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THE IMPACT OF TECHNOLOGY ON MATHEMATICS EDUCATION

Paul Fishback and Steven Schlicker

Most, if not all, of us are familiar with the image of the traditional mathematics classroom. Students sit while a teacher lectures about techniques needed to solve various types of problems. Homework then consists of a large number of repetitions of the techniques presented in class. Do students who are confronted with this type of mathematics instruction really learn mathematics as it is used in practice, e.g., as in industry, or do they learn something that is altogether different?

Several studies have shown that while there is a great deal of interest in mathematics on the part of students at the elementary level, this interest level declines drastically as these students reach the middle school and secondary levels. In fact, each year from high school through graduate school, of the remaining students studying mathematics, fifty percent stop doing so. This lack of interest has a dramatic impact on the science and mathematics programs in our colleges and universities and, ultimately, in the work force.

What are the causes of this decline in interest in mathematics between elementary school and high school? One of the major causes is likely to be the different methods of mathematics instruction between grade school and high school. In grade school, students are much more active in learning mathematics and the mathematics with which they are confronted is much more intuitive than is the mathematics they see later. High school and college mathematics instruction has traditionally been much more algorithmic, teaching techniques rather than ideas. The difference in approaches is likely due to content. High school mathematics centers on algebra, a more abstract topic than arithmetic. Traditional college mathematics instruction seems to have just gone with the flow, teaching mainly manipulation techniques with the lecture method because students are familiar with it from high school and because it is fairly easy and cost effective.

The effects on students of this traditional approach to secondary and post secondary mathematics instruction have been discouraging. Failure rates in beginning college mathematics courses have hovered around 50% in most major universities. Even more disastrous is the impression the general public has of such mathematics instruction. Many people wear their antipathy towards mathematics like a badge of honor. All too often the mere mention of mathematics in a social setting brings forth comments such as “Mathematics, that was always my worst subject.” or “Mathematics? I can’t even balance my checkbook.” The mathematics community is in large part responsible for this situation and, fortunately, we are beginning to attempt to address some of these problems. One of the main tools being used in this process is technology.
How is technology affecting mathematics education? In the last several years technology—primarily graphing calculators and personal computers—has raised questions and debate about what kind of mathematics is appropriate and necessary in the classroom. The technology has also had a tremendous impact on the way mathematics is being taught and learned. Let us illustrate with a couple of examples.

Here is a problem, chosen from a section of 60 similar problems in a high school level algebra textbook:

Find all real solutions of the equation \( \frac{1}{x} - \frac{1}{x-1} - \frac{2}{x} = 0 \).

Obviously, students are supposed to use algebraic methods to solve this equation. There are several possible approaches one could take, all of which lend themselves to simple arithmetic errors which lead to incorrect solutions.

A reasonable person might ask about the purpose of this problem. There are two possible answers. One is that such a problem increases the student’s conceptual understanding of algebraic equations. For example, the solutions to the above equation can be interpreted as the points where the graph of \( \frac{1}{x} - \frac{2}{x} \) crosses the x-axis. However, no research in mathematics education supports the view that the ability to solve algebraic equations by hand is a necessary prerequisite for the student to have a solid conceptual understanding of solutions of equations.

A second answer is that such problems are necessary drills for students to increase their manipulative skills. This is where the debate begins. Available today are several different computer programs, called computer algebra systems (CAS), which are capable of solving this problem. In fact, in one such CAS called MAPLE the simple one line command

\[ \text{solve} \left( \frac{1}{x} - \frac{1}{x-1} - \frac{2}{x} = 0, x \right) \]

gives the output -1, 2 as the solutions to the equation. If computers can so easily solve these kinds of algebra problems, should we be spending weeks if not months in drilling our students to solve them by hand?

Again one might argue that it is necessary for students to develop a facility for by hand manipulation so that they understand the techniques used to solve these problems. This is a reasonable argument. But how much drill is necessary to understand the techniques? We can all probably multiply 123 by 2,341 if we need to, but how many of us wouldn’t reach for a calculator to get the result? Technology has forced mathematics teachers to confront the issue of what manipulative skills in mathematics must be mastered by the student.

Technology has also opened up new avenues by which students can approach mathematics problems. Technology can create opportunities for students to devote more time to mastering concepts or ideas as opposed to simply building mechanical skills. For example, an expression can be investigated the resulting expression axes, whereas points, of its graph patterns. The computation that the manipulation frequently provides, could easily become lost sight, however, to suppose a necessary process.
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A few simple MAPLE commands allow one to simplify the same
expression above to the single fraction \( \frac{x^2-x-2}{(x^2-x-1)x} \) and to obtain the graph of
this resulting expression:

The numerator of \( \frac{x^2-x-2}{(x^2-x-1)x} \), namely \( x^2-x-2 \), is equal to zero at \( x = 2, -1 \)
and the denominator, \( (x^2-x-1)x \), is equal to zero at \( x = 0, \frac{1 \pm \sqrt{5}}{2} \). By
investigating the relation of these numbers to the graph and by considering other
examples if necessary, students are led to discover that the roots of the numerator of
an expression such as \( \frac{x^2-x-2}{(x^2-x-1)x} \) determine where its graph crosses the x-
axis, whereas the roots of the denominator determine the asymptotes, or breaking
points, of its graph.

Mathematics can be considered, in part, as the search for patterns like these, and
mathematics instruction should emphasize this type of discovery learning. The
discovery of patterns like these leads naturally into discussions of the reasons for the
patterns. The chief difficulty in the past of delivering this type of instruction has been
that the manipulations encountered on the way to making such discoveries have
frequently provided a roadblock for student understanding. Quite simply, students
easily become so lost in the morass (to them) of algebraic technicalities that they
often lose sight of the ultimate goal or process. There is a great deal of evidence,
however, to support the view that the ability to perform algebraic manipulations is not
a necessary prerequisite to understand the concepts or ideas inherent in a problem.
By removing the roadblock of rote manipulation that can obscure the ideas present in a problem, technology can be a tool through which students' conceptual understanding can be increased.

The use of technology provides additional advantages in mathematics instruction. Technology allows students to address problems that are realistic in nature. For example, the cost (in dollars) of driving a certain truck 100 miles at a speed of \( x \) miles per hour, assuming that driver wages are $15 per hour and that fuel costs $1.20 per gallon is represented by the expression

\[
100 \frac{15}{x} + 100 \frac{1.20}{8.3356 - 0.06414x}
\]

This expression can be derived from real world data by the students, and leads to a variety of interesting questions. What driving speed minimizes costs? Does this optimal driving speed change if wage costs or fuel costs change? All of these questions can be easily addressed by the student with the aid of technology. Such technology allows the student to test conjectures and search for patterns. These conjectures and patterns can then be used with the above expression and a strong conceptual understanding to answer all of these questions.

We have seen that technology provides both instructors and students with tools that allow a greater emphasis on real mathematics—the search for patterns and making conjectures and attempting to verify them—than was realistically possible before the technology was available. Curricula have been developed at several levels to take advantage of the widespread availability of technology: e.g., the Chicago series middle school and high school mathematics materials and the Harvard Calculus Consortium calculus texts. However, many questions that the use of technology in mathematics instruction raises have yet to be answered:

- Although a CAS can perform the algebraic manipulations that appear in mathematics problems, is it wise to eliminate drill and practice in such manipulations from the curriculum? Most scientists and mathematicians would agree that a proper balance between conceptual understanding and algebraic skill is desirable. However, what is the proper balance and how much drill is necessary to achieve that balance?
- When technology is involved, students often see it as a black box: whatever comes out must be correct. How do teachers use the technology to help students see technology as a tool, not as a replacement for critical thinking?
- Technology has developed to the stage where so much mathematics can be done with a machine that it can be overwhelming. At a given level of mathematical sophistication, what technology is appropriate? It would probably not, for example, be a good idea to use a CAS for third grade mathematics.
- Even with the appropriate technology, it is easy to be overwhelmed with the technology itself. How can technology be used most effectively to help students understand mathematics and not just the use of the technology?

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1 Everybody Counts, National Academy, March 1991.
There are many other questions to be addressed with the use of technology, but, overall, it appears that the impact technology has had on mathematics has been significant and positive. Mathematics instructors have been forced to face their own potential obsolescence. The technology has made us reevaluate and redesign our curricula and its place in the schools and society. It is a challenge that many mathematics teachers are facing. The ever increasing power of technology has taken the traditional, static mathematics which most of us grew up and transformed it into a dynamic, ever-changing field. New and exciting branches of mathematics have grown as a result of technology: for example, Chaos and Fractal Geometry, which have changed the way we look at the world, at least from a mathematical viewpoint. All in all, the advent of technology has made this a wonderful time to be a mathematics teacher.

References
