creating the need for this work (Romberg, 1988).

Likewise, F. Joe Crosswhite, former NCTM president applauded the effort. In an article for School Science and Mathematics he wrote, "Never before has a professional organization of teachers undertaken the task of specifying national standards for school curricula in its discipline" (1990). In the same article Mr. Crosswhite acknowledged that a goal of the Standards was to influence textbook companies to provide appropriate materials to support the needed reform.

The Arithmetic Teacher printed an article by Charles Thompson and Edward Rathwell (1988) in which the authors reiterated the goal of influencing the content of textbooks once teachers embraced the Standards. They noted that the NCTM Standards emphasized the need for students to be able to problem solve, represent mathematical ideas in several different ways, form conclusions based on given information and justify those conclusions using mathematical means (ex. theorems, definitions, representations), and use the available technology appropriately (Thompson & Rathnell, 1988). For students in the middle grades (5-8) this meant a shift from an emphasis on computation to one of application. Because the calculator can quickly add,
subtract, multiply, and divide, the teacher can present the student with some real-life problems that apply the concepts (Thompson & Rathnell, 1988).

The NCTM document itself reinforced the idea of using the mathematics not merely memorizing a method for doing the mathematics in its first four standards. These standards promote mathematics for problem solving, communication, reasoning, and making connections (NCTM, 1989).

In a report to the National Council of Teachers of Mathematics' Commission on Standards for School Mathematics J. Owens enforced the idea that problem solving involves using computational skills on real-world situations (Owens, 1988). The term "communication" includes presenting answers in a variety of formats and justifying those answers. Drawing valid conclusions from presented information and generalizing from examples combine communication with reasoning skills, while the ability to make connections is demonstrated by comparing and contrasting concepts that used to be taught as isolated units (Owens, 1988).

Ruth E. Parker provided insight into new methods needed to accomplish the standards in Mathematics Teacher. She felt it would require a change in the way mathematics had been taught. Traditionally, the teacher followed a
daily ritual of checking homework, explaining the lesson, and allowing students to work alone on assignments. Textbooks emphasized drill and practice (Parker, 1991).

"Course content must be restructured to reflect the dynamic, ever-expanding nature of mathematics, the availability of technology, and the study of mathematical topics relevant to the world of the 1990s" (Parker, 1990). Some suggestions presented by Ruth Parker include presenting problems with more than one solution or more than one method to find the solution, expecting students to communicate results in more than one form, and giving students problems that require students to make connections with previously learned material or with other subjects. Looking for, finding, and communicating patterns in open-ended problems gives students an opportunity to reason mathematically, gain a sense of power, and grow in self-confidence (Parker, 1991).

Results of the Fourth National Assessment of Educational Progress (NAEP) test in 1988, reported in Arithmetic Teacher, indicated that students were more successful with items that were more familiar (ex. feet and inches) than with those that were less familiar (ex. meters). Also indicated was a lack of understanding of the concepts underlying computation. The results pointed to a need for more hands-on, real-life activities before
expecting students to grasp abstract ideas (Kouba, et al., 1988).

State Influences

Taking a cue from the national studies and published works, the state of Michigan realized a need to assess its educational system. In their Position Paper on Core Curriculum (1991), written to comply with Public Act 25, the State Board of Education stated, "recent studies on the status of education in the State indicate that the knowledge and skills students receive in the course of elementary and secondary schooling needs to be strengthened to meet the demands today's graduates face in the changing workplace." In addition to the "basics of reading, comprehending, writing, computing, solving, learning and relating interpersonally, (the fundamental curriculum) develops individuals' aesthetic appreciation, makes them technologically literate, develops their personal management skills, their teamwork skills, makes them career ready, fosters self worth and social/personal values." The expectations of the state extend beyond those stated in publications on the national level.

the Appendix D and are aimed at preparing students to perform in a rapidly-changing technological world. According to the document, "to meet these needs, schools should provide a comprehensive mathematics curriculum that teaches the fundamentals of mathematics, develops critical thinking skills, and nurtures logical reasoning." At the beginning of the section specifying outcomes for mathematics is a description borrowed from the NCTM Standards and the Essential Goals and Objectives for Mathematics Education published by the Michigan State Board of Education (1988). It says a mathematically literate person is one who:

1. Values mathematics;
2. Has confidence in his/her ability to do mathematics, demonstrating conceptual understanding as well as proficiency in operations skills;
3. Applies conceptual understandings, operational skills, and technological tools to problem solving;
4. Communicates mathematical ideas orally and in writing; and
5. Reasons mathematically.
Again the emphases were on application, communication, and problem solving as opposed to pencil-and-paper manipulation of numbers.

From their outcomes the state Board of Education developed more specific expectations for each level—elementary, middle school, and high school. Published as Mathematics Content Standards and Benchmarks (1988), they were divided into six categories:

- Patterns, Relationships, and Functions
- Geometry and Measurement
- Data Analysis and Statistics
- Number Sense and Numeration
- Numerical and Algebraic Operations and Analytical Thinking
- Probability and Discrete Mathematics

To "encourage" districts to incorporate the state’s goals into their own curriculum, a state-mandated test was written and PA 335 defined the school district’s role in developing an aligned core curriculum. A state endorsed diploma would be given only to students who passed the test.

As previously stated, the NCTM and the authors of A Nation at Risk hoped to influence textbooks with their recommendations. In 1989, when the Mathematics Curriculum Committee at Kentwood Public Schools began the search for a
new textbook series for 1990, members were reminded by Chairperson Carole Johnson to keep in mind national studies, including A Nation at Risk and the NCTM Standards, as well as the budding Michigan laws when selecting texts.

UCSMP Background

Fortunately for Kentwood the national publications were beginning to affect textbooks. In 1983, Amoco Foundation funded a six-year project at the University of Chicago. Zalman Usiskin, in an article titled "Translating grades 7-12 Mathematics into Results," stated that the goal of the University of Chicago School Mathematics Project was to "improve school mathematics in grades K-12" (1986/87). To do this the UCSMP decided to review the studies and recommendations already in existence rather than develop their own. The task chosen instead was to translate those existing recommendations into a teachable text for the classroom (Usiskin, 1986/87).

To meet the demands of A Nation at Risk and other studies to incorporate algebra, functions, geometry, measurement, statistics, probability, estimation, computation and number concepts in the high school program required a realignment of the curriculum. The amount of time dedicated to review in the traditional mathematics program in grades K-8 amounted to between one and two complete years. To accomplish the needed changes "algebra
should be taught one year earlier to most students than is currently the case" (McConnell, et al., 1993, Teacher's Edition). Therefore, UCSMP Algebra is generally taught to eighth graders.


Planning involved 20-30 school personnel criticizing and evaluating ideas presented by Project members. The authors for the Algebra pilot were chosen from a national search. Seven out of eleven are teachers or administrators below college level. As stated earlier, one criticism of textbooks of the time was that the authors were not teachers. Once the Algebra pilot was written it was taught for a year by its authors, then rewritten to correct flaws found during this formative evaluation step. After rewriting it was taught by "regular" classroom teachers and again re-evaluated (Usiskin, 1988/89).

The field study for the algebra text "involved 61 teachers - 30 in UCSMP classes, 31 in comparison classes - at 38 urban, suburban, and rural schools in nine states across the country, and involved almost 2000 students."
When students in the study were given *American Testronics High School Subjects Test: Algebra*, those using UCSMP scored as well as those using traditional texts. On tests written to evaluate the goals of the UCSMP, the students using that text scored significantly better. Although some teachers felt the text should contain more drill-and-practice, test results indicated the students did not need it (UCSMP, 1989).

Eighty-one percent of teachers using the pilot text said it was better than the text they had been using and were especially pleased by the number of applications used. Students appreciated the real-world examples and the extensive calculator use (UCSMP, 1989). Actually, these two concepts are interrelated. The calculator allows students to solve actual problems by removing the difficulty some high schoolers have with complex manipulations of numbers. Exponential equations, statistics, and probability can become algebra topics for eighth graders with the use of technology (McConnell, 1987, in NCTM Yearbook).

A study reported by D. Hirschhorn in *Journal for Research in Mathematics Education* (1993) and one presented by J. Rivers to the American Educational Research Association (1990) demonstrate that the UCSMP textbooks compare favorably to traditional textbooks. In her report
Janelle Rivers (1990) compared five separate textbooks considered by schools in South Carolina. The Scott, Foresman (UCSMP) textbook was noted for its integration of topics. It also scored highest on having applications - 78 topics compared to second-high 45 topics.

Clearly, the UCSMP program has incorporated many of the recommendations in the NCTM Standards - written by teachers, less drill and practice, more real-life application, and incorporation of technology. A glimpse at the Algebra textbook also shows an emphasis on communicating answers in several forms. Consider Lesson 4-7. Two examples are presented, showing three ways of reporting the number of different ways of making one choice followed by a second choice (McConnell, et al., 1993).

After studying the goals of the UCSMP series and that of other textbook companies, Kentwood Public Schools adopted the UCSMP textbooks in 1990. They had as their focus many of the recommendations of the national and state publications.

Kentwood Background

Between 1990 and 1995, Kentwood updated its Exit Standards (Kentwood Board of Education, 1994). Kentwood expects its graduates to be:

1. Effective Communicators
2. Personal Managers
3. Quality Producers
4. Global Citizens
5. Creative/Critical Thinkers
6. Self-Directed Learners

Outcomes #1 and #5 fit directly into the NCTM standards for mathematics, while #2, #3, and #6 can be easily incorporated into a curriculum that follows the Standards. Because these Exit Standards are expected to be a combination of the teachings of all core classes it is not necessary for mathematics alone to stress all six. However, a program which follows NCTM Standards and Michigan’s Essential Goals and Objectives would contribute to most.

Kentwood has also updated its Program and Level Standards. That is, it has updated Mathematics Standards, in general, and Algebra Standards specifically (1995). However, to truly implement the mathematics Standards the teacher must also focus on the verbs that are used: gather, organize, apply, explain, interpret, and justify. Rubrics are provided for each verb. All relevant Kentwood Standards can be found in Appendix B. They indicate the emphasis placed on application of concepts, communication of results, and logical reasoning instead of rote memorization and pencil-and-paper manipulation.
The Program Standards for algebra contain an expectation that all verbs will be applied to the specific content of the algebra course. In writing the algebra outcomes, teachers continued to expect real-world application, as well as theoretical concepts.

Summary

After ten years of study the UCSMP shared some of their findings (Usiskin, 1993). Calculators are easily incorporated into the curriculum. They have made it possible for teachers to explore many new ideas with their students. Contrary to the belief of some teachers, applications help ALL students relate to the concepts being taught. Manipulating complicated expressions is unnecessary for most students. Therefore, the continued de-emphasizing of paper and pencil calculation continues to make sense (Usiskin, 1993).

It has been shown that the UCSMP and the Kentwood Public Schools Board of Education have very similar goals. The question of how the algebra Standards align with the Algebra text will be answered in Chapter Three, using primarily the stated objectives of the textbook and the written Kentwood Program and Level Standards for algebra. Recommendations from the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School
Mathematics and interpretations of those recommendations will be used to support supplementary lessons.
The literature review supports the importance of reforming the mathematics curriculum in secondary schools to better prepare students for further study or for contribution in the workplace. Several national publications, notably A Nation at Risk: The Imperative for Reform and the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics, have outlined the perceived needs and possible solutions. Both Kentwood Public Schools and the University of Chicago School Mathematics Project have used those publications and others to formulate Standards and objectives for their algebra program or text.

The purpose of this chapter, along with the over-all purpose of this research, is two-fold.

1. Compare Kentwood Public Schools' Standards for algebra with the objectives for the University of Chicago School Mathematics Project's Algebra textbook.

2. Provide supplementary units, including assessment, for those Standards not addressed at all or addressed in a cursory manner in the textbook.

When the Kentwood Mathematics Curriculum Committee undertook the task of writing Standards for the mathematics curriculum, they followed the format of the State of
Michigan’s Content Standards (Appendix E). Therefore, there are six Program Standards for all levels/classes regardless of grade or course. For each of the six Program Standards specific Level Standards were developed for algebra.

In this chapter each Level Standard will be considered separately. The textbook objectives which relate to each Standard will be listed and analyzed to determine if there is sufficient emphasis to accomplish the Standard. If needed, lessons or units will be included to supplement the text. In most instances the textbook coverage is excellent. Sometimes what is required by the district can be accomplished by modifying existing assignments. Occasionally a supplementary lesson is included.

Program Standard 1

The learner will explain, apply, and justify algebraic concepts (patterns, relationships and functions) and procedures in real-world and mathematical problem situations.

Level Standard A L 1.1

The learner will write and solve equations based on real-world applications translating among standard English, algebraic, and graphical representations.

The objectives from the text (Appendix A) that apply are listed below. The objectives are preceded by a key, which gives the chapter and the objective. For instance, 2B would be objective B from chapter 2.

1I Evaluate formulas in real situations.
4K Solve sentences of the form ax = b and ax < b to answer questions from real situations.
5H Solve percent and size change problems from real situations.
5I Solve problems involving proportions in real situations.
6H Answer questions involving linear sentence formulas.
6J Answer questions about situations combining addition and multiplication.
6L Translate balance scale models and rectangle area models into expressions and equations.
8H Use equations for lines to describe real situations.
8I Graph a straight line given its equation, or given a point on it and its slope.
8J Given data which approximates a linear graph, find a linear equation to fit the graph.
9H Use and simplify expressions with powers in everyday situations.
9I Graph exponential growth and decay relationships.
11F Use systems of equations to solve real world problems involving linear combinations.
12G Use the parabola and quadratic equations to solve real-world problems.

This standard is well-supported by the textbook. There are many examples of real-world applications and resulting problems. However, to comply with the National Council of Teachers of Mathematics’ recommendations that students represent mathematical ideas in several ways (Thompson & Rathwell, 1988), Kentwood’s Standard expects students to translate "among" different representations. While the text often asks the learner to make an equation from a graph or English question it does not ask the learner to develop a question to match an equation. To further emphasize the concept that equations arise from actual situations, teachers should require students to
create stories to explain theoretical equations in the text. For example, the text directions might be "Solve \( x + 6.1 = 10.2 \) for \( x \)." In addition to "solving," the student should be asked to state a problem, such as "Sharon had $6.10. She earned \( x \) dollars and now has $10.20. How much did she earn?" Although the textbook rarely asks students to do this, it would not be difficult to add this requirement frequently to two or three equations within an assignment and to expect the same type of reasoning on tests.

**Level Standard A1.2**

The learner will represent and analyze relationships by interpreting and translating among multiple representations of patterns including tables, rules, charts, and graphs.

Checking the text yields many objectives which fit this standard.

1L Interpret dot frequency graphs.
2K Plot points and interpret information on a coordinate graph.
3J Graph sets of ordered pairs \((x,y)\) where \( x + y = k \) and \( x - y = k \), where \( k \) is a real number.
7H Graph equations for straight lines by making a table of values.
7I Graph horizontal and vertical lines.
7J Find the lengths of the sides of a right triangle using the Pythagorean Theorem.
8A Find the slope of a line through two points.
8B Find equations for the line given its slope and any point on it.
8C Find an equation for a line through two given points.
8F Given an equation for a line, find its slope and \( y \)-intercept.
8I Graph a straight line given its equation, or given a point on it and its slope.
8K Graph linear inequalities.
9D Test a special case to determine whether a pattern is true.
9I Graph exponential growth and decay relationships.
11D Recognize sentences with no solution, one solution and all real numbers as solutions.
11E Determine whether a system has 0, 1, or infinitely many solutions.
11H Find solutions to systems by graphing.
11I Graphically represent solutions to systems of linear inequalities.
13A Find values of functions from their formulas.
13E Find the domain and range of a function from its formula, graph, or rule.

Except for those noted in Level Standard AL1.1 the Algebra text allows many varied opportunities for students to "translate among multiple representations of patterns." However, chances for analysis are less frequent. Chapters 7 and 12 offer a great opportunity to encourage analysis of graphs using technology. Graphing calculators or computers should be used to help the students understand what happens to a line of the form \( y = mx + b \) if \( m \) and \( b \) are changed (Ch. 7) and a parabola of the form \( y = ax^2 + bx + c \) if \( a \), \( b \), and \( c \) are changed (Ch. 12). Not only does this help the student analyze graphs and equations but also it provides an opportunity to comply with state recommendations that technology be used in the classroom (Michigan State Board of Education, 1988). Activities and worksheets (reprinted with permission), as well as suggested test questions, are provided in Appendix F.
Program Standard 2

The learner will explain, apply, and justify geometric and measurement concepts and procedures in real-world and mathematical problem situations.

Level Standard AL 2.3

The learner will use the concepts of rates and ratios to describe the real world and to solve problems by using proportional reasoning, indirect measurement, and computing with measurement units (e.g., rate factor model).

Much of the support for this standard occurs in Chapters 4 - 8 of the textbook. Real-life topics include populations, photographs, sale prices, taxes, maps, dart boards, and copy machines. There are several objectives which address this standard.

4G Apply area and rate factor models for multiplication.
5C Solve proportions.
5E Use the language of proportions and the Means-Extremes Property.
5F Use the rate model for division.
5G Use a ratio to compare two quantities.
5H Solve percent and size change problems from real situations.
5I Solve problems involving proportions in real situations.
5K Find missing lengths in similar figures.
5M Apply the Size Change Model for Multiplication on a coordinate plane.
6F Answer questions involving mark-ups and discounts.
8D Recognize slope as a rate of change.
8G Calculate rates of change from real situations.

While coverage of this standard does include a variety of real-world topics, allowing students to apply their knowledge in actual situations would improve understanding. The NCTM Standards note that middle school students "are
especially responsive to hands-on activities" (National Council of Teachers of Mathematics, 1989). These topics are often covered in December and January - a great time to involve students in some hands-on activities. Supplementary ideas for Chapter 5 are given. They include activities which use newspapers, convex lenses, and meter sticks. It would not be necessary for a student to do all of the activities, but some actual measuring is ideal for this Level Standard.

The UCSMP tests which accompany the textbook include sufficient questions to test for understanding after completion of the activities.

**Level Standard AL2.4**

The learner will use the coordinate plane to illustrate relationships by interpreting the graphs of points, lines, and curves.

A variety of objectives from the textbook meet Level Standard AL2.4. Some which merely ask students to graph (but not interpret) have been omitted here although they constitute a necessary background step.

1K Use graphs or symbols to describe intervals.
1L Interpret dot frequency graphs.
2K Plot points and interpret information on a coordinate graph.
2L Interpret two-dimensional slides on a coordinate graph.
3H Graph solution sets to inequalities.
7G Find distance on a number line.
7K Calculate distance in a plane.
8A Find the slope of the line through two points.
8H Use equations for lines to describe real situations.
11H Find solutions to systems of equations by graphing.
12D Recognize properties of the parabola.
13E Find the domain and range of a function from its formula, graph, and rule.

The verb "interpret" in the Program Standards can mean to "translate reciprocally from one form to another."

Enough opportunity is given in the textbook to graph equations and inequalities, as well as take points from a graph and write an equation. Chapter 12 includes equations for and graphs of parabolas that arise from real situations, giving students practice at translating between "concrete and representational forms," another definition of interpret. No lessons or units beyond the textbook are needed to accomplish this Lesson Standard.

**Lesson Standard A L 2.5**

The learner will translate between geometric figures and algebraic expressions, analyzing the effects of parametric changes.

Because one of the goals of this algebra text is to integrate mathematical topics (McConnell, et al., 1993), the instances of translating between geometric figures and algebraic expressions are numerous.

1K Use graphs or symbols to describe intervals.
2J Graph solutions to inequalities on a number line.
2K Plot points and interpret information on a coordinate graph.
2L Interpret two-dimensional slides on a coordinate graph.
3H Graph solution sets to inequalities.
3J Graph sets of ordered pairs (x,y) where x + y = k or x - y = k, where k is a real number.
4L Use areas, arrays and volumes to picture multiplication.
6J Translate balance scale models and rectangular area models into expressions and equations.

7H Graph equations for straight lines by making a table of values.

7I Graph horizontal and vertical lines.

8B Find the equation for the line given the slope and any point on it.

8F Given an equation for a line, find its slope and y-intercept.

8I Graph a straight line given its equation, or given a point on it and its slope.

8K Graph linear inequalities.

9I Graph exponential growth and decay relationships.

10J Represent areas of figures in terms of polynomials.

11H Find solutions to systems by graphing.

11I Graphically represent solutions to systems of linear inequalities.

There is no question that there is more than sufficient practice translating between geometric figures and algebraic expressions. However, there is very little emphasis on analyzing the effects of parametric changes.

To satisfy this requirement of Level Standard AL2.5 students should complete the activities provided in Appendix E which ask students look at graphs of lines of the form $y = mx + b$ and of parabolas of the form $y = ax^2 + bx + c$. Completing these activities along with the textbook will enable the student to satisfy this Standard.

**Program Standard 3**

The learner will gather, organize, and interpret data to explain, apply, and justify statistical and data analysis concepts and procedures in real-world and mathematical problem situations.
The learner will make predictions and decisions based on data the learner has gathered, organized, and interpreted, including interpolations and extrapolations.

The following text objectives might be needed for background for projects chosen to complete this Standard.

1L Interpret dot frequency graphs.
1H Calculate the mean, median, and mode for a set of data.
1L Interpret dot frequency graphs.
2K Plot points and interpret information on a coordinate graph.
6H Answer questions involving linear sentence formulas.
8A Find the slope of the line through two points.
8B Find the equation for a line given its slope and any point on it.
8C Find the equation for the line through two given points.
8D Recognize slope as a rate of change.
8H Use equations for lines to describe real situations.
8J Given data which approximates a linear graph, find a linear equation to fit the graph.

Standard AL3.6 goes beyond the requirements of any textbook. To achieve this Standard the student must gather his/her own data. The student may not rely on data from the text. The data must then be organized into a graph, chart, table or some other appropriate vehicle that will enable the student to explain the significance of the information. Instructions in the NCTM Standards (1989) recommend allowing students to pursue mathematical areas of interest to them which are not completely defined by the teacher. Suggested topics appear in Appendix F as well as an introductory probability activity. Evaluation will
depend on the topic chosen, but possible methods are included in Appendix F.

Program Standard 4

The learner will explain, apply, and justify number concepts (number sense and numeration) and procedures in real-world and mathematical problem situations.

Level Standard AL4.7

The learner will estimate and compute with real numbers (including numbers written in scientific notation) to evaluate and interpret algebraic expressions, explaining strategies used.

Although, in keeping with national recommendations, the authors of the Algebra text de-emphasized paper-and-pencil calculations, some skills are still recognized as important to the understanding of the subject. Numerical calculations need not be done manually, but concepts must be understood to apply them to algebraic expressions containing variables. The pertinent objectives follow.

1A Operate with and compare fractions.
1C Evaluate numeric and algebraic expressions.
1E Read and interpret set language and notations.
2A Add fractions.
2E Identify properties of addition.
2F Identify and apply properties used in solving equations and inequalities.
4A Multiply fractions.
4F Identify and apply properties of multiplication.
5A Divide real numbers and simplify division expressions.
5D Identify restrictions on a variable in a division situation.
6A Use the distributive property to remove parentheses and collect like terms.
6D Apply and recognize properties associated with linear sentences.
6E Use the Distributive Property to perform calculations in your head.
One advantage of calculators is the ease with which students can manipulate fractions and decimals. Therefore, these real numbers appear routinely throughout the UCSMP secondary math textbooks. In the algebra textbook, knowledge of scientific notation is assumed. However, it is presented as a lesson in Appendix B of the text if the teacher feels the class needs review. Therefore, although no objective directly addresses scientific notation, the textbook uses the concept and provides instruction, if required.

Of all the concepts covered in this Standard only estimation lacks emphasis in the text. This is a skill that needs constant repetition. Rather than teach a separate unit on estimation at this level, it would be more effective to approach this concept in the same manner as describing real-world examples for the equations. The teacher should regularly ask students to explain how they
know an answer is correct. The student should be expected to round numbers off to produce an estimate. It would be a simple matter to add that requirement once each week to four or five problems - some fractions, some decimals, some very large numbers. Estimation is the only section of this Standard that needs bolstering. The adjustment described will correct the situation. Test questions should reflect this expectation. Modification of tests should be the same as modification of homework.

Program Standard 5

The learner will apply a wide variety of problem solving concepts and procedures to explain and interpret solutions.

Level Standard AL5.8

The learner will apply mathematical problem solving strategies to solve problems from within and outside mathematics: formulate problems from situations, use mathematical modeling, make and test conjectures, formulate counter-examples, follow logical arguments, judge the validity of arguments, and construct simple valid arguments.

As has been stated in Chapter 2, the UCSMP series had problem solving as an emphasis. Although this Standard encompasses a wide range of ideas, many objectives and examples in the book stress problem solving.

1B Find solutions to open sentences using trial and error.
1J Use relative frequency to determine information about surveys.
2C Solve and check equations of the form $a + x = b$.
2F Identify and apply properties used in solving equations and inequalities.
3B Solve and check sentences involving subtraction.
4B Solve and check equations of the form \( ax = b \).
4C Solve and check equations of the form \( a - x = b \).
4D Solve and check inequalities of the form \( ax < b \).
4F Identify and apply properties and multiplication.
4G Apply area and rate factor models for multiplication.
4K Solve sentences of the form \( ax = b \) and \( ax < b \) to answer questions from real situations.
4L Use areas, arrays, and volumes to picture multiplication.
6D Apply and recognize properties associated with linear sentences.
6G Describe patterns and answer questions in repeated addition or repeated subtraction situations.
6J Translate balance scale models and rectangle area models into expressions and equations.
8G Calculate rates of change from real data.
8J Given data which approximates a linear graph, find a linear equation to fit the graph.
9G Test a special case to determine whether a pattern is true.
10C Recognize perfect squares and perfect square trinomials.
10E Factor perfect square trinomials and the difference of squares into a product of two binomials.
10H Recognize factors of polynomials using the properties of algebra, testing a special case and ruling out possibilities.
11F Use systems of equations to solve real-world problems involving linear combinations.
12F Determine whether a number is rational or irrational.

Because problem solving was perceived as a need by the authors, the text contains mathematical models in Chapters 2, 3, and 4, opportunity to guess and check beginning in Chapter 2, and to work with counter examples in Chapter 9. In the interest of incorporating other subjects into algebra, there are simple proofs using properties of numbers and equations. To completely satisfy this Standard students must formulate word problems from equations. It is one way for students to show they are not just
manipulating numbers, but actually understand the concepts, as recommended both nationally and on the state level.

As equations become more difficult, students should still be able to create some question to go with the numbers (Thompson and Rathwell, 1988). For instance, the text might ask students to "Solve for y: 3y + 14 = 35." In addition to solving, the student might describe this situation: "Kevin has 14 POGS. If he adds 3 FOGS to his collection every week how long will it take before he has 35 POGS?" The teacher should be requiring students to formulate questions for equations on an weekly basis (at least), because the text often does not. With that single exception and that simple solution Level Standard AL5.8 is well covered.

Program Standard 6

The learner will gather, organize, and interpret data to explain, apply, and justify probability and discrete mathematics concepts and procedures in real-world and mathematical problem situations.

Level Standard AL6.9

The learner will explain, interpret, and evaluate experimental and theoretical probabilities, including those with multiple conditions.

The concepts of probabilities are important for students because of the many applications of those concepts. These range from weather forecasting to insurance rates to playing poker. The appropriate topics appear in the text in several places.
1F Find and interpret the probability of an event when outcomes are assumed to occur randomly.
1J Use relative frequency to determine information about surveys.
3G Apply the formula for N(A∪B) and P(A∪B).
4H Apply the Multiplication Counting Principle.
4I Apply the Conditional Probability Formula.
4J Apply the Permutation Theorem.
5J Find probability involving geometric regions.
13F Determine values of probability functions.

As can be seen by the chapter numbers on the outcomes involving probability, the topic is developed throughout the text. There is ample opportunity for the student to interpret and evaluate probabilities. After completing Lesson 1-7, which includes theoretical probabilities for possible sums when two dice are tossed, the student should actually try the experiment using two dice. One possible method, described in Appendix F, would have students throw two dice a set number of times (for example, 25 times). Students would record the total dots on both dice (A four and a five would yield a nine). The results of the experiment would give data which the text refers to as "relative frequency." An activity for use with this data is included in Appendix F. Evaluation should require a student-generated experiment; rubrics for that experiment can be found with the activity. Mastering the textbook information and developing, organizing, and interpreting an "original" experiment will enable the student to accomplish this Level Standard.
Level Standard AL6.10

The learner will solve problems using simple discrete mathematical techniques such as Venn diagrams, factorials, tree diagrams, and the Fundamental Counting Principle.

Discrete mathematical techniques, while important, are generally isolated concepts, rather than building blocks to other skills. Therefore, each technique appears as a separate objective in the text.

3I Use Venn Diagrams to describe union and intersection.
4E Evaluate expressions containing the factorial symbol.
4H Apply the Multiplication Counting Principle.

Because discrete mathematical techniques are not building blocks there are not related objectives for these ideas. What is important is that Venn Diagrams, factorials, and the Fundamental Counting Principle (called Multiplication Counting Principle in this text) are included as objectives. Tree diagrams are obviously missing. However, a description and several examples are included within Chapter 4. The teacher should require students to answer questions involving combinations of outcomes in two or more ways.

Example: A high school student shops at the mall. The student buys two pair of jeans (blue and green), three shirts (solid, striped, and plaid) and two sweatshirts (white and beige). If all clothes are color-coordinated, how many outfits of jeans, shirts, and sweaters can be made?
Requiring the student to support the answer of 12, calculated using the Multiplication Counting Principle, by using one or two other means would force the student to learn tree diagrams. This would not require additional units or time, but would accomplish the Level Standard.

Summary
Fortunately, most of the Standards that Kentwood Public School teachers have defined for algebra are well covered in the UCSMP Algebra textbook. In cases when additional emphases or lessons are needed they can often be easily incorporated into lessons already appearing in the text. This means that concerns about finding time to supplement the textbook are unwarranted.

Satisfying AL1.1 and AL5.8 require the teacher to take equations from assignments in the text and ask students to create questions to fit the equations. AL4.7 can also be done using existing homework. It will merely be necessary to expect students to explain, using mathematically acceptable estimation, why a particular answer is correct. If these simple additions are made on a weekly or bi-weekly basis students would achieve Level Standards AL1.1, AL4.7, and AL5.8.

Because Kentwood teachers felt it was important for students to create their own problem, collect appropriate data, present that data in a clear format, and form
conclusions based on the information gathered, some time in the school year must be "found" for those activities. In addition students must be shown how to approach such a task. The probability activity in Appendix F would be appropriate for that purpose and could be substituted for Lesson 1-7 and 1-8, probability and relative frequency, in the text. This could mean only one additional day is needed, which might be created by combining Lessons 3-4 and 3-5, intersection and union of sets. The student generated experiment would be done outside of school. The teacher should reduce regular assignments during that time and allow more class time to complete them. These adjustments would allow students to satisfy AL3.6 and AL6.9.

To accomplish AL1.2 and AL2.5 requires finding time to teach and use graphing calculators or computers. This would require at least three days. It would be reasonable for the worksheet on graphing $y = mx + b$ to be substituted for Lesson 7-1, graphing lines, and the introduction to take the place of Lesson 7-8, chunking. The concept of "chunking" is covered adequately in each section where it occurs - square roots, absolute value, and evaluating expressions. The additional section merely gives extra practice and can therefore be eliminated if necessary. Finally, the worksheet analyzing the equation $y = ax^2 + bx + c$ could easily be substituted for
Lesson 12-2, using an automatic grapher. This means no additional days to complete AL1.2 and AL2.5.

As shown in this chapter, AL2.4 and AL6.10 are covered adequately in the textbook. For AL2.3 many appropriate problems are presented to the student in the book. However, actual hands-on activities should be added. Most of these are not time-consuming. Lessons 5-7 and 5-8, proportions and similar figures, include examples of indirect measurement similar to those activities. Instead of allotting an entire day for the supplementary worksheets provided in Appendix F, it would be appropriate to use them to introduce those lessons.

It can be seen that the textbook requires minimal supplemental material to meet Kentwood’s Standards. Using the suggestions and worksheets provided in this study and some creative scheduling, the algebra teacher can help students complete all ten Level Standards for that subject.
Conclusions

As expected from the study of the literature, the authors of the UCSMP Algebra textbook and the Kentwood Public School mathematics teachers emphasized the same general objectives. Both felt problem-solving and applications should have a high priority. Both included explaining and interpreting frequently in their requirements. Both de-emphasized the pencil-and-paper calculations so common in previous years, realizing that a calculator could be used for rote problems. For most of Kentwood’s Algebra Standards, following the UCSMP Algebra text is appropriate.

However, the Kentwood Public School District expects students to be creative thinkers and self-directed learners. To help them develop those skills requires some activities that are almost impossible to write into a textbook. Therefore, some supplementation is necessary. In some instances that might mean merely extending the expectation of the text. For example, to achieve the goal of translating from an algebraic equation to a real-world problem that the equation describes a teacher could simply expect the student to choose three or four equations from an assignment and make up a problem to fit the equations. If this were done regularly (weekly or bi-weekly) the student would learn the connection.
Sometimes the supplementation would need to be more involved. Use of technology other than the scientific calculator requires out-of-text activities. The UCSMP series does include a companion workbook with computer programs and accompanying worksheets. These can sometimes be used with a graphing calculator instead of a computer. Other activities of this type should also be done to enable the student to actually see what happens to equations when parameters are changed, as well as to help high schoolers become familiar with technology.

Finally, it is apparent that the national recommendations that encouraged Kentwood to change its curriculum standards have affected the textbook industry. The UCSMP series chosen by Kentwood has included the recommendations in its goals and outcomes, requiring minimal supplementation.
Recommendations

Clearly, for the current curriculum standards for algebra in Kentwood the UCSMP Algebra textbook remains an excellent choice. However, no textbook will meet all the needs of any district. Therefore, it is recommended that this research be followed by teachers of algebra in Kentwood and that they supplement the text with the activities included here or with similar activities.

It is recommended that activities be included that allow the student to think creatively - designing a survey, gathering data, and interpreting that data is a good example.

It is also recommended that technology, in addition to the scientific calculator, be introduced. The graphing calculator can be as effective as the computer at this level and is often more accessible.

And, it is recommended that the de-emphasis of pencil-and-paper calculation and the emphasis on thinking skills and real-world application, encouraged by the textbook authors, be adopted by the classroom teacher.

This research has shown that the University of Chicago School Mathematics Project Algebra text is an effective teaching tool for that subject. To accomplish all of their required Standards teachers of algebra should supplement it with lessons provided in this study.
Definition of Terms

NCTM - National Council of Teachers of Mathematics

objectives - Unless the context indicates otherwise, objectives in this study refer to those stated goals of lessons in the University of Chicago School Mathematics Project Algebra textbook.

Standards - Unless the context indicates otherwise, Standards refers to those outcomes written for Kentwood Public Schools. These are divided into three categories:

  District Exit Standards - those outcomes Kentwood expects of all its graduates, regardless of classes taken

  Program Standards - those outcomes Kentwood expects of all students in core areas, regardless of specific course (ex. English, mathematics)

  Level Standards - those outcomes Kentwood expects of all students in a certain course (ex. algebra)

UCSMP - University of Chicago School Mathematics Project
REFERENCES


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APPENDIX A
UCSMP ALGEBRA OBJECTIVES

Chapter 1

A Operate with, and compare fractions.
B Find solutions to open sentences using trial and error.
C Evaluate numerical and algebraic expressions.
D Give the output for or write short computer programs and statements that evaluate expressions.
E Read and interpret set language and notation.
F find and interpret the probability of an event when outcomes are assumed to occur randomly.
G In real situations, choose a reasonable domain for a variable.
H Calculate the mean, median, and mode for a set of data.
I Evaluate formulas in real situations.
J Use relative frequency to determine information about surveys.
K Use graphs or symbols to describe intervals.
L Interpret dot frequency graphs.

Chapter 2

A Add fractions.
B Combine like terms.
C Solve and check equations of the form \( a + x = b \).
D Solve inequalities of the form \( a + x < b \).
E Identify properties of addition.
F Identify and apply properties used in solving equations and inequalities.
G Use the Triangle inequality to determine possible lengths of sides of triangles.
H Use the models of addition to form and solve sentences involving addition.
I Apply the Triangle Inequality in real situations.
J Graph solutions to inequalities on a number line.
K Plot points and interpret information on a coordinate graph.
L Interpret two-dimensional slides on a coordinate graph.

Chapter 3

A find the union and intersection of sets.
B Solve and check sentences involving subtraction.
C Find the measure of the supplement or complement of an angle.
D Simplify expressions involving subtraction.
E Apply the algebraic definition of subtraction.
F Use models of subtraction to form expressions and solve sentences involving subtraction.
G Apply the formulas for \( N(A \cup B) \).
H Graph solution sets to inequalities.
I Use Venn diagrams to describe union and intersection.
J Graph sets of ordered pairs \((x, y)\) where \(x + y = k\) or
\(x - y = k\), where \(k\) is a real number.

**Chapter 4**

A Multiply fractions.
B Solve and check equations of the form \(ax = b\).
C Solve and check equations of the form \(a - x = b\).
D Solve and check inequalities of the form \(ax < b\).
E Evaluate expressions containing a factorial symbol.
F Identify and apply properties of multiplication.
   Commutative Property, Associative Property, Property of
   Reciprocals, Multiplication Property of Zero,
   Multiplication Property of Equality, Multiplication
   Property of Inequality, Multiplicative Identity Property
   of 1, Multiplication Property of -1.
G Apply the area and rate factor models for multiplication.
H Apply the Multiplication Counting Principle.
I Apply the Conditional Probability Formula.
J Apply the Permutation Theorem
K Solve sentences of the form \(ax + b\) and \(ax < b\) to answer
   questions from real situations.
L Use area, arrays, and volume to picture multiplication.

**Chapter 5**

A Divide real numbers and simplify division expressions.
B Solve percent problems using equations or in your head.
C Solve proportions.
D Identify restrictions on a variable in a division
   situation.
E Use the language of proportions and the Means-Extremes
   Property.
F Use the Rate Model for Division.
G Use a ratio to compare two quantities.
H Solve percent and size change problems from real
   situations.
I Solve problems involving proportions in real situations.
J Find probabilities involving geometric regions.
K Find missing lengths in similar figures.
L Use the IF-THEN command in computer programs.
M Apply the Size Change Model for Multiplication on the
   coordinate plane.

**Chapter 6**

A Use the Distributive Property to remove parentheses and
   collect like terms.
B Solve linear equations.
C Solve linear inequalities.
D Apply and recognize properties associated with linear
   sentences.
E Use the Distributive Property to perform calculations in
   you head.
Chapter 7

A Calculate absolute values.
B Evaluate and simplify expressions involving square roots.
C Solve equations involving squares and square roots.
D Use chunking to evaluate expressions and solve equations.
E Apply the Square Root of a Product Property.
F Use squares and square roots in measurement problems.
G Find distance on a number line.
H Graph equations for straight lines by marking a table of values.
I Graph horizontal and vertical lines.
J Find the lengths of the sides of a right triangle using the Pythagorean Theorem.
K Calculate distances in the plane.
L Graph solutions to sentences of the form $|x - a| < b$, or $|x - a| > b$, where $a$ and $b$ are real numbers.

Chapter 8

A Find the slope of the line through two given points.
B Find an equation for a line given its slope and any point on it.
C Find an equation for a line through two given points.
D Recognize slope as a rate of change.
E Rewrite an equation for a line in standard form or slope-intercept form.
F Given an equation for a line, find its slope and y-intercept.
G Calculate rates of change from real data.
H Use equations for lines to describe real situations.
I Graph a straight line given its equation, or given a point on it and its slope.
J Given data which approximates a linear graph, find a linear equation to fit the graph.
K Graph linear inequalities.

Chapter 9

A Evaluate integer powers of real numbers.
B Simplify products, quotients, and powers of powers.
C Rewrite powers of products and quotients.
D Test a special case to determine whether a pattern is true.
E Identify properties of exponents.
F Calculate compound interest.
G Solve problems involving exponential growth and decay.
H Use and simplify expressions with powers in everyday situations.
I Graph exponential growth and decay relationships.

Chapter 10

A Add and subtract polynomials.
B Multiply polynomials.
C Recognize perfect squares and perfect square trinomials.
D Find common monomial factors of polynomials.
E Factor perfect square trinomials and the difference of squares into a product of two binomials.
F Apply \((a + b)^2 = a^2 + 2ab + b^2\) and \((a - b)(a - b) = a^2 - b^2\) to multiply numbers in your head.
G Recognize and use the Zero Product Property to solve equations.
H Recognize factors of polynomials using the properties of algebra, testing a special case and ruling out possibilities.
I Translate real situations into polynomials.
J Represent areas of figures in terms of polynomials.

Chapter 11

A Solve systems using substitution.
B Solve systems by addition.
C Solve systems by multiplying.
D Recognize sentences with no solutions, one solution, or all real numbers as solutions.
E Determine whether a system has 0, 1, or infinitely many solutions.
F Use systems of equations to solve real world problems involving linear combinations.
G Calculate and use weighted averages.
H Find solutions to systems of equations by graphing.
I Graphically represent solutions to systems of linear inequalities.

Chapter 12

A Solve quadratic equations using the Quadratic Formula.
B Factor quadratic trinomials.
C Solve quadratic equations by factoring.
D Recognize properties of the parabola.
E Recognize properties of quadratic equations.
F Determine whether a number is rational or irrational.
G Use the parabola and quadratic equations to solve real world problems.
H Graph parabolas.
Chapter 13

A Find values of functions from their formulas.
B Find and analyze values of functions.
C Determine whether a set of ordered pairs is a function.
D Classify functions.
E Find the domain and range of a function from its formula, graph, or rule.
F Determine values of probability functions.
G Find lengths and angle measures in triangles using the tangent function.
H Determine whether or not a graph represents a function.
I Graph functions.
APPENDIX B
# MATHEMATICS PROGRAM OUTCOMES

## CRITERIA AND CONTENT STANDARDS

### INTERPRETS DATA

#### A. INTERPRETS DATA BY TRANSLATING RECIPROCALLY FROM ONE FORM TO ANOTHER

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to translate among concrete and representational forms.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally translates among concrete and representational forms with increasing complexity.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually can translate among concrete and representational forms with increasing emphasis on representational.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently translates among concrete and representational forms with primary emphasis on representational.</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to discuss conclusions and predict future trends with significant assistance.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally able to discuss conclusions and predict future trends with appropriate assistance.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually able to understand the data, discuss conclusions, and predict future trends in real-world situations.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently uses an efficient approach to drawing conclusions in a clear, well-organized, detailed manner and can predict future trends in real-world situations.</td>
</tr>
</tbody>
</table>
## EXPLAINS CONCEPTS

### A. EXPLAINS USING PROPER TERMINOLOGY, DEFINITIONS, NOTATION, AND STRUCTURE

<table>
<thead>
<tr>
<th>Grade Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to use vocabulary, notation and structure to express ideas and relationships.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally uses appropriate vocabulary, notation, and structure to express ideas and relationships. Beginning to use definitions.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually uses appropriate and effective vocabulary, notation, and structure to express ideas and relationships. Generally uses definitions.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently uses rich, precise, effective vocabulary, notation, and structure to express ideas and relationships. Definitions used and extended.</td>
</tr>
</tbody>
</table>

### B. EXPLAINS USING REPRESENTATIONS: pictures, diagrams, graphs, models, tables, charts, manipulatives, technology, drawings, constructions, equations, or numerical examples

<table>
<thead>
<tr>
<th>Grade Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to use representations.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally uses representations. Representations used are increasingly accurate, complete, and directly related to the solution(s).</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually uses accurate and appropriate representations. Representations used are directly related to the solution(s) and generally clear and unencumbered.</td>
</tr>
<tr>
<td>8-12</td>
<td>Perceptive use of representations. All representations used are directly related to the solution(s) and enhance or clarify by their accuracy, completeness, and simplicity.</td>
</tr>
</tbody>
</table>
C. **EXPLAINS USING EFFECTIVE ORGANIZATION:** identifies all the important elements of the problem, chooses appropriate criteria and applies consistently, organizes information so that any pattern present could be observed, presents in a logical sequence, makes connections among concrete, representational, and abstract ideas.

<table>
<thead>
<tr>
<th>Age Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Organization is often unclear, disorganized, and incomplete.</td>
</tr>
<tr>
<td>2-6</td>
<td>Organization has some clear parts. Gives solution(s) with observations.</td>
</tr>
<tr>
<td>5-9</td>
<td>Organization is mostly clear. Gives solution(s) with connections or applications.</td>
</tr>
<tr>
<td>8-12</td>
<td>Organization is clear, well-organized, complete and detailed. Solution(s) have synthesis, generalization, or abstraction.</td>
</tr>
</tbody>
</table>

D. **EXPLAINS CONCEPTS BY ADAPTING CONTENT TO THE RECEIVER AND THE SITUATION:** gives similarities and differences, identifies and generates examples and non-examples, uses counter examples, explores different approaches, restates complex problems in an easier form, relates a given problem to a simpler or similar problem that can be solved or that has already been solved.

<table>
<thead>
<tr>
<th>Age Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to verbalize and demonstrate concepts. Will often be unclear and show gaps.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally can communicate different approaches to concepts with some clarity.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually can communicate a variety of approaches to concepts clearly and logically.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently can communicate a variety of approaches to concepts with precision.</td>
</tr>
</tbody>
</table>
GATHERS DATA

A. GATHERS DATA BY DECIDING THE QUESTIONS TO BE ANSWERED.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to show an understanding of the questions.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally shows an understanding of the questions.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually shows an understanding of the questions, mathematical ideas, and processes.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently shows an understanding of the questions, mathematical ideas, and processes.</td>
</tr>
</tbody>
</table>

B. GATHERS DATA USING DIVERSE RESOURCES: calculators, videos, computers, media, business and community resources, charts, graphs, etc.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to explore a variety of resources.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally uses a variety of resources. Beginning to make independent choices.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually demonstrates appropriate and effective use of a variety of resources. Generally makes independent choices.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently demonstrates precise and effective use of appropriate resources. Consistently makes independent choices.</td>
</tr>
</tbody>
</table>

C. GATHERS DATA USING A VARIETY OF METHODS: surveys, interviews, random samples, research, observations, questionnaires, etc.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to explore a variety of methods.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally uses a variety of methods. Beginning to make appropriate independent choices.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually demonstrates appropriate and effective use of a variety of methods.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently demonstrates precise and effective use of appropriate methods.</td>
</tr>
</tbody>
</table>
**MATHEMATICS PROGRAM OUTCOMES**

**CRITERIA AND CONTENT STANDARDS**

**APPLIES CONCEPTS**

A. **APPLIES KNOWLEDGE EFFECTIVELY BY DEVISING A PLAN:** identifies the problem, chooses an appropriate strategy.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to show understanding of the question being asked, mathematical ideas, and processes.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally identifies the problem and chooses a strategy from a limited list of strategies.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually can identify the problem and choose an appropriate strategy from an extended list.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently identifies the problem and chooses appropriate strategies.</td>
</tr>
</tbody>
</table>

B. **APPLIES KNOWLEDGE TO STRUCTURED AND UNSTRUCTURED, REAL-WORLD SITUATIONS BY EXTENDING AND REFINING:** creates, predicts, infers, formulates hypotheses, adapts, makes connections.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Through modeling, begins to make connections between mathematical concepts and real-world situations.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally making connections between mathematical concepts and real-world situations. Beginning to create similar problems and solutions and to predict simple trends.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually makes connections between mathematical concepts and real-world situations. Can create similar problems, but may not always be clear and accurate. Can usually predict outcomes and trends with accuracy.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently makes connections between mathematical concepts and real-world situations. Can devise original real-world problems and make inferences about outcomes if variables are changed.</td>
</tr>
</tbody>
</table>
A. **JUSTIFIES SOLUTIONS USING SUPPORTING EVIDENCE**: definitions, theorems, properties, representations, models, diagrams, etc.

<table>
<thead>
<tr>
<th>K-3</th>
<th>Beginning to give reasons for solutions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-6</td>
<td>Generally gives reasons, both orally and written, for solutions. Begins to question appropriate form for the solution (estimate, actual, rounded, fraction, decimal, etc.) as prescribed by the context.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually can validate and explain solutions using appropriate mathematical methods. Generally uses the form of the solution prescribed by the context.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently can validate and explain solutions using appropriate mathematical methods. Consistently uses the form of the solution prescribed by the context.</td>
</tr>
</tbody>
</table>

B. **JUSTIFIES BY CONFIRMING CONCLUSIONS IN ANOTHER WAY**: eliminating possibilities, induction, deduction, indirect proof, experimentation, counter examples, etc.

<table>
<thead>
<tr>
<th>K-3</th>
<th>Beginning to check answers and develop a sense of their reasonableness.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-6</td>
<td>Generally able to solve problems in diverse ways such as eliminating possibilities and experimenting.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually can demonstrate alternate approaches to the solution. The explanation may not be completely clear.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently can demonstrate alternate approaches to the solution. The explanation is clear and precise.</td>
</tr>
</tbody>
</table>
### MATHEMATICS PROGRAM OUTCOMES

### CRITERIA AND CONTENT STANDARDS

#### ORGANIZES DATA

**A. ORGANIZES DATA BY ARRANGING, COMPARING, AND CLASSIFYING:** uses methods such as listing, matching, and sorting to recognize patterns and distinguish connections in the data.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to organize concrete data with appropriate guidance.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally organizes concrete and abstract data with appropriate guidance.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually organizes data with increasing independence.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently organizes data independently.</td>
</tr>
</tbody>
</table>

**B. ORGANIZES DATA USING REPRESENTATIONS:** pictures, diagrams, graphs, models, tables, charts, manipulatives, technology, drawings, constructions, equations, numerical examples, etc.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-3</td>
<td>Beginning to use representations to organize data after appropriate modeling.</td>
</tr>
<tr>
<td>2-6</td>
<td>Generally, after modeling, can independently use a similar representation to organize data.</td>
</tr>
<tr>
<td>5-9</td>
<td>Usually has accurate and appropriate use of representations, but some less important variables may be missing.</td>
</tr>
<tr>
<td>8-12</td>
<td>Consistently has accurate and appropriate use of representations that are clear, well organized and detailed.</td>
</tr>
</tbody>
</table>
ALGEBRA

PROGRAM STANDARD 1

TLW explain, apply, and justify algebraic concepts (patterns, relationships and functions) and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL1.1

TLW write and solve equations based on real-world applications translating among standard English, algebraic, and graphical representations.

VOCABULARY

variable, equality, open sentence, solution, solution set, solve, factor

FACTS/RULES/CONCEPTS

Methods and properties used to solve linear equations; use of graphing calculator; use of quadratic formula.

LESSON STANDARDS

1. Given a word problem involving constant rate of increase or decrease (ex: saving $5/mo. or losing 2lb./wk.), TLW represent the information in an equation and solve the equation.

2. Given a linear equation, TLW describe a real-world situation that could be represented using that equation.

3. Given a real-world situation involving a constant rate of increase or decrease (ex: distance covered over time at a constant rate of 50 mph), TLW explain how the graph would be affected if the rate were increased or decreased.

4. Given three graphs representing three different linear relationships and the real-world situations that are represented by the graphs, TLW determine which graph represents which situation and justify their conclusions.

5. Given an equation representing projectile motion (i.e., quadratic equation), TLW use a graphing calculator to graph the equation and find solutions. TLW explain the significance and/or appropriateness of the solutions.
ALGEBRA

LEVEL STANDARD AL1.1

LESSON STANDARDS

6. Given the graph of a real-world situation, TLW explain the trend and make predictions about future results.

7. Given a quadratic equation arising from a real-world situation, TLW solve, explaining and justifying their method.
ALGEBRA

PROGRAM STANDARD 1

TLW explain, apply, and justify algebraic concepts (patterns, relationships and functions) and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL1.2

TLW represent and analyze relationships by interpreting and translating among multiple representations of patterns including tables, rules, charts, and graphs.

VOCABULARY

variable, open sentence, equality, inequality, coincident, parallel, intersecting, instance, slope, intercept, solution set

FACTS/RULES/CONCEPTS

systems of equations; properties of equality; meaning of \( y = mx + b \)

LESSON STANDARDS

1. Given a rule containing a variable, TLW list three applications of the rule.

2. Given three instances of a rule involving a variable, TLW write the rule.

3. Given an equation of the form \( y = mx + b \), TLW explain the changes to the graph that would result from changing \( m \) and \( b \), including large and small values, negative numbers and fractions.

4. Given the graph of a linear relationship, TLW write the equation for the relationship in the form \( y = mx + b \).

5. TLW justify steps in the solution of a linear equation using properties of the real numbers.

6. Given two linear equations, TLW explain how to tell if the graphs will be intersecting, parallel, or coincident and justify by graphing.
PROGRAM STANDARD 2

TLW explain, apply, and justify geometric and measurement concepts and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL2.3

TLW use the concepts of rates and ratios to describe the real world and to solve problems by using proportional reasoning, indirect measurement, and computing with measurement units (e.g., rate factor model).

VOCABULARY

rate, ratio, rate factor model, proportion, reciprocal

FACTS/RULES/CONCEPTS

Scale on a map is a ratio.

LESSON STANDARDS

1. TLW convert measurements to larger and/or smaller units (ex: feet to inches).

2. Given maps, TLW calculate actual distances using the scale provided.

3. TLW determine tree heights by measuring shadows and comparing ratios.

4. TLW determine the probability of an event using ratios and use that probability to predict the number of occurrences in a large number of events.

5. TLW use exchange rates between the dollar and foreign currency to determine whether travel to foreign countries is getting cheaper or more expensive.

6. TLW use ratios in practical situations and demonstrate the meaning by using models relating measurement units.

7. TLW explain the relationships between the areas, the perimeters, and the volumes of similar figures.
ALGEBRA

PROGRAM STANDARD 2

TLW explain, apply, and justify geometric and measurement concepts and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL2.4

TLW use the coordinate plane to illustrate relationships by interpreting the graphs of points, lines, and curves.

VOCABULARY

coordinate system, x-axis, y-axis, origin, quadrant, midpoint, latitude, longitude, ordered pair

FACTS/RULES/CONCEPTS

Ordered pairs are written (x,y).

LESSON STANDARDS

1. TLW use informal and formal coordinate systems on lines and planes to specify locations and distances.

2. TLW describe in detail using compass directions how to travel from one location to another (ex: school to home; New York to Boston).

3. Given two linear equations, TLW graph them on the coordinate plane and compare and contrast the graphs (slopes, y-intercepts, etc.).

4. TLW use the coordinate plane to describe the shape of a building to someone on the other end of a phone line.

5. TLW visualize, sketch, and construct geometric shapes or relationships on the coordinate plane.

6. TLW locate cities on a world map by longitude and latitude and use that information to predict which cities might have similar temperatures.
PROGRAM STANDARD 2

TLW explain, apply, and justify geometric and measurement concepts and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL2.5

TLW translate between geometric figures and algebraic expressions, analyzing the effects of parametric changes.

VOCABULARY

slope (including zero and no), intercepts (x and y), image, pre-image, scale factor, transformation, parabola, orientation

FACTS/RULES/CONCEPTS

\[ y = mx + b \] graphs as a line; \[ y = ax^2 + bx + c \] graphs as a parabola

LESSON STANDARDS

1. Given an equation of the form \( y = mx + b \), or \( y = ax^2 + bx + c \), TLW graph the equation on the coordinate plane.

2. Given a closed geometric figure defined by ordered pairs \((x_i, y_i)\), TLW describe the figure defined by the ordered pairs \((ax_i, ay_i)\), including size, shape, and orientation.

3. Given an equation of the form \( y = ax^2 + bx + c \), TLW describe the effect on the graph of changing \( a, b, \) and \( c \).

4. Given the equation of a parabola, TLW graph the equation with a graphing calculator. TLW give the equation of another parabola that would lie between the given one and the x-axis and justify his/her new equation.

5. TLW explain the slopes of equations of the form \( x = a \), and \( y = bx + h \), using algebraic calculations, and describe real situations they could represent.

6. Given an image defined by the ordered pairs \((x_i, y_i)\), TLW describe the pre-image defined by \((x^*, y^*)\) including size, shape, and orientation.
PROGRAM STANDARD 3

TLW gather, organize, and interpret data to explain, apply, and justify statistical and data analysis concepts and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL3.6

TLW make predictions and decisions based on data the learner has gathered, organized, and interpreted, including interpolations and extrapolations.

VOCABULARY

scattergram, line of best fit, box and whisker plot, quartile, extrapolate, interpolate, circle graph, stem and leaf plot

FACTS/RULES/CONCEPTS

Students must be able to order real numbers. A circle contains 360 degrees.

LESSON STANDARDS

1. TLW make a list of heights of 24 classmates, organize the information into a box and whisker plot, explain the significance of the sizes of the box, the whiskers, and the outliers, if any.

2. TLW keep track of how he/she spends time on a typical weekday and a typical weekend day, organize the data into two circle graphs and compare and contrast those graphs.

3. TLW gather information about arm span and foot length of 24 classmates, organize the information in a scattergram and determine a line of best fit. Given the arm span of 5 other students, TLW give the approximate foot size of those students.

4. TLW gather information about test scores on a recent test (NO NAMES) and organize it into a stem and leaf plot. TLW write an explanation of the significance of the distribution.

5. TLW use the information from #4 to make a box plot. TLW compare and contrast the two different representations.
PROGRAM STANDARD 4

TLW explain, apply, and justify number concepts (number sense and numeration) and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL4.7

TLW estimate and compute with real numbers (including numbers written in scientific notation) to evaluate and interpret algebraic expressions, explaining strategies used.

VOCABULARY

square, square root, scientific notation, density of real numbers, opposites, inverses, base, exponent, radical sign, identities

FACTS/RULES/CONCEPTS

Increase level of generality using concrete models and examples and finding results using compact algebraic symbolism.

LESSON STANDARDS

1. TLW demonstrate the use of real numbers using models and generalizations to solve problems.

2. TLW recognize and use the patterns of square and roots to solve problems.

3. TLW recognize and use function concepts to solve problems.

4. TLW estimate and compute sums, differences, products, and quotients using square roots, powers, and scientific notation, explaining when such operations can actually be done and when they can only be written symbolically.

5. TLW estimate the square root of a non-perfect square number (without using a calculator) to the nearest whole number, justify his/her estimate with a calculator.

6. Given a non-perfect square number, TLW simplify that number so the smallest possible integer remains under the radical sign, explaining his/her process (ex: prime factors, finding perfect square factors).
ALGEBRA

PROGRAM STANDARD 5

TLW apply, a wide variety of problem solving concepts and procedures to explain and interpret solutions.

LEVEL STANDARD ALS.8

TLW apply mathematical problem solving strategies to solve problems from within and outside mathematics: formulate problems from situations, use mathematical modeling, make and test conjectures, formulate counter examples, follow logical arguments, judge the validity of arguments, and construct simple valid arguments.

VOCABULARY

rule, element, counter example

FACTS/RULES/CONCEPTS

A formal rule can be stated in a variety of ways: in a table, as a formula, or with written statements.

LESSON STANDARDS

1. TLW create a pattern, determine a missing element, give a formal rule, and extrapolate the information for extended solutions.

2. TLW formulate problems for mathematical expressions or number sentences.

3. TLW make, or use a drawing, graph, or physical model to solve a problem.

4. TLW evaluate and interpret the solution to a problem and decide if the answer is reasonable.

5. TLW find and evaluate alternative processes for solving problems and provide extensions to the solutions.

6. TLW provide counter examples to justify the conclusion that an algebraic equation is false.
PROGRAM STANDARD 6

TLW gather, organize, and interpret data to explain, apply, and justify probability and discrete mathematics concepts and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL6.9

TLW explain, interpret and evaluate experimental and theoretical probabilities, including those with multiple conditions.

VOCABULARY

probability, event, mutually exclusive, theoretical probability, empirical probability, relative frequency

FACTS/RULES/CONCEPTS

A certain event has a probability of one. An impossible event has a probability of zero. All other events have probabilities between zero and one. Computer simulations can be run to generate a large number of trials quickly, once a few simulations have been done by the students.

LESSON STANDARDS

1. TLW name two events which are certain and two events which are impossible, and justify that they are.

2. Given devices such as coins, discs, spinners, dice, playing cards, marbles, and random digits, TLW determine the probability of simple and compound events, using calculators, computers, and pencil/paper techniques to record results.

   EX 1: (simple event) The probability of throwing a 7 with two dice is 1/6. What is the probability of not throwing a 7?

   EX 2: (compound event) What is the probability of selecting the right colored hat and the right colored marble?

3. TLW use probability devices to simulate real-world events.

   EX: Suppose a family has four children. Use a coin toss for each birth. Heads = boys; tails = girls. How many boys and how many girls does the family have as shown by the coins?

4. TLW explain similarities and differences between experimental probabilities (relative frequencies) and theoretical probabilities.
PROGRAM STANDARD 6

TLW gather, organize, and interpret data to explain, apply, and justify probability and discrete mathematics concepts and procedures in real-world and mathematical problem situations.

LEVEL STANDARD AL6.10

TLW solve problems using simple discrete mathematical techniques such as Venn diagrams, factorials, tree diagrams, and the Fundamental Counting Principle.

VOCABULARY

if-then, Venn diagram, factorial (!), generalization, insufficient information

FACTS/RULES/CONCEPTS

n! means n x (n-1) x (n-2) x ... 3 x 2 x 1

LESSON STANDARDS

1. Given a Venn diagram with three circles inside a rectangle, where A contains multiples of 2, B contains multiples of 3, and C contains multiples of 4, TLW place the numbers from 1-24 in the appropriate region, and interpret the results by asking and answering questions, justifying their responses.

2. Using a tree diagram, TLW determine all possible combinations of tossing a coin three times, make a list of them, and interpret the results.

3. TLW justify the number of combinations found in #2, using the Fundamental Counting Principle.

4. If a classroom has 25 students and 25 desks, TLW determine how many different seating charts are possible.

5. TLW make a chart, a tree diagram, and the Fundamental Counting Principle to determine how many different combinations of classes one could have from 2 languages (Spanish and French), 3 music classes (Band, Orchestra, and Choir), 2 art classes (Beginner and Intermediate) if one must have a language, a music class, and an art class. TLW compare and contrast the three methods of organizing the data.
APPENDIX C
Dear Professor Usiskin,

I am writing to ask your help in obtaining information for my master’s thesis. I am a middle school teacher in Kentwood, Michigan and have taught pre-algebra, algebra, and geometry from the UCSMP series since it was adopted by the district in 1990. At the time it was the series that best addressed the goals and objectives of Kentwood’s mathematics program.

This winter, Kentwood’s Mathematics Curriculum Committee completed the task of writing outcomes for all grades/levels from kindergarten through twelfth grade. For my thesis I have chosen to cross-reference our current algebra outcomes with the objectives of the UCSMP Algebra text. To make my study valid I must have statistics to show that the book’s stated objectives are actually achieved by students using it. That is, I must demonstrate that the objectives are actually attainable outcomes.

Would you send me any such statistics that you have compiled from pilot schools or other sources? I will need more than one source to make the study credible. If you have other information that you feel is pertinent to this paper, I would appreciate your including it also. I will be happy to send you a copy of my completed thesis. Thank you in advance for your consideration.

Sincerely,

Carol Farrer
June 21, 1995

Ms. Carol Farrer  
1643 Whitfield, S.E.  
Grand Rapids, MI 49506

Dear Ms. Farrer:

I am catching up on correspondence; I am sorry it has taken so long to respond to your letter of May 31st.

I cannot answer the question you ask the way you seem to want it. We have never gathered data on each objective individually to collect data on it. In fact, it would not be so easy to do so. UCSMP Algebra has perhaps 150 objectives, and to test all 150 at the end of the year would take an enormous amount of time, yet to test each objective only in the chapter it occurred would not provide any picture of retention or of cumulative growth.

When we test our materials, we ask the field sites to examine our tests to determine whether or not they are of appropriate difficulty. We rarely find that one of our objectives is considered too difficult. In fact, difficulty is seldom the criterion by which mathematics teachers judge an objective. The appropriateness of the objective is far more important. Most teachers believe as we do that any of these objectives can be attained by virtually any student provided that (1) the student has the prerequisites for the course, (2) adequate time was given in the course to cover the objective, and (3) the student has done the requisite homework.

For example, as you know, we suggest a grading scale of 85-100 for an A, and so on down to 50 for passing. It is not uncommon for teachers to believe that our scale is too easy and to use 90-100 for an A, and so on down to 60 for passing. This to me indicates not only that these teachers believe our objectives are attainable but also that they think we are being too easy on students.

As you also know, each of our chapter tests is keyed to the objectives, and every objective is covered. You could use test data from your own classes to confirm which objectives were reached by the students in your classes. You would need to identify the criteria you would use to decide that an objective had been reached by your students, but it could be done.

Having said all this, I still am unclear that I understand the reason for your request. Do you believe some of the UCSMP Algebra objectives are...
unattainable? If so, which ones? I can assure you that we have never kept any
topic in our books that teachers have thought their students could not learn.

I would be interested in receiving a copy of your master’s thesis if it has
anything to do with the use of UCSMP materials.

Sincerely,

Zalman Usiskin
Professor of Education
Director, UCSMP
The advances of technology have profoundly influenced society. The world today increasingly depends upon information processing to solve problems. This revolution in technology will create professions requiring abilities and skills in mathematics and in decision making. To meet these needs, schools should provide a comprehensive mathematics curriculum that teaches the fundamentals of mathematics, develops critical thinking skills, and nurtures logical reasoning.

A mathematically literate person is one who:

1. Values mathematics;
2. Has confidence in his/her ability to do mathematics demonstrating conceptual understanding as well as proficiency in operations skills;
3. Applies conceptual understandings, operational skills, and technological tools to problem solving;
4. Communicates mathematical ideas orally and in writing; and
5. Reasons mathematically. *

The following topics of study are recommended with specific outcomes enumerated for each level of schooling (elementary, middle/junior high, high school).

At the middle/junior high school level, students will:

1. Estimate and mentally compute with selected whole numbers;
2. Add, subtract, multiply and divide whole numbers;
3. Use multiples and factors; and
4. Express whole numbers in scientific notation.

At the elementary school level, students will:

1. Read, write, compare, order, and round numbers;
2. Add, subtract, multiply, and divide using models and computational algorithms with specified numbers;
3. Estimate and mentally compute with selected whole numbers; and
4. Identify and use properties of whole numbers.

FRACTIONS, DECIMALS, RATIO, AND PERCENT

At the middle/junior high school level, students will:

1. Compute with fractions/decimals;
2. Identify and find equivalent ratios;
3. Use a calculator to compare and order fractions using decimal equivalents;
4. Demonstrate the meaning of percent;
5. Find a percent of a number; and
6. Express ratios as percents, fractions, or decimals and relate each form to the other two.

At the elementary school level, students will:

1. Demonstrate and use the meaning of fractions/decimals;
2. Find equivalent fractions/decimals using models and generalizations for equivalent fractions/decimals;
3. Use models to compare and order fractions/decimals; and
4. Add, subtract, multiply, and divide selected fractions/decimals.

MEASUREMENT

At the middle/junior high school level, students will:

1. Measure length, area, volume, and angles;
2. Measure liquid capacity, mass, time, temperature, and monetary values using metric units; and
3. Apply knowledge of the relationships among the basic metric units to solve multi-step problems.

At the elementary school level, students will:

1. Identify and describe the concept of the length, area, and volume and standard units of measurement;
2. Estimate the length, area, and volume of familiar objects;
3. Measure length by relating objects to given units;
4. Measure area by the process of covering and measure volume by the process of filling; and
5. Use the concepts of capacity, mass, time, temperature, and money to solve problems.

GEOMETRIC CONCEPTS

At the high school level, students will

1. Represent problem situations with geometric models and apply properties of figures;
2. Classify figures in terms of congruence and similarity and apply these relationships;
3. Deduce properties of, and relationships between, figures from given assumptions;
4. Translate between geometric and algebraic representations;
5. Deduce properties of figures using transformations and using coordinate and
genetic methods;
6. Identify congruent and similar figures using transformations.
At the middle/junior high school level, students will:

1. Identify and use shapes in one, two, and three dimensions;
2. Identify and apply properties of one, two, and three dimensional shapes, such as equal sides, equal angles, and symmetry;
3. Solve problems using the relationships of congruence, similarity, intersection, parallelism, and perpendicularity for appropriate figures in one, two, and three dimensions.
4. Identify and apply transformations (reflections, translation, rotation, and size change) to problem solving;
5. Use informal and formal coordinate systems on lines and planes to specify locations and distances; and
6. Visualize, sketch, and construct geometric objects.

At the elementary school level, students will:

1. Identify and use selected common shapes;
2. Solve problems using the properties of selected common shapes;
3. Identify and use the properties of similarity and congruence;
4. Specify locations and distances using informal and formal coordinate systems on lines and planes; and
5. Identify and apply the transformations of flip, slide, and turn.

At the high school level, students will:

1. Construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
2. Use curve-fitting to predict from data;
3. Apply measure of central tendency, variability, and correlation;
4. Explain the concepts of sampling and random variable as used in statistics and the importance of each concept;
5. Design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes;
6. Use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
7. Use simulation to estimate probabilities; and
8. Create and interpret discrete probability distributions.

At the middle/junior high school level, students will:

1. Generate tables using calculators and computers, including the use of spreadsheets;
2. Construct, read, and interpret circle graphs, line plots, stem-and-leaf plots, box plots, and scatter plots;
3. Read, interpret, determine and apply descriptive statistics, including the concepts of outlier and quartile;
4. Read, interpret, determine, and apply probabilities, including applications that make use of calculators and computers; and
5. Use tables, graphs, descriptive statistics, and probabilities to determine patterns, identify trends, predict outcomes, and make wise choices.

At the elementary school level, students will:

1. Construct, read, and interpret tables;
2. Construct, read, and interpret picture graphs, bar graphs, and line graphs;
At the high school level, students will:

1. Represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
2. Use tables and graphs as tools to interpret expressions, equations, and inequalities;
3. Operate on expressions and matrices, and solve equations and inequalities;
4. Model real-world phenomena with a variety of functions;
5. Represent and analyze relationships using tables, rules, and graphs;
6. Translate among tabular, symbolic, and graphical representations of functions;
7. Recognize that a variety of problem situations can be modeled by the same type of function; and
8. Analyze the effects of parameter changes on the graphs of functions.

At the middle/junior high school, students will:

1. Use variables to evaluate and estimate expressions;
2. Solve problems represented physically, symbolically, or verbally;
3. Use variables to write and solve open sentences;
4. Apply the distributive property to problem solving;
5. Compute with integers;
6. Identify and apply the concepts of exponents, powers, and roots to solve problems;
7. Identify and apply function concepts in problem situations; and
8. Identify and interpret graphs representing situations, tables of values, or sentences.

At the elementary school level, students will:

1. Use expressions containing variables written as letters;
2. Use variables to write and solve open sentences;
3. Use variables in translating among verbal expressions, symbols, and situations that are pictorial or practical when solving problems;
4. Identify, compare and use concepts of exponents, powers, and roots;
5. Identify and use the distributive property;
6. Identify and use integers in simple applications; and
7. Identify and use function concepts that represent a relationship between two sets.

At the high school level, students will:

1. Apply mathematical problem solving strategies to solve problems from within and outside of mathematics: formulate problems from situations, use mathematical modeling, make and test conjectures, formulate counter examples, follow logical arguments, judge the validity of arguments, and construct simple valid arguments.
At the middle/junior high school level, students will:

1. Apply the strategy of working backward to solve a problem;
2. Interpret and use statements involving logical operations and quantifiers (and, or, not, if...then, every, all, some, no, at least, at most, each, exactly); and
3. Use logical reasoning to draw valid conclusions from given information.

At the elementary school level, students will:

1. Identify, use, and construct patterns;
2. Demonstrate an understanding of a problem by organizing information to formulate a solution;
3. Select and apply appropriate problem-solving strategies;
4. Interpret and evaluate the solution to a problem; and
5. Use logical reasoning.

At the high school level, students will:

1. Use scientific calculators to learn mathematics; and
2. Use scientific calculators and computers to investigate and solve mathematical applications.

At the middle/junior high school level, students will:

1. Use the calculator to compute percents, including percent of increase or decrease;
2. Use the calculator and computer to solve problems; and
3. Describe certain common limitations to calculators and interpret selected calculator-displayed symbols.

At the elementary school level, students will:

1. Identify specific calculator and computer keys as they relate to basic mathematical operations;
2. Use a calculator to compute sums, differences, products, and quotients using whole numbers, fractions, and decimals;
3. Interpret the calculator display;
4. Explain common limitations of the calculator regarding decimal number display and order of operations; and
5. Use a computer as a tool for learning and problem solving.
APPENDIX E
**Mathematics Content Standards and Benchmarks**

### Patterns, Relationships, and Functions

**Content Standard 1:** Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships (Patterns). [C3, C8] [L1] [P11] [T4, T5] [W1, W6]

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
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<tbody>
<tr>
<td>- Recognize, describe, and extend numerical and geometric patterns.</td>
<td>- Describe, analyze, and generalize patterns arising in a variety of contexts and express them in general terms.</td>
<td>- Analyze and generalize mathematical patterns including sequences, series, and recursive patterns.</td>
</tr>
<tr>
<td>- Represent and record patterns and relationships in a variety of ways including tables, charts, and pictures.</td>
<td>- Represent and record patterns in a variety of ways including tables, charts, and graphs, and translate between various representations.</td>
<td>- Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations, or relations.</td>
</tr>
<tr>
<td>- Use patterns to describe real world phenomena.</td>
<td>- Use patterns and their generalizations to make and justify inferences and predictions.</td>
<td>- Use patterns and reasoning to solve problems and explore new content.</td>
</tr>
<tr>
<td>- Explore various types of patterns (repeating, growing, shrinking).</td>
<td>- Explore and describe patterns for linear expressions and other near-linear patterns such as step and constant functions.</td>
<td>- Apply their experiences with patterns to help solve problems and explore new content.</td>
</tr>
<tr>
<td>- Apply their experiences with patterns to help solve problems and explore new content.</td>
<td>- Use patterns and generalizations to solve problems and explore new content.</td>
<td>- Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations, or relations.</td>
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**Content Standard 2:** Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, and compare patterns of change (Variability and Change). [A2] [C1, C3] [T4, T5] [W1]

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<tr>
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<tr>
<td>- Recognize change when it occurs in a variety of settings.</td>
<td>- Identify and describe the nature of change; recognize change in more abstract and complex situations and introduce different kinds of change, such as continuous and non-continuous variation.</td>
<td>- Begin to identify and describe the nature of change in the more formal language of change such as rate of change, continuity, and limits.</td>
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**DRAFT**

Mathematics Content Standards and Benchmarks 32
- Recognize that change is often predictable and that patterns emerge that help to describe change.

- Connect an initial state to a final state and generalize a rule that describes a pattern of change.

- Develop a mathematical concept of function and recognize that functions display characteristic patterns of change (e.g., linear, quadratic, exponential).

- Explore change, and realize that change is frequently interdependent—for instance, changing the number of cookies also changes the weight of the cookies.

- Begin to investigate applications in bivariate data and linear relationships and explore questions of what will happen to one quantity if another variable is changed.

- Expand their understanding of function to include non-linear functions, composition of functions, inverses of functions, and piecewise- and recursively-defined functions.

- Use tables, charts, open sentences, and hands-on models to physically represent change.

- Represent variability or change by ordered pairs, tables, graphs, and equations.

- Represent functions using symbolism such as matrices, vectors, and functional representation (f(x)).

- Begin to describe and differentiate between types of relationships, especially repeating, growing, and shrinking patterns.

- Differentiate between types of relationships such as linear vs. not linear, direct vs. indirect, and continuous vs. non-continuous.

- Differentiate and analyze classes of functions including linear, power, quadratic, exponential, circular, and trigonometric functions and realize that many different situations can be modeled by a particular type of function.

- Begin to explore relationships in various contexts and see that patterns help to solve mathematical problems.

- Continue to explore relationships arising from interesting contexts and use variables and relationships to solve mathematical problems.

- Use functions and mathematical models to solve problems in context.

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**Geometry and Measurement**

**Content Standard 3:** Students develop spatial sense, identify characteristics and define shapes, identify properties and describe relationships among shapes (Shape and Shape Relationships). [P11] [T3, T4, T5] [W1]

**Elementary**

- Recognize and name familiar shapes in one, two, and three dimensions such as lines, rectangles, and spheres and informally discuss the shape of a graph.

**Middle School**

- Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.

**High School**

- Generalize the characteristics of shapes and apply their generalizations to classes of shapes.

- Determine necessary and sufficient conditions for the existence of a particular shape and apply those conditions to analyze shapes.

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A = Arts Education  
C = Career and Employability  
H = Health Education  
P = Physical Education  
T = Technology  
W = World Languages  
L = Life Management  

**Mathematics Content Standards and Benchmarks** 33
Content Standard 4: Students identify locations of objects, identify location relative to other objects, and describe the effects transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object (Position). [C3] [P111] [T3, T4, T5] [W1]

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<tr>
<th>Elementary</th>
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<tr>
<td>Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between, and locations on the number line, on a coordinate graph, and on a map.</td>
<td>Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude, and midpoints.</td>
<td>Locate and describe objects in terms of their position, including polar coordinates, cylindrical coordinates, spherical coordinates, three-dimensional Cartesian coordinates, vectors, and limits.</td>
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- Draw and build familiar shapes.
- Explore ways to combine, dissect, and transform shapes.
- Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.
- Use concepts of shapes and their properties and relationships as studied at the elementary level to describe the physical world and to solve problems.
- Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.
- Construct familiar shapes using coordinates, and appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.
- Generalize about the common properties of similar, congruent, parallel, and perpendicular shapes and verify their generalizations informally.
- Use concepts of shapes and their properties and relationships as studied at the middle level to describe the physical world and to solve problems.

Locate and describe objects in terms of their orientation, direction and relative position, including up, down, front, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.

Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N-S-E-W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.

Locate and describe objects in terms of their orientation and relative position, including displacement (vectors), phase shift, maxima, minima, and inflection points; give precise mathematical descriptions of symmetries.

- Analyze shapes and their properties and relationships to describe the physical world and to solve problems.

- Use transformations, coordinate, or synthetic methods to verify (prove) the generalizations about properties of classes of shapes.

- Draw and construct shapes in two and three dimensions and analyze and justify the steps of their constructions.

- Study transformations of shapes using isometries, size transformations, and coordinate mappings.

- Compare and analyze shapes and formally establish the relationships among them, including congruence, similarity, parallelism, perpendicularity, and incidence.

A = Arts Education  
C = Career and Employability  
H = Health Education  
L = Life Management  
P = Physical Education  
T = Technology  
W = World Languages  
1-26 = Content standard number
<table>
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<tr>
<th>Content Standard 5: Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision (Measurement). [C1, C3, C4] [L8] [P10] [T3, T4, T5] [W1]</th>
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<tr>
<td>• Explain what happens to the size, shape and position of an object after sliding, flipping, turning, enlarging, or reducing it.</td>
<td>• Describe translations, reflections, rotations, and dilations using the language of transformations and employ transformations to verify congruence of figures.</td>
<td>• Give precise mathematical descriptions of transformations and describe the effects of transformations on size, shape, position, and orientation.</td>
</tr>
<tr>
<td>• Locate the position of points or objects described by two or more conditions: locate all the points (locus) that satisfy a given condition.</td>
<td>• Describe the locus of a point by a rule or mathematical expression; trace the locus of a moving point.</td>
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<tr>
<td>• Use concepts of position, direction, and orientation to describe the physical world and to solve problems.</td>
<td>• Use concepts of position, direction, and orientation to describe the physical world and to solve problems.</td>
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<th><strong>High School</strong></th>
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<tr>
<td>• Measure common objects by comparing them to other objects, to non-standard units and to standard units and order objects according to their size.</td>
<td>• Measure objects using standard units in both the metric and common systems, and measure angles in degrees.</td>
<td>• Make accurate measurements using both metric and common units, and measure angles in degrees and radians.</td>
</tr>
<tr>
<td>• Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), area, perimeter, capacity, time, temperature, and money.</td>
<td>• Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume, and angle.</td>
<td>• Continue to make and apply measurements of length, mass (weight), time, temperature, area, volume, angle; classify objects according to their dimensions.</td>
</tr>
<tr>
<td>• Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is &quot;a good estimate.&quot;</td>
<td>• Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is close enough.&quot;</td>
<td>• Select and use appropriate tools to measure with increased precision.</td>
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<td>• Develop and use their own measuring units and tools, and progress to selecting and using standard tools for measurement.</td>
<td>• Select and use appropriate tools to measure length, mass, time, temperature, perimeter, area, volume, and angle.</td>
<td>• Interpret measurements and explain how changes in one measure may affect other measures (e.g., what happens to the volume and surface area of a cube when the side of the cube is halved?)</td>
</tr>
<tr>
<td>• Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.</td>
<td>• Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area), but not necessarily on another (e.g., perimeter).</td>
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</table>
• Explore scale drawings, models, and maps and relate them to measurements of real objects.

• Use proportional reasoning and indirect measurements to draw inferences, such as measuring the thickness of a book to estimate the thickness of one page.

• Use proportional reasoning and indirect measurements, including applications of trigonometric ratios, to measure inaccessible distances and to determine derived measures such as density.

• Apply measurement to describe the real world and to solve problems.

• Apply measurement to describe the real world and to solve problems.

• Apply measurement to describe the real world and to solve problems.

**Data Analysis and Statistics**

**Content Standard 6**: Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats (Collection, Organization and Presentation of Data). [C2, C3, C4, C6] [H2] [L8] [P9, P10] [T3, T4, T5] [W1, W8, W9]

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<tr>
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<th>High School</th>
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<tbody>
<tr>
<td>Collect and explore data through counting, measuring, and conducting surveys and experiments.</td>
<td>Collect and explore data through observation, measurement, surveys, sampling techniques, and simulations.</td>
<td>Collect and explore data through observation, measurement, surveys, sampling techniques, and simulations.</td>
</tr>
<tr>
<td>Organize data using concrete objects, pictures, tallsies, tables, charts, diagrams, and graphs.</td>
<td>Organize data using tables, charts, graphs, box plots, tree diagrams, stem-and-leaf plots, spreadsheets, and data bases.</td>
<td>Organize data using tables, charts, graphs, box plots, tree diagrams, stem-and-leaf plots, spreadsheets, and data bases.</td>
</tr>
<tr>
<td>Present data using a variety of appropriate representations, and explain the meaning of the data.</td>
<td>Present data using a variety of appropriate representations, and explain why one representation is preferred over another or how a particular representation may bias the presentation.</td>
<td>Present data using the most appropriate representation and give a rationale for their choice; show how certain representations may skew the data or bias the presentation.</td>
</tr>
<tr>
<td>Identify what data are needed to answer a particular question or solve a given problem; use objects, pictures, tallsies, tables, charts, diagrams, and graphs and design and implement strategies to obtain, organize, and present those data.</td>
<td>Identify what data are needed to answer a particular question or solve a given problem; use tables, charts, graphs, box plots, tree diagrams, stem-and-leaf plots, spreadsheets, and data bases and design and implement strategies to obtain, organize, and present those data.</td>
<td>Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize, and present those data.</td>
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**Mathematics Content Standards and Benchmarks**
**Content Standard 7:** Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively (Description and Interpretation). [C1, C2, C3, C4, C8] [L8] [P7, P9, P10] [T3, T4] [W1, W8, W9]

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<tr>
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<th>High School</th>
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<tbody>
<tr>
<td><strong>• Read and explain data they have collected and organized themselves and progress to reading data from other sources.</strong></td>
<td><strong>• Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.</strong></td>
<td><strong>• Critically question the source of data as well as the collection, organization, and presentation of data and the inferences drawn from the data.</strong></td>
</tr>
<tr>
<td><strong>• Raise and answer questions about data and how the data were collected and represented, and how their explorations led to new questions.</strong></td>
<td><strong>• Describe the shape of a data distribution (e.g., rectangular, symmetric, and bimodal) and identify the center, the spread, correlation, and any outliers.</strong></td>
<td><strong>• Describe the shape of a data distribution (e.g., normal, skewed, symmetric) and determine measures of central tendency, variability, and correlation.</strong></td>
</tr>
<tr>
<td><strong>• Describe the shape of the data using informal language.</strong></td>
<td><strong>• Draw, explain, and justify conclusions based on data.</strong></td>
<td><strong>• Use the data and their characteristics (e.g., regression model, correlation, and measures of center) to draw and support conclusions.</strong></td>
</tr>
<tr>
<td><strong>• Explore bias by discussing problems they encounter when they collect data.</strong></td>
<td><strong>• Recognize bias in data and critique presentations of data such as in advertisements or survey results.</strong></td>
<td><strong>• Identify sources of bias in data and describe strategies for eliminating such bias.</strong></td>
</tr>
<tr>
<td><strong>• Formulate questions and problems, and gather and interpret data to answer those questions.</strong></td>
<td><strong>• Formulate questions and problems, and gather and interpret data to answer those questions.</strong></td>
<td><strong>• Formulate questions and problems, and gather and interpret data to answer those questions.</strong></td>
</tr>
</tbody>
</table>

**Content Standard 8:** Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions (Inference and Prediction). [C2, C4] [L8] [T3, T4, T5] [W4]

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>• Make and test hypotheses.</strong></td>
<td><strong>• Make and test hypotheses.</strong></td>
<td><strong>• Make and test hypotheses.</strong></td>
</tr>
<tr>
<td><strong>• Conduct surveys, samplings, and experiments to solve problems and answer questions of interest to them.</strong></td>
<td><strong>• Design experiments to model and solve problems using sampling, simulations, and controlled investigations.</strong></td>
<td><strong>• Design investigations to model and solve problems; also employ confidence intervals and curve fitting in analyzing the data.</strong></td>
</tr>
</tbody>
</table>
- Formulate and communicate arguments and conclusions based on data, and evaluate their arguments and those of others.

- Make and explain predictions based on data.

- Make predictions to answer questions and solve problems.

- Make and explain predictions based on data.

- Formulate and communicate arguments and conclusions based on data, and evaluate their arguments and those of others.

- Make predictions and decisions based on data, including interpolations and extrapolations.

- Employ investigations, mathematical models, and simulations to make inferences and predictions to answer questions and solve problems.

- Make predictions and decisions based on data, including interpolations and extrapolations.

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Develop an understanding of whole numbers, and read, write, and count using whole numbers; investigate basic concepts of fractions and decimals.</td>
<td>• Develop an understanding of integers and rational numbers, and represent rational numbers in both fraction and decimal form.</td>
<td>• Develop an understanding of irrational, real, and complex numbers.</td>
</tr>
<tr>
<td>• Investigate and develop an understanding of numeration systems, in particular the base-10 place-value system.</td>
<td>• Extend their understanding of numeration systems to include decimal numeration and scientific numeration.</td>
<td>• Use the ((a+bi)) and polar forms of complex numbers.</td>
</tr>
<tr>
<td>• Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.</td>
<td>• Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0 and 1, and the additive and multiplicative inverses.</td>
<td>• Develop an understanding of the properties of the real and complex number systems and of the properties of special numbers including (\pi), i.e., and conjugates.</td>
</tr>
<tr>
<td>• Apply their understanding of number systems to model and solve problems.</td>
<td>• Apply their understanding of number systems, including integers and rational numbers to model and solve mathematical and applied problems.</td>
<td>• Apply their understanding of number systems, including irrational, real, and complex numbers to model, and solve mathematical and applied problems.</td>
</tr>
</tbody>
</table>

---

**Number Sense and Numeration**

**Content Standard 9**: Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers (Concepts and Properties of Numbers). [C1] [P7, P9, P10] [T3, T4, T5] [W1, W4, W7]

**Elementary**
- Develop an understanding of whole numbers, and read, write, and count using whole numbers; investigate basic concepts of fractions and decimals.
- Investigate and develop an understanding of numeration systems, in particular the base-10 place-value system.
- Apply their understanding of number systems to model and solve problems.

**Middle School**
- Develop an understanding of integers and rational numbers, and represent rational numbers in both fraction and decimal form.
- Extend their understanding of numeration systems to include decimal numeration and scientific numeration.
- Apply their understanding of number systems, including integers and rational numbers to model and solve mathematical and applied problems.

**High School**
- Develop an understanding of irrational, real, and complex numbers.
- Use the \((a+bi)\) and polar forms of complex numbers.
- Develop an understanding of the properties of the real and complex number systems and of the properties of special numbers including \(\pi\), i.e., and conjugates.
- Apply their understanding of number systems, including irrational, real, and complex numbers to model, and solve mathematical and applied problems.

---

**DRAFT**

**Mathematics Content Standards and Benchmarks**
Content Standard 10: Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations (Representation and Uses of Numbers). [C1] [T3, T4, T5] [W1, W4]

<table>
<thead>
<tr>
<th><strong>Content Standard 10</strong></th>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Represent whole numbers, fractions and decimals using concrete, pictorial, and symbolic representations.</td>
<td>Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.</td>
<td>Give decimal representations of rational and irrational numbers and coordinate and vector representations of complex numbers.</td>
</tr>
<tr>
<td></td>
<td>Explore and recognize different representations for the same number and explain why they are the same.</td>
<td>Recognize equivalent representations of a number, especially fractions, decimals, and percents, and translate freely among representations.</td>
<td>Develop an understanding of more complex representations of numbers, including exponential (e.g., (4^2)) and logarithmic (e.g., (\log 100)) expressions, and select an appropriate representation to facilitate problem solving.</td>
</tr>
<tr>
<td></td>
<td>Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).</td>
<td>Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring, and numbers that are used for naming.</td>
<td>Distinguish between rational approximations and the exact values of numbers such as (e), (\pi) and the irrationals.</td>
</tr>
<tr>
<td></td>
<td>Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.</td>
<td>Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.</td>
<td>Develop and use increasingly sophisticated estimation strategies.</td>
</tr>
<tr>
<td></td>
<td>Select appropriate numbers and representations in order to solve problems.</td>
<td>Select appropriate representations for numbers, including integers and rational numbers in order to simplify and solve problems.</td>
<td>Select appropriate representations for numbers, including representations of rational and irrational numbers and coordinate and vector representations of complex numbers in order to simplify and solve problems.</td>
</tr>
<tr>
<td></td>
<td>Compare and order numbers using &quot;equal,&quot; &quot;less than&quot; or &quot;greater than.&quot;</td>
<td>Compare and order integers and rational numbers using relations of equality and inequality.</td>
<td>Compare and order real numbers and compare rational approximations to exact values.</td>
</tr>
</tbody>
</table>

Content Standard 11: Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers (Number Relationships). [C1] [L8] [T3, T4, T5]

<table>
<thead>
<tr>
<th><strong>Content Standard 11</strong></th>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Compare and order numbers using &quot;equal,&quot; &quot;less than&quot; or &quot;greater than.&quot;</td>
<td>Compare and order integers and rational numbers using relations of equality and inequality.</td>
<td>Compare and order real numbers and compare rational approximations to exact values.</td>
</tr>
</tbody>
</table>
• Use part-whole relationships to explore numbers, develop number concepts, and understand computation.

• Express numerical comparisons as ratios and rates.

• Express numerical comparisons as ratios and rates.

• Classify numbers as even or odd and explore concepts of factors and multiples.

• Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.

• Express number relationships using positive and negative rational exponents, logarithms, and radicals.

• Apply their understanding of number relationships in solving problems.

• Apply their understanding of number relationships in solving problems.

• Apply their understanding of number relationships in solving problems.

**Numerical and Algebraic Operations and Analytical Thinking**

**Content Standard 12:** Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems (Operations and their Properties). [C1, C4, C8] [L8] [T3, T4, T5] [W1, W2, W5]

**Elementary**

• Use manipulatives to model operations with numbers; develop their own methods of recording operations and relate their models and recordings to standard symbolic expressions and algorithms.

**Middle School**

• Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.

**High School**

• Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators, and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.

• Present and explain geometric and symbolic models for operations with real and complex numbers and algebraic expressions.

• Describe the properties of operations with rational numbers and simple algebraic expressions and give examples of how they use those properties.

• Compute with real numbers, complex numbers, algebraic expressions, matrices and vectors using technology and, for simple instances, with paper-and-pencil algorithms.

• Describe the properties of operations with numbers, algebraic expressions, vectors, and matrices, and make generalizations about the properties of given mathematical systems.

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**Draft**

**Mathematics Content Standards and Benchmarks**
- Apply operations efficiently and accurately in solving problems.
- Efficiently and accurately apply operations with integers, rational numbers, and simple algebraic expressions in solving problems.
- Efficiently and accurately apply operations with real numbers, complex numbers, algebraic expressions, matrices, and vectors in solving problems.

Content Standard 13: Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems (Algebraic and Analytic Thinking). [C4, C8] [H2] [L3, L8] [T3, T4, T5]

**Elementary**
- Write and solve open sentences (e.g. \(0 + \square = 5\)) and write stories to fit the open sentence.
- Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.
- Use analytic thinking and algebraic models and representations to describe situations and solve problems.
- Find replacements for open sentences.
- Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables, and graphs; and write algebraic expressions to correspond to the multiple representations.

**Middle School**
- Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.
- Represent algebraic concepts with geometric models (e.g., balance beam), tables, and graphs; and write algebraic expressions to correspond to the multiple representations.
- Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.
- Explore problems modeled by linear functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.

**High School**
- Identify important variables in a context, symbolize them, and express their relationships algebraically.
- Represent algebraic concepts and relationships with matrices, spreadsheets, diagrams, graphs, tables, physical models, vectors, equations and inequalities; and translate among the various representations.
- Solve linear relations with no technical assistance and non-linear relations using graphing, symbol-manipulating or spreadsheet technology; and solve linear and non-linear systems using appropriate methods.
- Analyze problems that can be modeled by functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.
- Explore problems that reflect the contemporary uses of mathematics in significant contexts drawn from many fields of work, study and recreation; use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.
### Probability and Discrete Mathematics

**Content Standard 14:** Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations (Probability). [C2, C4, C8] [L8] [T3, T4, T5] [W5, W9]

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Explain the difference between chance and certainty and give examples to illustrate their understanding.</td>
<td>• Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.</td>
<td>• Develop an understanding of randomness and chance variation and describe chance and certainty in the language of probability.</td>
</tr>
<tr>
<td>• Compare events and describe them as &quot;more likely&quot; or &quot;less likely&quot; and use the language of fractions to describe simple probabilities.</td>
<td>• Describe probability as a measure of certainty ranging from 0 to 1, and conduct activities that allow them to express probabilities of simple events in mathematical terms.</td>
<td>• Give a mathematical definition of probability and determine the probabilities of more complex events, and generate and interpret probability distributions.</td>
</tr>
<tr>
<td>• Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.</td>
<td>• Conduct experiments and give examples to illustrate the difference between dependent and independent events.</td>
<td>• Analyze events to determine their dependence or independence and calculate probabilities of compound events.</td>
</tr>
<tr>
<td>• Conduct probability experiments and simulations to model and solve problems.</td>
<td>• Conduct probability experiments and simulations to model and solve problems.</td>
<td>• Conduct probability experiments and simulations, to model and solve problems, including problems involving compound events.</td>
</tr>
</tbody>
</table>

**Content Standard 15:** Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design (Discrete Mathematics). [C2, C3, C4, C6, C8, C9] [L8] [T3, T4, T5] [W1]

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Middle School</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Use manipulatives and diagrams to explore problems involving counting and arranging objects.</td>
<td>• Use manipulatives, diagrams, and the fundamental theorem of counting to count permutations and combinations.</td>
<td>• Derive and use formulas for calculating permutations and combinations.</td>
</tr>
</tbody>
</table>
• Explore sets and set relationships by sorting and classifying objects.

• Use sets and set relationships to explore and solve simple algebraic and geometric problems.

• Use sets and set relationships to represent algebraic and geometric concepts.

• Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.

• Solve problems involving networks, for example planning delivery routes or counting paths between points.

• Solve network problems including circuits, critical paths, minimum spanning trees, and adjacency matrices.

• Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.

• Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.

• Design efficient algorithms to accomplish a task or solve a mathematical problem such as to construct a geometric figure, to develop a flow diagram for a calculator or computer program, to sum a finite series, or to determine a critical path through a network.

• Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).

• Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).

• Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).
YOU ARE TRAVELLING IN EUROPE. THE CURRENCY EXCHANGE RATE FOR TODAY IS GIVEN AT THE RIGHT. YOU HAVE $1000 AMERICAN MONEY. THE FOLLOWING IS YOUR ITINERARY. CHANGE THE CURRENCY AS INDICATED AND GIVE YOUR REMAINING MONEY AS ASKED.

1. Start with $1000 US money.
2. Convert to Spanish pesetas.
5. Convert to French franc.
6. Spend 1,000 francs in France.
7. Convert remainder to US money.
8. How much do you have left?

If you are finished, figure out how to convert Spanish pesetas to French francs. What do you have to multiply by one peseta to change it to francs?

Compared to US currency, which country has the cheapest money?

Which countries have money that is worth more than US currency?
NAME ______________________

RATIO ACTIVITY WITH A CONVEX LENS

Students should work in pairs or groups of three.

Equipment: convex lens, 5x8 plain white card, metric ruler, meterstick, sunshine

Purpose: Students will use ratio to determine the magnitude of the lens.

DIRECTIONS

1. Determine the height of an object (tree, post, building) using indirect measurement.* OR Measure the height of an object that is at least one meter high.

2. Take lens and card and hold them in this order:

   ![Diagram of light, object, lens, card]

3. Move the lens and card until you can see the image of the object on the card. It will be upside down and in color. This is how a camera lens works!

4. Have a partner measure the height of the image.
   - Height of image ______
   - Height of object ______
   - Ratio of image to object (magnification) ______

5. Repeat with other size lenses if available.

* 1. Find the length of the shadow of the meterstick and of the object.
   2. Use this proportion to find the height of the object:
      \[
      \frac{\text{height of object}}{100 \text{ cm}} = \frac{\text{length of shadow of object}}{\text{length of shadow of meterstick}}
      \]
NAME ____________________

USING RATIOS AND PROPORTIONS WITH SHADOWS

Students should work in pairs or groups of three.

Equipment: meterstick, sunshine, and various outdoor objects

Purpose: Students will use indirect measurement to calculate heights of objects.

FOR THIS ACTIVITY USE ONE OF THE FOLLOWING PROPORTIONS:

\[
\frac{\text{length of object } #1}{\text{length of object } #2} = \frac{\text{length of object } #1\text{'s shadow}}{\text{length of object } #2\text{'s shadow}}
\]

OR

\[
\frac{\text{length of object } #1\text{'s shadow}}{\text{length of object } #2\text{'s shadow}} = \frac{\text{length of object } #1}{\text{length of object } #2}
\]

DIRECTIONS

1. This activity must be done on level ground on a sunny day. Stand a meterstick straight up (vertical). This will be considered object #1. Measure the length of its shadow in cm. Record in the chart below.

2. Measure the shadows of the objects listed in the chart below.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>LENGTH OF OBJECT</th>
<th>HEIGHT OF OBJECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. meterstick</td>
<td>100 cm</td>
<td></td>
</tr>
<tr>
<td>2. tree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. building</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. light pole</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. partner #1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. partner #2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. yourself</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Use proportions to calculate the height of #2 - #7. Record above. Show work on the back of this sheet.

4. Check your own height in centimeters to see if calculations are correct.
Activity 1  ►  Graphing Lines of the Form $y = mx$

Name _____________________________________________________

In this lesson we see how the value of $m$ affects the graph of a straight line.

1. Use a graphing calculator to graph each function. Then complete the following chart.

<table>
<thead>
<tr>
<th>Function</th>
<th>Value of $m$</th>
<th>Sketch</th>
<th>$x$-intercept</th>
<th>$y$-intercept</th>
<th>Is this graph the same, steeper, or flatter than the graph of $y = x$ (or $y = -x$)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 2x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = -2x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 0.5x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = -0.2x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 0.06x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Use the results to answer the following items.

a. What point does every graph in Exercise 1 have in common?

b. Does the value of $m$ affect the location of the $x$- or $y$-intercept?

c. If the graph lies in quadrants I and III, then $m$ is

d. If the graph lies in quadrants II and IV, then $m$ is

e. If $m$ is positive, predict what will happen to the graph as $m$ gets larger.

f. If $m$ is negative, predict what will happen to the graph as $|m|$ increases.

g. Make a conjecture about the graph when $m = 0$. Check your prediction by graphing the function on your calculator.

h. Summarize the role of $m$ in the graph of $y = mx$.

3. For each of the following, write an equation of a line that fits the characteristics. Verify your answers with the calculator.

a. A straight line in quadrants I and III steeper than $y = 2x$. Equation: _______________

b. A straight line in quadrants II and IV flatter than $y = -0.3x$.
   Equation: _______________

c. A straight line with the graphs shown below.

   ![Graph 1](image1)
   ![Graph 2](image2)

   Equation: _______________
   Equation: _______________

d. A straight line that lies between the graphs of $y = 3.1x$ and $y = 0.5x$.
   Equation: _______________
Activity 2 ▶ Graphing Lines of the Form $y = mx + b$

*In this lesson we see how the constant $b$ affects the graph of a straight line.*

1. Use a graphing calculator to graph each equation and complete the following chart. The first entry has been completed for you.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value of $m$</th>
<th>Value of $b$</th>
<th>Sketch</th>
<th>$y$-intercept</th>
<th>$x$-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x + 3$</td>
<td>1</td>
<td>3</td>
<td></td>
<td>(0, 3)</td>
<td>(-3, 0)</td>
</tr>
<tr>
<td>$y = x - 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 2x + 6.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 2x - 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 2x + 4.7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 3x - 2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Use the results to answer the following questions.
   a. For positive values of \( b \), is the \( y \)-intercept above or below the \( x \)-axis? __________________
   b. For negative values of \( b \), is the \( y \)-intercept above or below the \( x \)-axis? __________________
   c. What is the \( y \)-intercept for the equation \( y = 3x + 1 \)? __________________
   d. What is the \( y \)-intercept for the equation \( y = mx + b \)? __________________

3. Answer the following items about the first three entries in Exercise 1.
   a. What is the same about all three graphs? __________________
   b. These lines never intersect so we say they are __________________
   c. What is the relationship between \( b \) and the \( x \)-intercept in these equations? __________________
   d. What are the \( x \)- and \( y \)-intercepts of \( y = x - 7 \)? __________________
   e. How does changing the value of \( b \) affect graphs of the form \( y = x + b \)? __________________

4. Describe and compare the graphs of \( y = 4x - 1 \) and \( y = 4x + 3 \). (Use the calculator if you have trouble visualizing the graphs.) ______________________________________________________

5. Write an equation whose graph is a horizontal line. __________________

6. Write an equation whose graph is a horizontal line through (0, 2.5). __________________

7. Write an equation whose graph is a line parallel to and between the graphs of \( y = 2x + 3 \) and \( y = 2x + 4.7 \). __________________

8. Write an equation whose graph is a line parallel to the graph of \( y = -3x + 1 \), but with \( y \)-intercept (0, -5). __________________
Computer Master 24

Graphing $y = ax^2 + bx + c$

The shape of the graph of $y = ax^2 + bx + c$ is called a parabola. In this activity you will use your computer grapher to learn about properties of parabolas and how the variables $a$, $b$, and $c$ affect the location and shape of a parabola.

1. Set the window to $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. Use the grapher to draw the graph of $y = x^2 - 2x - 3$.

   a. At what points does the parabola intersect the $x$-axis? ____________
   b. At what point does the parabola intersect the $y$-axis? ____________
   c. Does the parabola open up or down? ____________
   d. What are the coordinates of the vertex of the parabola? ____________
   e. Is the parabola symmetric to the $x$-axis, the $y$-axis, or some other line? Explain. ____________

2. Clear the axes and use the grapher to draw $y = x^2$. Describe the graph of $y = x^4$ in terms of its intercepts, vertex, axis of symmetry, and whether it opens up or down.

3. On the same set of axes as $y = x^2$, graph $y = x^2 + 4$ and $y = x^2 - 5$. Generalize about the effect of the variable $c$ on the graph of $y = x^2 + c$.

4. Clear the axes, then graph $y = x^2$ and $y = -x^2$ on the same set of axes. Compare and contrast the graphs.

5. What is the equation for each of the following? Use your grapher to verify your answer by reproducing each graph:

   a. ____________
   b. ____________

   (1, 2) (1, 2)

   (-4, 3) (-4, 3)

(Continued)
6. Clear the previous graphs and then graph \( y = x^2 \), \( y = 2x^2 \), and \( y = \frac{1}{3}x^2 \) on the same set of axes. Generalize about the effect of the variable \( a \) on the graph of \( y = ax \).

7. What is the equation for each of the following? Use your grapher to verify your answer by reproducing each graph:

   a. ![Graph A](image)

   b. ![Graph B](image)

8. Clear the axes and graph \( y = x^2 \), \( y = x^2 + 2x \), and \( y = x^2 - 2x \) on the same set of axes. Compare and contrast the three parabolas.

9. Without graphing, choose which of the following equations determines which parabola shown below: \( y = -x^2 - 2x + 6 \), \( y = -2x^2 + 4x + 5 \), \( y = x^2 + 6 \), \( y = 2x^2 - 3x \), \( y = -x^2 + x - 6 \), \( y = 2x^2 + 6x \)

   a. ![Parabola A](image)

   b. ![Parabola B](image)
EVALUATION QUESTIONS FOR ACTIVITY 1 AND 2

These evaluation questions could easily be added to an existing test. It would not be necessary to include all of them. One would be sufficient to assure the comprehension of the concept.

1. Compare and contrast the graphs of these equations: $y = 2x$, $y = -2x$, and $y = \frac{1}{2} x$. Consider shape, slope, and y-intercept.

2. Compare and contrast the graphs of $y = 5x + 6$, $-\frac{2}{3} x + 6$, and $x + 6$. Consider shape, slope, and y-intercept.

3. Give the equation of a line whose graph
   a. lies between $y = 3x$ and the x-axis.
   b. lies between $y = -6x + 6$ and the y-axis.

EVALUATION QUESTIONS FOR COMPUTER MASTER 24

1. Compare and contrast the graphs of $y = 2x^2$, $y = 3x^2$, and $y = -3x^2$. Consider shape (including amount of opening), intercepts, vertex, direction, and axis of symmetry.

2. Compare and contrast the graphs of $y = x^2 + 7$, $x^2 + 4$, and $2x^2 + 4$. Consider shape (including amount of opening), intercepts, vertex, direction, and axis of symmetry.

3. Compare and contrast the graphs of $y = x^2$, $y = x^2 + 2x$, and $y = x^2 - 2x$. Consider shape (including amount of opening), intercepts, vertex, direction, and axis of symmetry.
DIRECTIONS FOR DICE PROBABILITY EXPERIMENT

1. Check the textbook on p. 38. Using the picture of the possible sums when throwing two dice, find \( P(\text{throwing a sum of 2}) \), \( P(\text{throwing a sum of 3}) \), \( P(4) \), etc. Change fraction to a percent and record in column #1 on your worksheet.

2. Make a coordinate graph. Put "Sums" across the bottom and "Probability" on the left side. Number every other LINE on the bottom (What numbers should you use?) Number the left side by ones. Put the information from Column #1 on the graph using a line graph. Connect the dots. This is the theoretical graph. Give the graph an appropriate title and begin a key.

3. Get two dice from Mrs. Farrer. Throw the dice EXACTLY 25 times and record the tally in Column #2. Count carefully!

4. Find the percent for each possible sum and record in Column #7. Graph your results, using a different color. How does this graph compare to the theoretical graph? Answer in complete sentences on a separate sheet. Call it question 1.

5. Check with three other students in the room. Record their tallies in Columns 3, 4, and 5.

6. Add your tallies from Columns 2, 3, 4, and 5. Put the totals in Column #6. The sum of all the numbers should be 100. Double check. Find the probability for each sum and change to a percent. Record in Column #8. Graph this information in a third color. How does this graph compare to the other two? Make this question 2 on the sheet.

7. If time allows Mrs. Farrer will help you tally the probability for the class. Record this in column #9 and graph in a fourth color. How does this graph compare to the other three graphs? This is question 3.

8. Which graph is closest to the theoretical graph? Explain your results. Guess what question this is?

9. Reread the directions to make sure you did everything. Put your name on your graph, your tables, and your question sheet. Staple them together with the graph on top. Hand in.
Quantitative Study: Teaching Aid 7 (for use beginning with Chapter 2 Refresher)
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- Maintenance Frequency (in %)
- Data Sheet
- Name
INDEPENDENT STUDENT PROJECT

NOTE: Kentwood Public Schools requires its algebra students to "make predictions and decisions based on data the learner has gathered, organized, and interpreted, including interpolations and extrapolations." There are several sections of the textbook that might lend themselves to this activity. However, two of them seem most appropriate. Therefore, some ideas are presented here for those sections.

CHAPTER 1 - Basic Concepts

This chapter contains sections on probability and relative frequency. The Dice Probability activity presented in Appendix F could be followed by one of these probability topics and could be reported in a manner similar to the activity.

1. Differences or products of numbers on the dice.

2. Frequency of finding various colors of M & Ms in a package.

3. Frequency of drawing each number (Ace - King) in a deck of cards.

4. Frequency of the digits 0 - 9 at the end of a residential phone number.

5. A study of children in families to determine the frequency of boys and girls.

6. An experiment with a dart board to determine the frequency of the dart landing in each section, assuming it lands somewhere on the board and the dart is randomly thrown.

7. An experiment similar to #6 but using Pin the Tail on the Donkey.

8. Experiments with spinners in games to determine frequency of each possibility compared to relative area.

Many other ideas are possible as long as the student can give a theoretical probability for each event to compare with the experimental frequency.
This chapter contains sections on the Means-Extremes Property and similar figures. Many of the problems given in the textbook could be expanded into a project. Some suggestions follow.

1. Measure the length of feet compared to height.

2. Measure circumference of trees compared to height (determine by indirect measurement).

3. Measure arm span compared to height.

4. Compare number of pennies to their weight.

5. Compare height of shadows to height of objects.

6. Compare temperatures of cities found in the newspaper to latitudes found in an atlas.

7. Report on homework time and television time of several classmates.

8. If classmates are willing report study time (or homework completed) compared to grade on a test - no names, of course.

Other ideas are possible. Successful completion of the project depends on how the information was gathered, organized, and interpreted and on the conclusions drawn.
To comply with the Level Standard the student must gather, organize, and interpret information and then draw some conclusions. Evaluation must include all of the requirements. Each teacher can allot points as appropriate, but these ideas should be evaluated:

1. Has the student defined in writing what is to be done and the expected result? ___

2. Has the student gathered sufficient information to accomplish the stated purpose? ___

3. Was that information recorded in an organized manner? ___

4. Was that information then reported in some mathematically appropriate form (table, graph, equation)? ___

5. Was the table, graph, or equation explained accurately and completely, using correct terminology? ___

6. Were conclusions drawn from the information? These might include trends, comparisons and contrasts, variations from expectations, or other applications as appropriate. ___

7. Was the presentation neat and did it show careful planning? ___
To Whom It May Concern,

I am a mathematics teacher in Kentwood, Michigan. In 1990, Kentwood Public Schools, a district of about 4500 students, adopted the UCSMP series for grades 7 - 12.

Currently I am writing my master's thesis, matching the objectives of the UCSMP Algebra textbook with the outcomes of the Kentwood Mathematics curriculum. To accomplish this effectively requires reprinting objectives, lessons and/or examples from the textbook and teacher support materials. Therefore, I request permission to reprint those items needed to support the points of my thesis.

I have prepared the form at the bottom of the page. Please note that the thesis will be placed on microfilm by UMI Dissertation Services and that they make single copies available upon request. Thank you for your time and consideration.

Sincerely,

Carol Farrer

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- Elem Ed
- Elem LD
- Read/Lang Arts
- Ed Leadership
- G/T Ed
- Sec LD
- Sec/Adult
- Early Child
- SpEd PPI

TITLE: A Comparison of Kentwood Public School's Standards and the UCSMP Algebra Objectives

PAPER TYPE: (Choose only 1)

- Project
- Thesis

SUPERVISOR'S SIGNATURE OF APPROVAL

Using the ERIC thesaurus, choose as many descriptors (5 - 7 minimum) to describe the contents of your paper.

1. UCSMP
2. NCTM and Standards
3. A Nation at Risk
4. Middle School and Mathematics
5. Middle School and Standards
6. Textbooks and Mathematics
7. 
8. 
9. 
10. 

ABSTRACT: Two to three sentences that describe the contents of your paper.

This paper compares Kentwood Public School's Algebra Standards (Outcomes) with the University of Chicago School Mathematics Project Algebra objectives. The research examines national and state influences on the district as well as on the textbook authors. Where the book fails to address a Standard, or addresses it incompletely, supplementary materials are presented in the paper itself or in the appendix.

**Note: This page must be included as the last page in your master's paper.**

rev 5/94
CHAPTER ONE
(Objective 2 An understanding of current issues in their field.)

Demonstrates in-depth understanding of at least one issue in their field.

1. Describe why this problem/issue is worthy of study.
   1

2. Identify resources that strengthen the case.
   2-5

   1-4

4. Describe options for addressing the problem/issue.
   1-2, 4

5. Articulate why the option selected was chosen.
   5

CHAPTER TWO
(Objective 1 An understanding of the recent research in their field.)
(Objective 4 Demonstrate the ability to critically analyze and synthesize existing and emerging knowledge and pedagogy in their major area.)

6. Includes recent and seminal sources in review of literature.
   10-24

7. Includes statement of generalization that are supported by the cited research.
   10-24

8. Finds, integrates and evaluates related work (compare/contrast, categorize).
   19-24

9. Provides a summary of their literature.
   24-25

CHAPTER THREE
(Objective 3 Demonstrate the effective use of research methods appropriate to their field of concentration.)
(Objective 5 The ability to communicate concepts germane to their major area effectively to others through various kinds of literacy e.g. numeracy, graphics, printed text, computers and electronic data.)
(Objective 6 Demonstrate the ability to adapt their work to the needs of their clients.)

10. Gives insight into the methods of other researchers.
    20-22

11. Systematic and comprehensive description of the development/design of the project.
    26-27

12. Communicates findings in the context of past work.
    28, 30, 32, 35, 36, 40

13. Written in a style that addresses the needs of the clients (students, teachers, administrators, parents, school board, etc.)
    26-52, Appendix F

14. Provides recommendations for further research and/or dissemination.
    48

Faculty Signature
Date Reviewed 8/8/94

Approved SOE Graduate Committee 2/14/94